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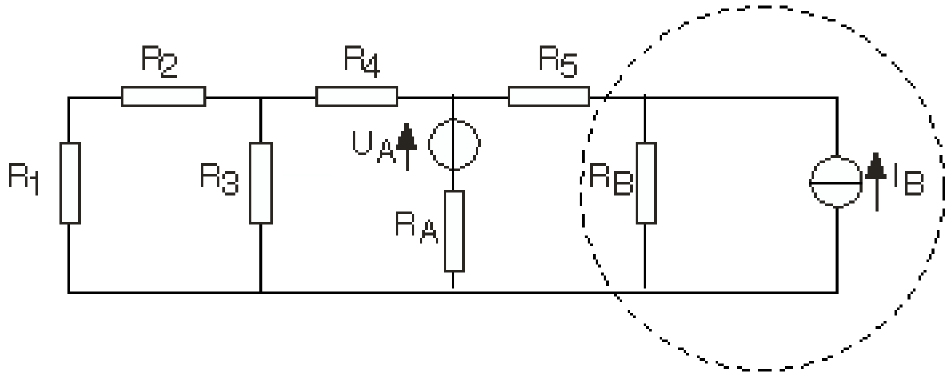
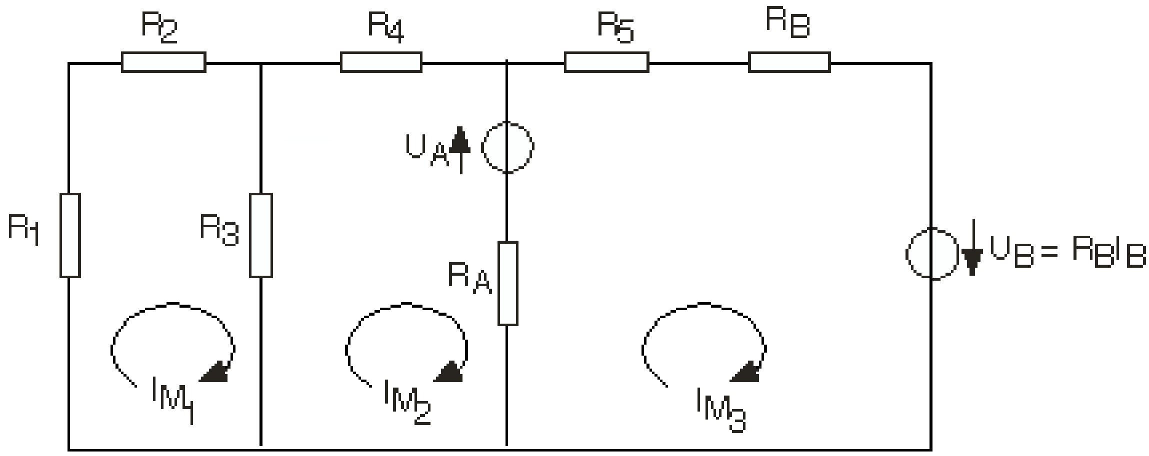
Lösung Aufgabe 1a

Siehe Abb.1.1:

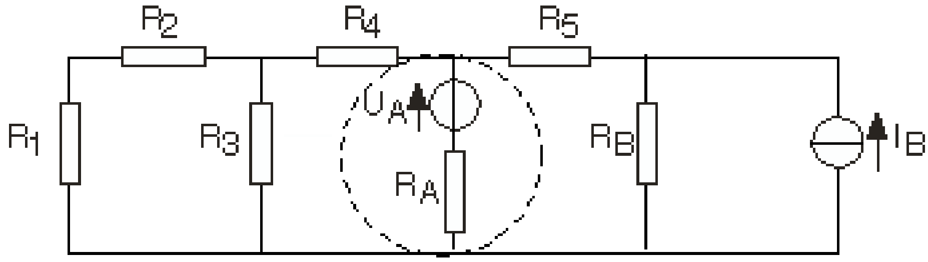


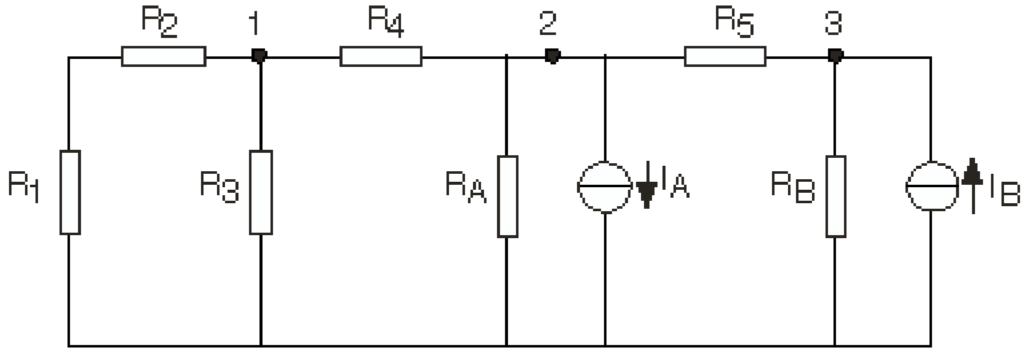


$$\begin{bmatrix} R_1+R_2+R_3 & -R_3 & 0 \\ -R_3 & R_3+R_4+R_A & -R_A \\ 0 & -R_A & R_A+R_5+R_B \end{bmatrix} \begin{bmatrix} I_{M1} \\ I_{M2} \\ I_{M3} \end{bmatrix} = \begin{bmatrix} 0 \\ U_A \\ -(U_A+R_B I_B) \end{bmatrix}$$

Lösung Aufgabe 1b

Siehe Abb.1.1:





$$\begin{bmatrix} \frac{1}{R_1+R_2}+\frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} & 0 \\ -\frac{1}{R_4} & \frac{1}{R_4}+\frac{1}{R_A}+\frac{1}{R_5} & -\frac{1}{R_5} \\ 0 & -\frac{1}{R_5} & \frac{1}{R_5}+\frac{1}{R_B} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_A \\ I_B \end{bmatrix}$$

Lösung Aufgabe 1c

Werte in Matrix aus Aufgabe 1a) einsetzen:

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 50 & -30 & 0 \\ -30 & 55 & -10 \\ 0 & -10 & 35 \end{bmatrix} \begin{bmatrix} I_{M1} \\ I_{M2} \\ I_{M3} \end{bmatrix} = \begin{bmatrix} 0 \\ 2{,}5 \\ -5 \end{bmatrix}$$

aus 1:

$$50 I_{M1} = 30 I_{M2}$$

$$\Rightarrow \quad I_{M1} = \frac{3}{5} I_{M2}$$

aus 2:

$$(-30)\left(\frac{3}{5}\right) I_{M2} + 55 I_{M2} - 10 I_{M3} = 2{,}5$$

$$\Leftrightarrow (-18 + 55) I_{M2} - 10 I_{M3} = 2{,}5$$

$$\Leftrightarrow 37 I_{M2} - 10 I_{M3} = 2{,}5$$

$$\Leftrightarrow I_{M3} = 3{,}7 I_{M2} - 0{,}25$$

aus 3:

$$-10 I_{M2} + 35(3{,}7 I_{M2} - 0{,}25) = -5$$

$$\Leftrightarrow \quad 119{,}5 I_{M2} = 3{,}75$$

$$\Leftrightarrow \quad I_{M2} = 0{,}0314\text{A}$$

$$\Rightarrow I_{M3} = -0{,}1339\text{A}$$

$$\Rightarrow I_{M1} = 0{,}0188\text{A}$$

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Lösung Aufgabe 2a)

(2.1)

$$\frac{\underline{U}_a}{\underline{U}_e} = -\frac{R_1 + \frac{1}{j\omega C_1}}{R_1} = -\frac{1 + j\omega C_1 R_1}{j\omega C_1 R_1}$$

$$\omega_{01} = \frac{1}{C_1 R_1} \quad \Omega_1 = \frac{\omega}{\omega_{01}}$$

(2.2)

$$\frac{\underline{U}_a}{\underline{U}_e} = \frac{R_2}{R_2 + \frac{1}{j\omega C_2}} = \frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2}$$

$$\omega_{02} = \frac{1}{R_2 C_2} \quad \Omega_2 = \frac{\omega}{\omega_{02}}$$

(2.3)

