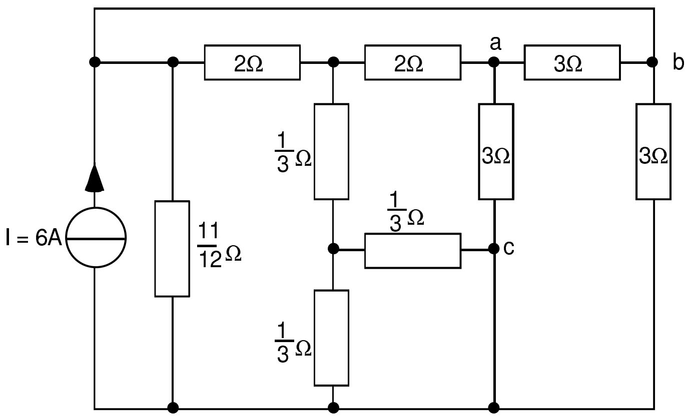
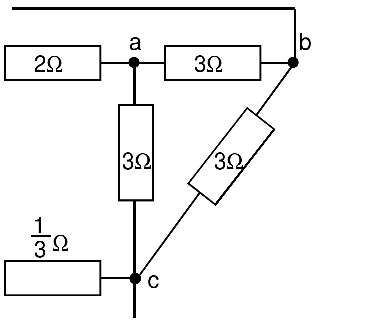
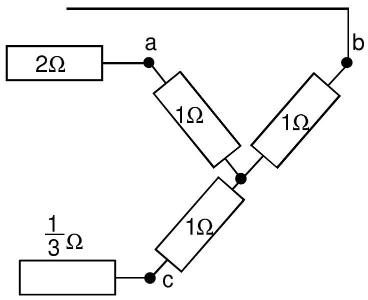
Aufgabe 1

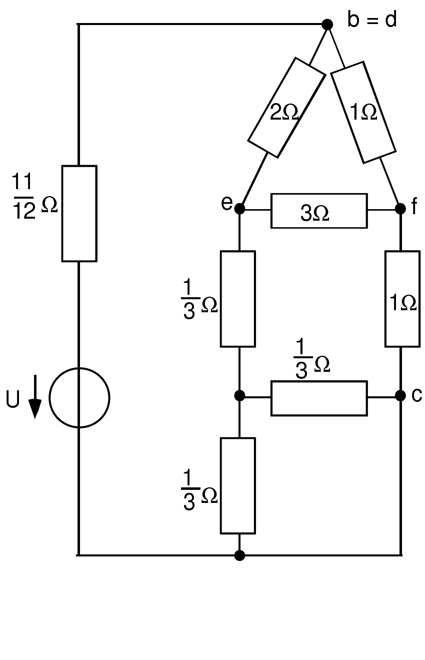






Dreieck-Stern Umwandlung

$$R_a = \frac{3\Omega \cdot 3\Omega}{3\Omega + 3\Omega + 3\Omega} = R_b = R_c = 1\Omega$$



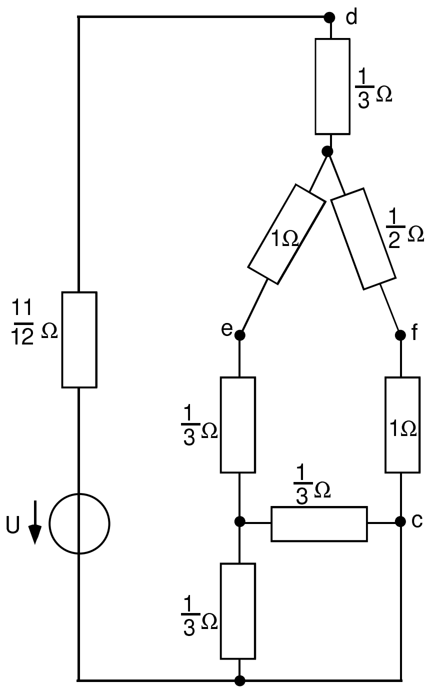
$$R_d = \frac{2\Omega \cdot 1\Omega}{2\Omega + 3\Omega + 1\Omega} = \frac{2}{6}\Omega = \frac{1}{3}\Omega$$

