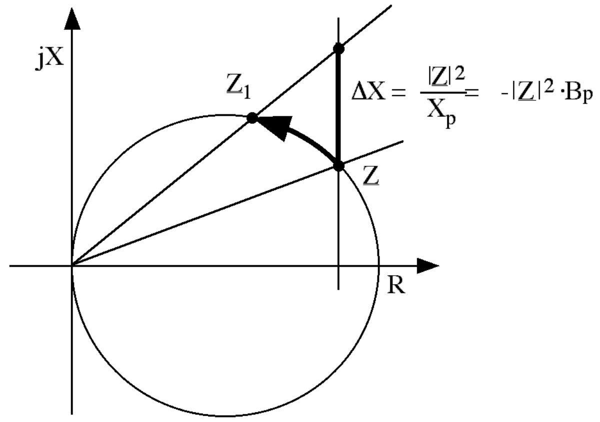
Formelblatt zur Vorlesung 'Grundlagen der Hochfrequenztechnik'

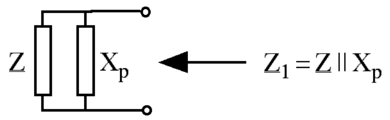
Impedanz $\xleftrightarrow{Z=1/Y}$ Admittanz

$$\underline{Z} = R + jX \qquad \underline{Y} = G + jB$$

$$\underline{Z} = \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2} \qquad \underline{Y} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

Hilfskonstruktion zur Transformation





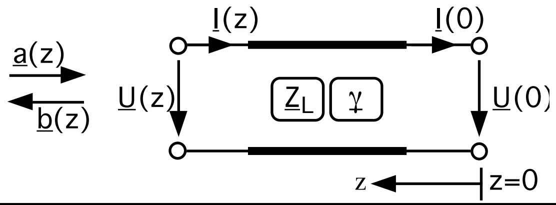
Kompensation mit dualen Elementen

Bedingungen für Kompensation: $X_s = R^2 \cdot B_p$

Frequenzfaktor: $F(f) = \sqrt{X_s \cdot B_p}$

krit. Frequenz, Grenzfrequenz: $|F(f_k)| = 1$

Leitungen



$$\underline{U}(z) = \underline{U}_H(0)e^{\underline{\gamma}z} + \underline{U}_R(0)e^{-\underline{\gamma}z} = \sqrt{\underline{Z}_L}\left(\underline{a}(z) + \underline{b}(z)\right)$$

$$\underline{I}(z) = \frac{\underline{U}_H(0)}{\underline{Z}_L}e^{\underline{\gamma}z} - \frac{\underline{U}_R(0)}{\underline{Z}_L}e^{-\underline{\gamma}z} = \frac{1}{\sqrt{\underline{Z}_L}}\left(\underline{a}(z) - \underline{b}(z)\right)$$

$$\underline{\gamma} = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}; \quad \underline{Z}_L = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Koaxialleitung

$$Z_L = \frac{1}{2\pi}\cdot\sqrt{\frac{\mu}{\varepsilon}}\cdot\ln\left(\frac{\rho_2}{\rho_1}\right)$$

ungedämpfte Leitung (homogenes Dielektrikum und konst. Querschnitt)

$$\beta = \omega\cdot\sqrt{L'C'} = \omega\cdot\sqrt{\mu\varepsilon}\ ; \quad \lambda = \frac{2\pi}{\beta}\ ; \quad C' = \frac{\sqrt{\mu\varepsilon}}{Z_L}\ ; \quad L' = Z_L\cdot\sqrt{\mu\varepsilon}\ ; \quad v_\varphi = \frac{\omega}{\beta}$$

schwach gedämpfte Leitungen ($R' \ll \omega L'$; $G' \ll \omega C'$)

$$\alpha \approx \frac{1}{2}\left(\frac{R'}{Z_L} + G'\cdot Z_L\right)\ ; \quad G' = \omega C'\cdot\tan(\delta_c)\ ; \quad R' \sim \frac{1}{\kappa\cdot s}$$

Dämpfung einer Leitung der Länge ℓ (für hinlaufende Welle a)

$$D/\mathrm{dB} = 10\cdot\log\left(\frac{P_a(\ell)}{P_a(0)}\right) = 10\cdot\log\left(e^{2\alpha\ell}\right)$$

Eindringtiefe s

$$s = \sqrt{\frac{2}{\omega\kappa\mu}}$$

Reflexionsfaktor $\underline{r}$

$$\underline{r}(z) = \frac{\underline{U}_R(z)}{\underline{U}_H(z)} = \frac{\underline{b}(z)}{\underline{a}(z)} = \frac{\underline{b}(0)}{\underline{a}(0)}\cdot e^{-2\underline{\gamma}z}$$