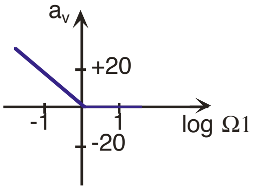
$$\frac{\underline{U}_a}{\underline{U}_e} = \frac{\frac{1}{j\omega C_3}}{R_3 + \frac{1}{j\omega C_3}} = \frac{1}{1 + j\omega R_3 C_3}$$

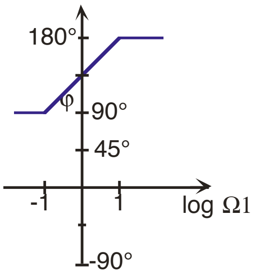
$$\omega_{03} = \frac{1}{R_3 C_3} \quad \Omega_3 = \frac{\omega}{\omega_{03}}$$

Lösung Aufgabe 2b)

2.1

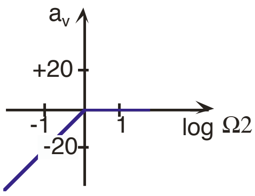
$$-\frac{1+j\Omega_1}{j\Omega_1}$$

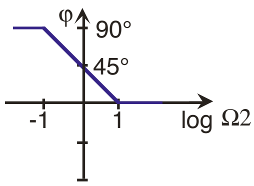




2.2

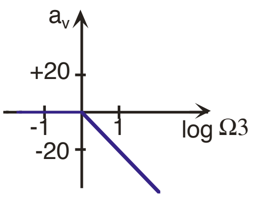
$$\frac{j\Omega_2}{1+j\Omega_2}$$

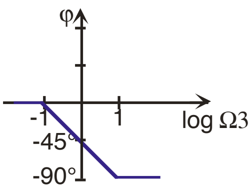




2.3

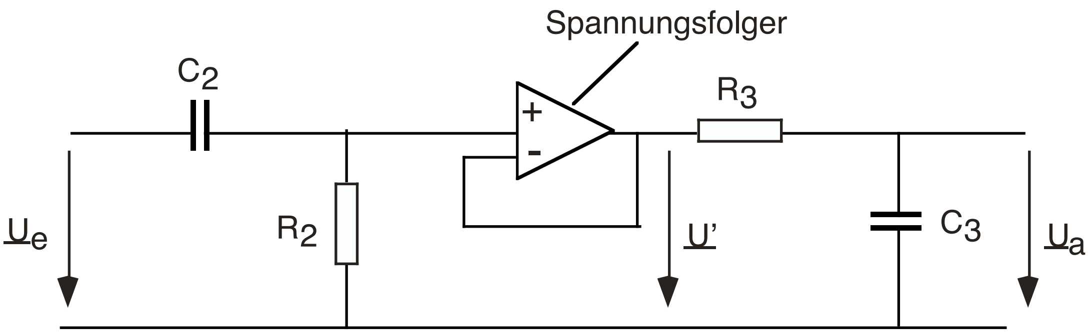
$$\frac{1}{1+j\Omega_3}$$





Lösung Aufgabe 2c)

1)