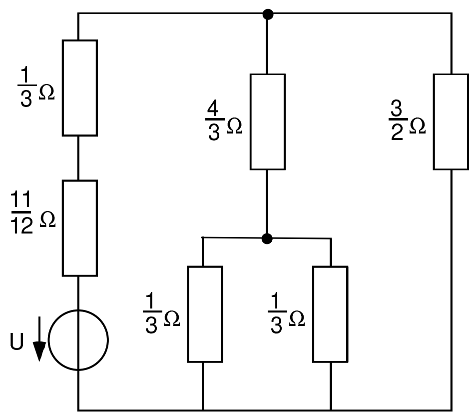
$$R_e = \frac{2\Omega \cdot 3\Omega}{2\Omega + 3\Omega + 1\Omega} = \frac{6}{6}\Omega = 1\Omega$$

$$R_f = \frac{3\Omega \cdot 1\Omega}{2\Omega + 3\Omega + 1\Omega} = \frac{3}{6}\Omega = \frac{1}{2}\Omega$$

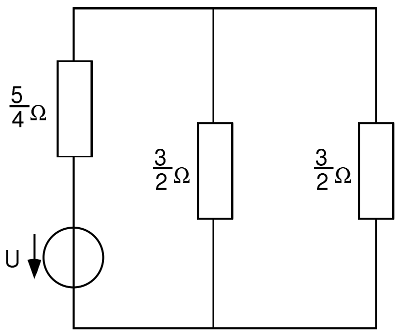
$$U = R \cdot I = \frac{11}{12}\Omega \cdot 6A$$

$$U = \frac{11}{2}V = 5{,}5V$$

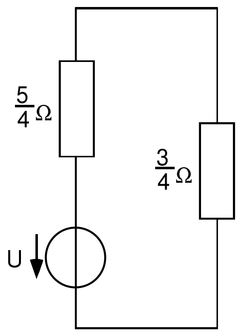


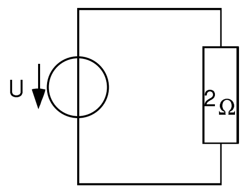


$$\frac{1}{R} = (3+3)\frac{1}{\Omega} = 6\frac{1}{\Omega} \Rightarrow \frac{1}{6}\Omega$$



$$\frac{1}{R} = \frac{2}{3}\frac{1}{\Omega} + \frac{2}{3}\frac{1}{\Omega} = \frac{4}{3}\frac{1}{\Omega} \Rightarrow \frac{3}{4}\Omega$$





Leistung:

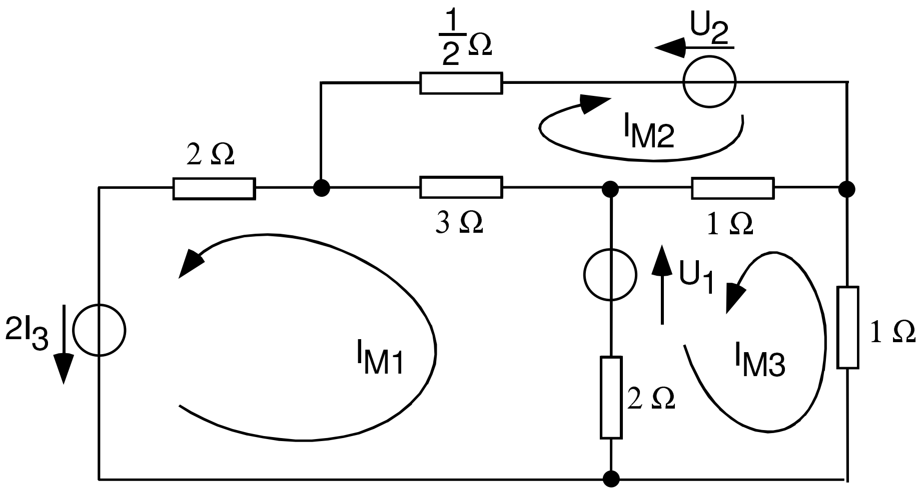
$$I = \frac{U}{R} = \frac{11}{2}\cdot\frac{1}{2} = \frac{11}{4}A$$

