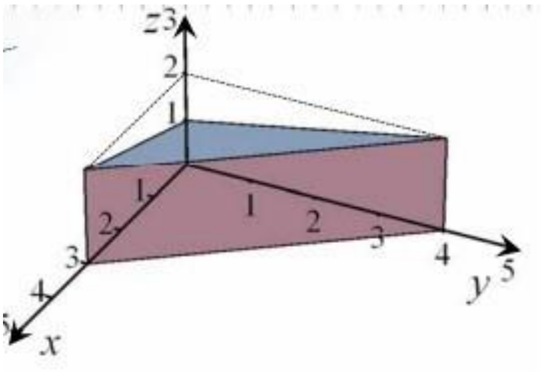
# Volumenintegrale

$$vol(A) = \iiint_A 1\, d(x,y,z) = \int_{x_u}^{x_o} \int_{y_u(x)}^{y_o(x)} \int_{z_u(x,y)}^{z_o(x,y)} 1\, dz\, dy\, dx \qquad f(x,y,z) = 1$$

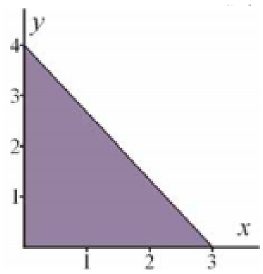
Beispiel:



$0 \leq x \leq 3$

$0 \leq y \leq 4 - \frac{4}{3}x$

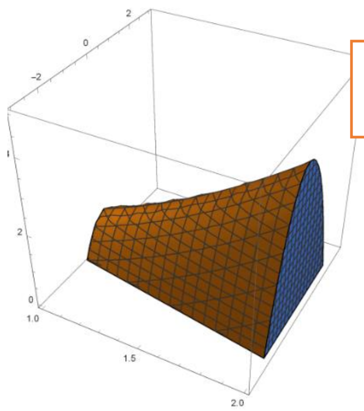
$0 \leq z \leq 1 + \frac{1}{3}x + \frac{1}{4}y$



$$vol(A) = \iiint_A 1\, d(x,y,z) = \int_{x_u}^{x_o} \int_{y_u(x)}^{y_o(x)} \int_{z_u(x,y)}^{z_o(x,y)} 1\, dz\, dy\, dx = \int_0^3 \int_0^{4-\frac{4}{3}x} \int_0^{1+\frac{1}{3}x+\frac{1}{4}y} 1\, dz\, dy\, dx$$

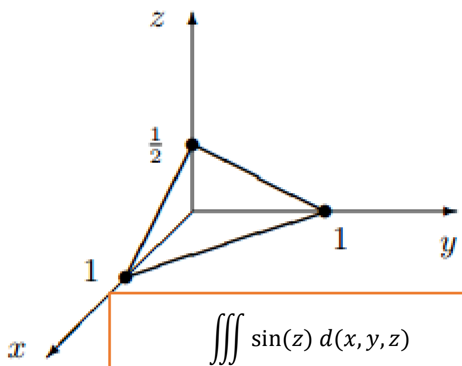
Tipps

4 a) $A = \{(x,y,z) \in \mathrm{R3} : 1 \leq x \leq 2 \;,\; 0 \leq z \leq x^2 - y^2\}$



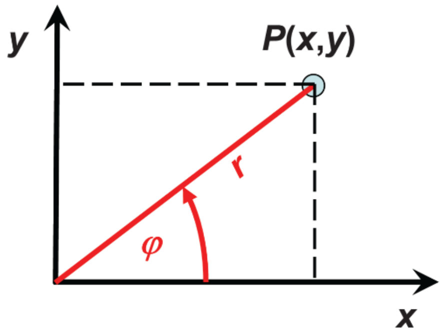
Im Zweiten Teil stecken Infos zu Grenzen von y und z

4 b) $B$: $x = 0, y = 0, z = 0, x + y + 2z = 1$




$$\iiint_B \sin(z)\, d(x,y,z)$$

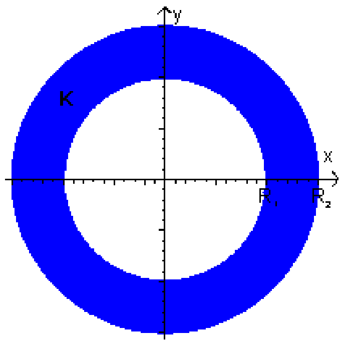
## Polarkoordinaten




*$\varphi$ immer in Radian!*

*$\varphi$ beginnt bei $x-$Achse, eine Umdrehung sind $2\pi$*

Beispiel:




Kartesische Koordinaten

$$K := \{ (x,y) \in \mathbb{R}^2 \mid R_1^2 \leq x^2 + y^2 \leq R_2^2 \}$$

Polar Koordinaten

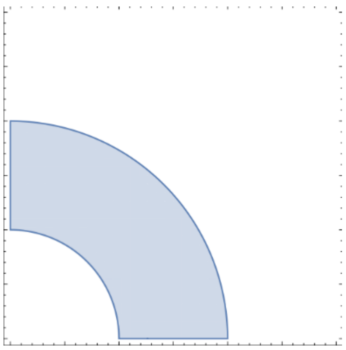
$$Q := \{ (r\cos(\varphi), r\sin(\varphi)) \mid R_1 \leq r \leq R_2, 0 \leq \varphi \leq 2\pi \}$$

Aufgabe 5

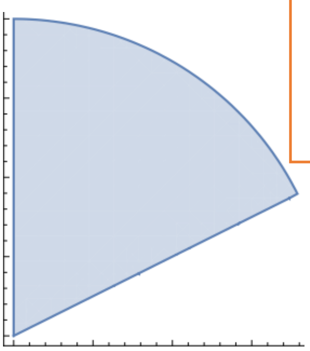
$$A = \{(x,y) \in \mathbb{R}^2 : r^2 \leq x^2 + y^2 \leq R^2, x \geq 0, y \geq 0\} \; (R \geq r \geq 0)$$

$$B = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq R^2, x \geq 0, y \geq ax\} \; (R \geq 0, a > 0)$$

A:



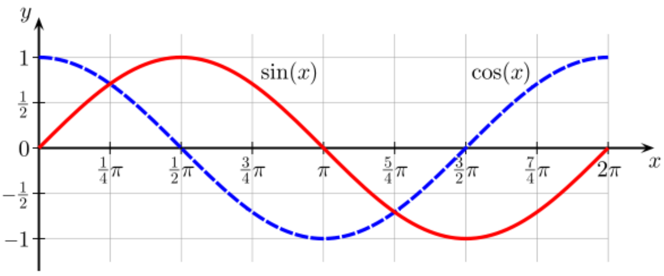
B:



$$\text{für } x = \rho\cos(\varphi)$$

$$\text{für } y = \rho\sin(\varphi)$$

Einsetzen und Ungleichungen lösen



# Transformationsformel

$$\int_M f(x,y,z)\,dzdydx \;=\; \int_K f\big(\Phi(u,v,w)\big)\;\;|det\,(\Phi'\,(u,v,w))|\,dwdvdu$$

$M = \Phi(K)$

$K := \{(u,v,w) \in \mathbb{R}^3 : u^2 + v^2 + w^2 \le 1\}$

$M := \left\{(x,y,z) \in \mathbb{R}^3 : \frac{1}{16}x^2 + \frac{1}{8}y^2 + z^2 \le 1\right\}$

$\rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \Phi \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4u \\ \sqrt{8}v \\ w \end{pmatrix}$

**Aufgabe 6**: Die seit Jahren bewährte Marzipankartoffel

$$M = \left\{(x,y,z) \in \mathbb{R}^3 : \tfrac{1}{16}x^2 + \tfrac{1}{8}y^2 + z^2 \le 1\right\}$$

muss sich dieses Jahr der neuen Marzipankartoffel

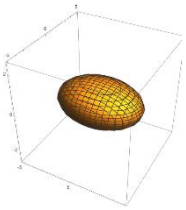
$$\widetilde{M} = \left\{(x,y,z) \in \mathbb{R}^3 : |z| \le \sqrt{3-x^2}\,\sqrt{3-y^2}\right\}$$

erwehren, die 3 Prozent billiger angeboten wird. Welche der beiden würden Sie kaufen?

Um das Volumen auszurechnen: $f(x,y,z) = f\big(\Phi(u,v,w)\big) = 1$

Tipp 5a:

$$\int_{Kugel} 1\,du\,dv\,dw = \frac{4}{3}\pi$$



Tipp 5b:

$welche\ Zahlen\ machen\ Sinn\ für\ x, y\ als\ Grenzen?$