Reflexionsfaktor → Impedanz

$$\underline{r}(\ell) = \frac{\underline{Z}(\ell) - \underline{Z}_L}{\underline{Z}(\ell) + \underline{Z}_L}\ ; \quad \underline{Z}(\ell) = \frac{\underline{U}(\ell)}{\underline{I}(\ell)} = \frac{1 + \underline{r}(\ell)}{1 - \underline{r}(\ell)}\cdot\underline{Z}_L$$

Anpassungsfaktor, Stehwellenverhältnis

$$m = \frac{1}{VSWR} = \frac{1 - |\underline{r}|}{1 + |\underline{r}|} = \frac{U_{min}}{U_{max}}$$

Dem Verbraucher zugeführte Wirkleistung P_W

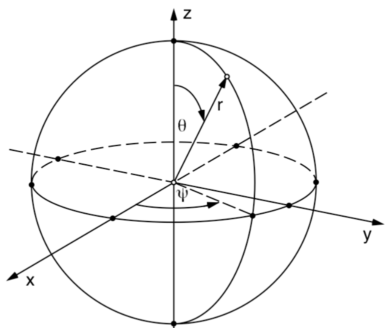
$$P_W = P_a(0) - P_b(0) = \frac{1}{2}\left(|\underline{a}(0)|^2 - |\underline{b}(0)|^2\right) = \frac{1}{2}|\underline{a}(0)|^2\cdot\left(1 - |\underline{r}(0)|^2\right)$$

mit: $\underline{a}(z) = \frac{\underline{U}_H(z)}{\sqrt{\underline{Z}_L}} = \sqrt{\underline{Z}_L}\cdot\underline{I}_H(z)$

Transformation durch Kettenschaltung einer Leitung

$$\underline{Z}(\ell) = \underline{Z}_L\cdot\frac{\underline{Z}(0) + \underline{Z}_L\tanh(\underline{\gamma}\ell)}{\underline{Z}_L + \underline{Z}(0)\tanh(\underline{\gamma}\ell)} = \underline{Z}(0)\cdot\left.\frac{1 + j\frac{\underline{Z}_L}{\underline{Z}(0)}\cdot\tan(\beta\ell)}{1 + j\frac{\underline{Z}(0)}{\underline{Z}_L}\cdot\tan(\beta\ell)}\right|_{\alpha=0}$$

Kugelkoordinaten



Azimuth: ψ Elevation: θ

Volumen: $V = \frac{4}{3}\pi r^3$ Oberfläche: $F = 4\pi r^2$

Konstanten

$$Z_{F0} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi\ \Omega$$

$$c_o = 2{,}997925\cdot 10^8\ \frac{m}{s}$$

$$k = 1{,}38065\cdot 10^{-23}\ \frac{Ws}{K}$$

$$\mu_0 = 4\pi\cdot 10^{-7}\ \frac{Vs}{Am}$$

$$\varepsilon_o = 8{,}854\cdot 10^{-12}\ \frac{As}{Vm}$$

Anmerkung: **Dieses Doppelblatt wird Ihnen zur Prüfung als einziges Hilfsmittel ausgeteilt. Versuchen Sie deshalb zumindest in der Endphase der Prüfungsvorbereitung, alle Aufgaben ohne weitere Hilfen zu lösen!**

Umrechnung verschiedener Zweitorparameter

	[S]	[Z]	[Y]	[A] (ABCD)	[T]
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{(Z_{11}+Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{(Y_{11}+Y_0)(Y_{22}+Y_0)-Y_{12}Y_{21}}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$	$\frac{T_{12}}{T_{22}}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{(Z_{11}+Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}$	$\frac{-2Y_{12}Y_0}{(Y_{11}+Y_0)(Y_{22}+Y_0)-Y_{12}Y_{21}}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$	$\frac{T_{11}T_{22}-T_{12}T_{21}}{T_{22}}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{(Z_{11}+Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}$	$\frac{-2Y_{21}Y_0}{(Y_{11}+Y_0)(Y_{22}+Y_0)-Y_{12}Y_{21}}$	$\frac{2}{A+B/Z_0+CZ_0+D}$	$\frac{1}{T_{22}}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{(Z_{11}+Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{(Y_{11}+Y_0)(Y_{22}+Y_0)-Y_{12}Y_{21}}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$	$\frac{-T_{21}}{T_{22}}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$\frac{A}{C}$	
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$\frac{AD-BC}{C}$	
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$\frac{1}{C}$	
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$\frac{D}{C}$	
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	Y_{11}	$\frac{D}{B}$	
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	Y_{12}	$\frac{BC-AD}{B}$	
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	Y_{21}	$\frac{-1}{B}$	
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	Y_{22}	$\frac{A}{B}$	
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A	
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B	
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}}$	C	
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D	
T_{11}	$\frac{S_{12}S_{21}-S_{11}S_{22}}{S_{21}}$				T_{11}
T_{12}	$\frac{S_{11}}{S_{21}}$				T_{12}
T_{21}	$\frac{-S_{22}}{S_{21}}$				T_{21}
T_{22}	$\frac{1}{S_{21}}$				T_{22}