$$P = U \cdot I = \frac{11}{2}\cdot\frac{11}{4} = \frac{121}{8}W$$

Lösung Aufgabe 2

a) Alle Stromquellen müssen in Spannungsquellen umgewandelt werden.

b) Es ergibt sich somit folgendes Bild:



Die Maschenströme sind bereits eingetragen.

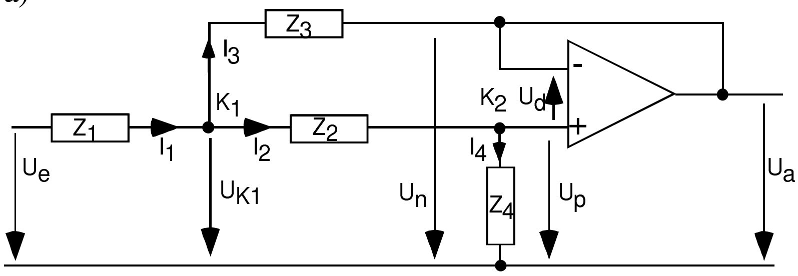
Die Aufstellung der Matrixgleichung erfolgt nach dem Verfahren beschrieben in der Formelsammlung:

$$\begin{pmatrix} 2\Omega+3\Omega+2\Omega & 3\Omega & -2\Omega \\ 3\Omega & 3\Omega+1\Omega+\frac{1}{2}\Omega & 1\Omega \\ -2\Omega & 1\Omega & 2\Omega+1\Omega+1\Omega \end{pmatrix} \begin{pmatrix} I_{M1} \\ I_{M2} \\ I_{M3} \end{pmatrix} = \begin{pmatrix} -2I_3\Omega-U_1 \\ +U_2 \\ +U_1 \end{pmatrix}$$

$$\begin{pmatrix} 7\Omega & 3\Omega & -2\Omega \\ 3\Omega & +\frac{9}{2}\Omega & 1\Omega \\ -2\Omega & 1\Omega & 4\Omega \end{pmatrix} \begin{pmatrix} I_{M1} \\ I_{M2} \\ I_{M3} \end{pmatrix} = \begin{pmatrix} -2I_3\Omega-U_1 \\ +U_2 \\ +U_1 \end{pmatrix}$$

Lösung Aufgabe 3

a)



$$K_1:\quad \frac{U_e - U_{K1}}{Z_1} - \frac{U_{K1} - U_a}{Z_3} - \frac{U_{K1} - U_a}{Z_2} = 0$$

$$K_2:\quad \frac{U_{K1} - U_a}{Z_2} - \frac{U_a}{Z_4} = 0$$

$$\frac{U_{K1}}{Z_2} = \frac{U_a}{Z_2} + \frac{U_a}{Z_4}$$

$$U_{K1} = \left(1 + \frac{Z_2}{Z_4}\right) U_a$$

$$K_1:\quad \frac{U_e}{Z_1} + U_{K1}\left(-\frac{1}{Z_1} - \frac{1}{Z_3} - \frac{1}{Z_2}\right) + U_a\left(\frac{1}{Z_3} + \frac{1}{Z_2}\right) = 0$$

$$\frac{U_e}{Z_1} + \left[\left(1 + \frac{Z_2}{Z_4}\right)\left(-\frac{1}{Z_1} - \frac{1}{Z_3} - \frac{1}{Z_2}\right) + \frac{1}{Z_3} + \frac{1}{Z_2}\right] U_a = 0$$

