$U(V,S)$

$$dU = dq_{rev} + dw_{rev}$$

$$dU = dq_{rev} - pdV$$

1.HS: $dU = \delta q + \delta w$, reversible Bedingungen

wenn nur Volumenarbeit: $dw_{rev} = -pdV$

2.HS: $dq_{rev} = TdS$

$$\boxed{dU = TdS - pdV}$$

Fundamentalgleichung (Mastergleichung)

dU ist totales Differential:

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$$

Ableiten nach V bei T = const

formal – Mastergleichung durch dV dividieren

$$\boxed{\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p}$$

Thermodynamische Zustandsgleichung

$$\Rightarrow T = \left(\frac{\partial U}{\partial S}\right)_V \qquad p = -\left(\frac{\partial U}{\partial V}\right)_S$$

gemischte 2. Ableitung, Schwarzscher Satz

$$\left(\frac{\partial^2 U}{\partial V\,\partial S}\right) = \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V}$$

Maxwell-Beziehung

$H(S,p)$

$$H = U + pV$$

$$dH = dU + pdV + Vdp$$

$$dH = dq_{rev} + \cancel{dw_{rev}} + \cancel{pdV} + Vdp$$

$$dH = dq_{rev} + Vdp$$

1.HS: $dU = \delta q + \delta w$, reversible Bedingungen

nur Volumenarbeit: $dw_{rev} = -pdV$

2.HS: $dq_{rev} = TdS$

$$\boxed{dH = TdS + Vdp}$$

Fundamentalgleichung

$$dH = \left(\frac{\partial H}{\partial S}\right)_p dS + \left(\frac{\partial H}{\partial p}\right)_S dp$$

Ableiten nach p bei T = const

$$\boxed{\left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial S}{\partial P}\right)_T + V}$$

Thermodyn. Zustandsgleichung

$$\Rightarrow T = \left(\frac{\partial H}{\partial S}\right)_p \qquad V = \left(\frac{\partial H}{\partial p}\right)_S$$

$$\left(\frac{\partial^2 H}{\partial p \partial S}\right) = \boxed{\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p}$$

Maxwell-Beziehung

$A(V,T)$

$$A := U - TS$$

$$dA = dU - TdS - SdT$$

$$dA = dq_{rev} + dw_{rev} - TdS - SdT$$

$$dA = \cancel{dq_{rev}} - pdV \cancel{- TdS} - SdT$$

1.HS: $dU = \delta q + \delta w$, reversible Bedingungen

nur Volumenarbeit: $dw_{rev} = -pdV$

2.HS: $dq_{rev} = TdS$

$$\boxed{dA = -pdV - SdT}$$

Fundamentalgleichung

$$dA = \left(\frac{\partial A}{\partial V}\right)_T dV + \left(\frac{\partial A}{\partial T}\right)_V dT$$

Ableiten nach V bei S = const

$$\boxed{\left(\frac{\partial A}{\partial V}\right)_S = -p - S\left(\frac{\partial T}{\partial V}\right)_S}$$

Thermodyn. Zustandsgleichung

$$\Rightarrow p = -\left(\frac{\partial A}{\partial V}\right)_T \qquad S = -\left(\frac{\partial A}{\partial T}\right)_V$$

$$\left(\frac{\partial^2 A}{\partial V \partial T}\right) = \boxed{\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$$

Maxwell-Beziehung

$G(p,T)$

$$G := H - TS$$

$$dG = dH - TdS - SdT = dU + pdV + Vdp - TdS - SdT$$

$$dG = dq_{rev} + \cancel{dw_{rev}} + \cancel{pdV} + Vdp - TdS - SdT$$

$$dG = \cancel{dq_{rev}} + Vdp \cancel{- TdS} - SdT$$

nur Volumenarbeit: $dw_{rev} = -pdV$

2.HS: $dq_{rev} = TdS$

$$\boxed{dG = Vdp - SdT}$$

Fundamentalgleichung

$$dG = \left(\frac{\partial G}{\partial p}\right)_T dp + \left(\frac{\partial G}{\partial T}\right)_p dT$$

Ableiten nach p bei S = const

$$\boxed{\left(\frac{\partial G}{\partial p}\right)_S = V - S\left(\frac{\partial T}{\partial p}\right)_S}$$

Thermodyn. Zustandsgleichung

$$\Rightarrow V = \left(\frac{\partial G}{\partial p}\right)_T \qquad S = -\left(\frac{\partial G}{\partial T}\right)_p$$

$$\left(\frac{\partial^2 G}{\partial p \partial T}\right) = \boxed{\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T}$$

Maxwell-Beziehung

Fundamentalgleichungen (Mastergleichungen)

$U(S,V) \rightarrow dU = TdS - pdV$

$H(S,p) \rightarrow dH = TdS + Vdp$

$A(T,V) \rightarrow dA = -SdT - pdV$

$G(T,p) \rightarrow dG = -SdT + Vdp$

Maxwell-Beziehungen

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial p}\right)_S = +\left(\frac{\partial V}{\partial S}\right)_p$$

$$\left(\frac{\partial S}{\partial V}\right)_T = +\left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

Thermodynamische Zustandsgleichungen

siehe Kap 2.5, "Verknüpfung von U mit leicht messbaren Größen"

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$\left(\frac{\partial H}{\partial p}\right)_T = T\left(\frac{\partial S}{\partial P}\right)_T + V = -T\left(\frac{\partial V}{\partial T}\right)_p + V$$

$$\left(\frac{\partial A}{\partial V}\right)_S = -p - S\left(\frac{\partial T}{\partial V}\right)_S = S\left(\frac{\partial p}{\partial S}\right)_V - p$$

$$\left(\frac{\partial G}{\partial p}\right)_S = V - S\left(\frac{\partial T}{\partial p}\right)_S = -S\left(\frac{\partial V}{\partial S}\right)_p + V$$