$$\frac{U_e}{Z_1} + \left[-\frac{1}{Z_1} - \frac{1}{Z_3} - \frac{1}{Z_2} - \frac{Z_2}{Z_4 Z_1} - \frac{Z_2}{Z_4 Z_3} - \frac{Z_2}{Z_4 Z_2} + \frac{1}{Z_3} + \frac{1}{Z_2}\right] U_a = 0$$

$$\frac{U_e}{Z_1} = U_a\left[\frac{1}{Z_1} + \frac{Z_2}{Z_4 Z_1} + \frac{Z_2}{Z_4 Z_3} + \frac{Z_2}{Z_4 Z_2}\right]$$

$$\frac{U_e}{U_a} = 1 + \frac{Z_2}{Z_4} + \frac{Z_1 Z_2}{Z_4}\left(\frac{1}{Z_3} + \frac{1}{Z_2}\right)$$

$$\frac{U_a}{U_e} = \frac{1}{1 + \frac{Z_2}{Z_4} + \frac{Z_1 Z_2}{Z_4}\left(\frac{1}{Z_3} + \frac{1}{Z_2}\right)}$$

Einsetzen der komplexen Bauteile:

$$\frac{U_a}{U_e} = \frac{1}{1 + jR_2\omega C_4 + jR_1R_2\omega C_4\left(\frac{1}{j\omega L_3} + \frac{1}{R_2}\right)}$$

$$= \frac{1}{1 + R_1R_2\frac{C_4}{L_3} + j\omega C_4 R_2\left(1 + \frac{R_1}{R_2}\right)}$$

b)

Einsetzen der Werte:

$C_4 = 1pF$

$L_3 = 10mH$

$R_1 = 5\Omega$

$R_2 = 10\Omega$

Daraus folgt

$$R_1 R_2 \frac{C_4}{L_3} = 5 \cdot 10^{-9}$$

gegenüber 1 vernachlässigbar!

Somit ergibt sich:

$$\frac{U_a}{U_e} = \frac{1}{1 + j\omega C_4 R_2 \left(1 + \frac{R_1}{R_2}\right)}$$

mit:

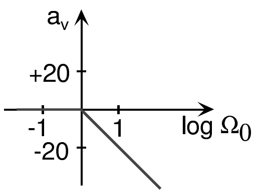
$$\Omega_o = \omega C_4 R_2 \left(1 + \frac{R_1}{R_2}\right)$$

folgt:

$$\frac{U_a}{U_e} = \frac{1}{1 + j\Omega_0}$$

Betrag:

$$a_V = 20 \log \left| \frac{1}{1 + j\Omega_0} \right|$$



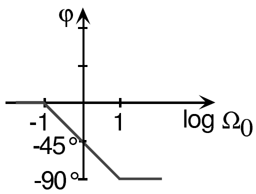
Phase:

$$\varphi = \arctan \frac{Im}{Re}$$

Zähler: $\varphi_Z = \arctan \frac{0}{1} = 0$

Nenner: $\varphi_N = \arctan \frac{\Omega_0}{1} = \arctan \Omega_0$

$$\varphi_{ges} = \varphi_Z - \varphi_N = -\arctan \Omega_0$$

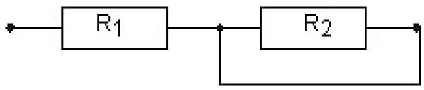


Lösung Aufgabe 4

a)

Für

$\omega \to \infty \Rightarrow$ Kurzschluss des Kondensators $\to$

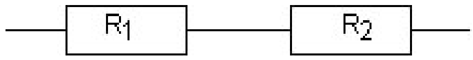


$Z(\infty) = R_1 = 200\Omega$

b)

für $\omega \to 0 \;\Rightarrow$ C kann durch Leerlauf ersetzt werden

$Z(0) = 900\Omega$



$Z(0) = R_1 + R_2 = 200\Omega + R_2 = 900\Omega$

$\Rightarrow R_2 = 700\Omega$

c)

$$Z = R_1 + \left(R_2 \| Z_c\right)$$

$$= R_1 + \frac{1}{\frac{1}{R_2} + j\omega C} = R_1 + \frac{R_2}{1 + j\omega C R_2}$$

$$= R_1 + \frac{R_2\left(1 - j\omega C R_2\right)}{1 + \omega^2 C^2 R_2^2}$$

d)

$$\mathrm{Im}\{Z\} = \frac{-\omega C R_2^2}{1 + \omega^2 C^2 R_2^2}$$

$$\frac{u}{v} = \frac{v \cdot u' - u \cdot v'}{v^2}$$

$$\frac{d\,\mathrm{Im}\{Z\}}{d\omega} = \frac{\left(1+\omega^2 C^2 R_2^2\right)\cdot\left(-C R_2^2\right) - \left(-\omega C R_2^2\right)\left(2\cdot C^2 R_2^2 \omega\right)}{\left(1+\omega^2 C^2 R_2^2\right)^2}$$

$$= \frac{-C R_2^2\left[1 + \omega^2 C^2 R_2^2 - 2 C^2 R_2^2 \omega^2\right]}{\left(1+\omega^2 C^2 R_2^2\right)^2}$$

$$= \frac{-C R_2^2\left[1 - \omega^2 C^2 R_2^2\right]}{\left(1+\omega^2 C^2 R_2^2\right)^2}$$

$$\text{Zähler} \overset{!}{=} 0$$

$$0 = 1 - \omega^2 C^2 R_2^2$$

$$1 = \omega^2 C^2 R_2^2$$

$$\frac{1}{C^2 R_2^2} = \omega^2$$

Alternativ Lösung:
Die Ortskurve ist ein Halbkreis.
Somit hat der maximale Betrag vom $\mathrm{Im}\{Z\}$ seinen reellen Teil im Zentrum des Halbkreises. Deshalb kann auch gerechnet werden: $Z(Mitte) = 550\Omega$
Und die gesuchte Frequenz aus $\mathrm{Re}\{Z\} = 550\Omega$ errechnet werden.

$$Z = R_1 + \frac{R_2}{1+\omega^2 C^2 R_2^2} - \frac{j\omega C_1 R_2^2}{1+\omega^2 C^2 R_2^2}$$

$$550\Omega = R_1 + \frac{R_2}{1+\omega^2 C^2 R_2^2}$$

$$550\Omega - R_1 = \frac{R_2}{1+\omega^2 C^2 R_2^2}$$

$$1 + \omega^2 C^2 R_2^2 = \frac{R_2}{550\Omega - R_1} = 2$$

$$\omega^2 C^2 R_2^2 = 1$$

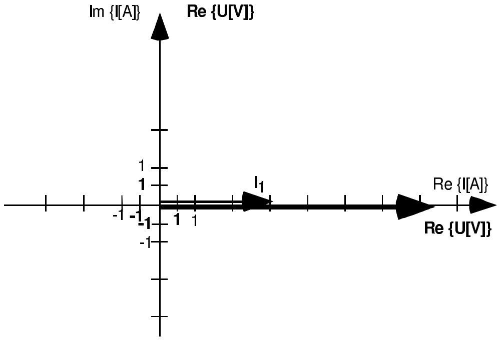
$$\omega^2 = \frac{1}{C^2 R_2^2}$$

$$\omega = \pm \frac{1}{C R_2}$$ nur postive Frequenzen

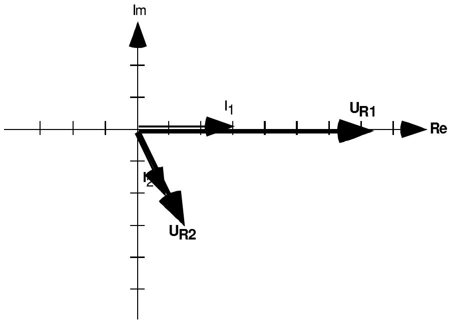
$$\omega = + \frac{1}{1\mu F \cdot 700\Omega} = 1428{,}57 \frac{1}{s}$$

Lösung Aufgabe 5

a)



b)



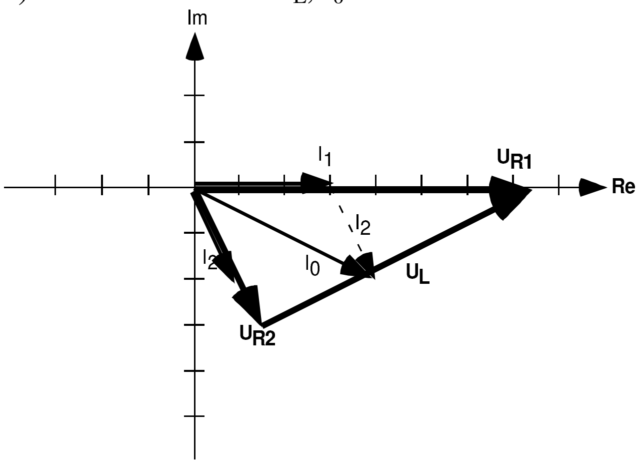
$$U_{R1} = U_L + U_{R2}$$

$$U_{R1} = I_2(R_2 + jX_L)$$

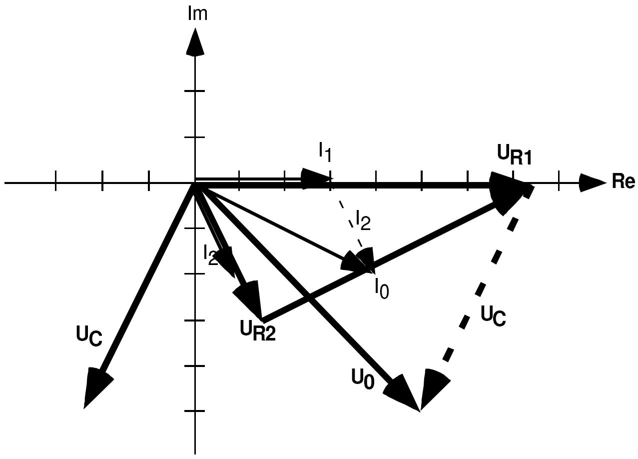
$$I_2 = \frac{U_{R1}}{R_2 + jX_L} = \frac{15}{3 + j6}\frac{V}{\Omega} = \frac{15(3 - j6)}{9 + 36}A = \frac{45 - j90}{45}A = (1 - j2)A$$

$$U_{R2} = (3 - j6)V$$

c) Konstruktion von U_L, I_0



Konstruktion von U_C, U_0



d) Aus Bild c), $\underline{U}_C = -5 - j10$ und $\underline{I}_0 = 4 - j2$

$$\frac{U_C}{I_0} = Z_C = \frac{-5 - j10}{4 - j2} = \frac{(-5 - j10)(4 + j2)}{16 + 4} = \frac{-j50}{20} = -j\frac{5}{2}$$

e)

Aus Bild c) $U_C = -5 - j10$

Phase: Achtung Quadrant III

$$\varphi = \pi + \arctan\frac{-10}{-5}$$

$$= \pi + 1{,}107 \ \text{rad}$$

$$= 4{,}248 \ \text{rad}$$

$$\hat{=}\ 243{,}43^{\circ}$$

f)

Aus Bild c)

$$I_0 = 4 - j2A \rightarrow \varphi = \arctan\frac{-2}{4} = -0{,}4636 rad = -26{,}6°$$

$$U_0 = 10 - j10V \rightarrow$$

$$\varphi = \arctan\left(-\frac{10}{10}\right) = 0{,}7853 rad = -45°$$

Phasenwinkel zwischen U_0 und I_0

$$\varphi = \left|-0{,}7853 - (-0{,}4636)\right| = 0{,}3217 rad = 18{,}4°$$

(hier ist +/- 18,4° richtig!)

Lösung Aufgabe 6

Abb. 1

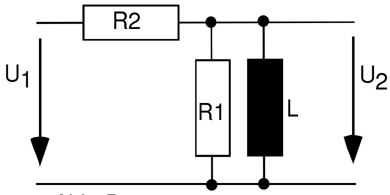
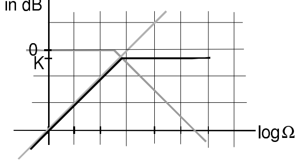


Abb. D



$$\frac{U_2}{U_1}=\frac{\frac{1}{\frac{1}{R_1}+\frac{1}{j\omega L}}}{R_2+\frac{1}{\frac{1}{R_1}+\frac{1}{j\omega L}}}=\frac{1}{R_2\left(\frac{1}{R_1}+\frac{1}{j\omega L}\right)+1}=\frac{1}{\frac{R_2\left(j\omega L+R_1\right)+jR_1\omega L}{jR_1\omega L}}$$

$$=\frac{jR_1\omega L}{R_1R_2+j\omega L\left(R_2+R_1\right)}$$

$$=\frac{R_1R_2}{R_1R_2}\cdot\frac{j\omega\frac{L}{R_2}}{1+j\omega\frac{L}{R_2}\left(\frac{R_2+R_1}{R_1}\right)}$$

$\frac{R_2+R_1}{R_1}=1.027>1\Rightarrow$ Nenner Kurve nach links verschoben

Abb. 2

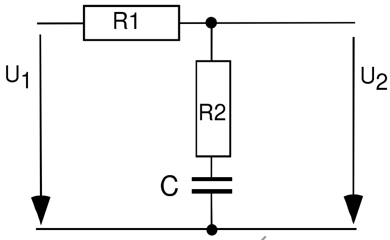
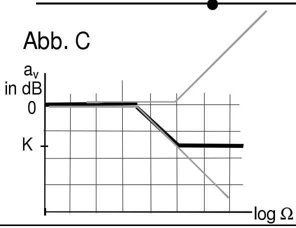


Abb. C



$$\frac{U_2}{U_1}=\frac{R_2+\frac{1}{j\omega C}}{R_1+R_2+\frac{1}{j\omega C}}=\frac{1+j\omega CR_2}{1+j\omega C\left(R_1+R_2\right)}$$

$$\Omega=\omega CR_2$$

$$=\frac{1+j\Omega}{1+j\Omega\frac{R_1+R_2}{R_2}}$$

$$\frac{R_1+R_2}{R_2}=38{,}03>1\Rightarrow$$

Nenner Kurve verschoben nach links (~1,58)

Abb. 3

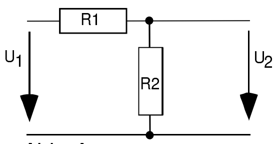
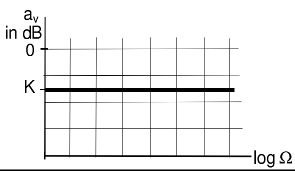


Abb. A



$$\frac{U_2}{U_1}=\frac{R_2}{R_2+R_1}$$

$$a_V=20\log\left|\frac{R_2}{R_1+R_2}\right|=-31{,}604$$

Abb. 4

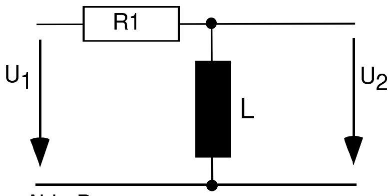
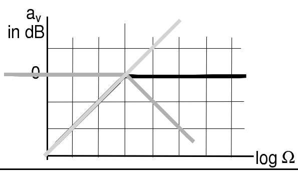


Abb. B



$$\frac{U_2}{U_1}=\frac{j\omega L}{R_1+j\omega L}=\frac{j\omega\frac{L}{R_1}}{1+j\omega\frac{L}{R_1}}$$