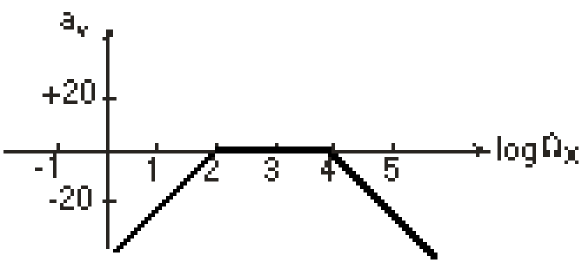


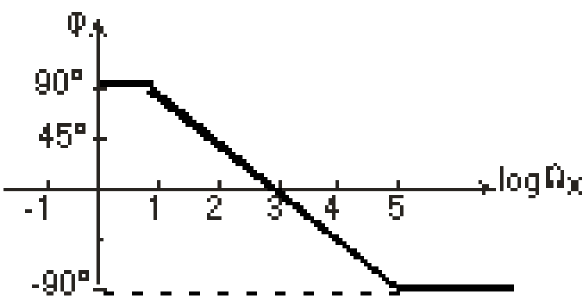
$$\Omega_2 = \Omega_x \cdot 10^{-2}$$

$$\Omega_3 = \Omega_x \cdot 10^{-4}$$

$$\frac{\underline{U}'}{\underline{U}_e} = \frac{j10^{-2}\Omega_x}{1 + j10^{-2}\Omega_x} \qquad \frac{\underline{U}_a}{\underline{U}'} = \frac{1}{1 + j10^{-4}\Omega_x}$$

$$\frac{\underline{U}_a}{\underline{U}_e} = \frac{j10^{-2}\Omega_x}{\left(1 + j10^{-2}\Omega_x\right)\left(1 + j10^{-4}\Omega_x\right)}$$





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3.a)

Gesamtimpedanz:

$$\underline{\underline{Z}} = \left(R_1 + j\omega L + \frac{1}{j\omega C}\right) || R_2$$

$$= \frac{R_1 R_2 + j\left(\omega L - \frac{1}{\omega C}\right) R_2}{R_1 + R_2 + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{\left[R_1 R_2 + j\left(\omega L - \frac{1}{\omega C}\right) R_2\right]\left[(R_1 + R_2) - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{(R_1 + R_2)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \left\{\frac{R_1 R_2 (R_1 + R_2) + \left(\omega L - \frac{1}{\omega C}\right)^2 R_2}{(R_1 + R_2)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right\} + j\left\{\frac{(R_1 + R_2) R_2 \left(\omega L - \frac{1}{\omega C}\right) - R_1 R_2 \left(\omega L - \frac{1}{\omega C}\right)}{(R_1 + R_2)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right\}$$

Resonanzfrequenz ω_o gegeben wenn $\mathrm{Im}\{\underline{\underline{Z}}\} = 0$

$$\Rightarrow 0 = \frac{(R_1+R_2)R_2\left(\omega_0 L - \frac{1}{\omega_0 C}\right) - R_1 R_2\left(\omega_0 L - \frac{1}{\omega_0 C}\right)}{(R_1+R_2)^2 + \left(\omega_0 L - \frac{1}{\omega_0 C}\right)^2}$$

$$\Leftrightarrow 0 = (R_1+R_2)R_2\left(\omega_0 L - \frac{1}{\omega_0 C}\right) - R_1 R_2\left(\omega_0 L - \frac{1}{\omega_0 C}\right)$$

$$\Leftrightarrow 0 = \left[(R_1+R_2)R_2 - R_1 R_2\right]\left(\omega_0 L - \frac{1}{\omega_0 C}\right)$$

$$\Leftrightarrow 0 = \omega_0 L - \frac{1}{\omega_0 C}$$

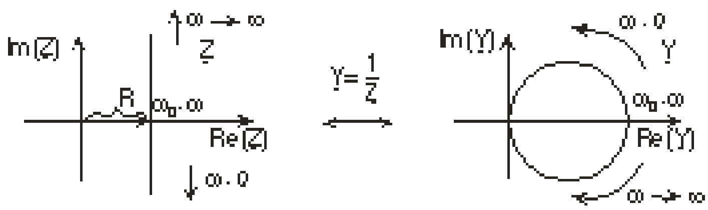
$$\Leftrightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\Leftrightarrow \omega_0^2 = \frac{1}{LC}$$

$$\Leftrightarrow \omega_0 = \sqrt{\frac{1}{LC}}$$

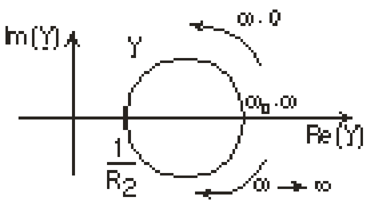
3.b-c)

Ortskurve $\underline{Z}$ und $\underline{Y}$:



Man muss eine Spiegelung am Einheitskreis durchführen.

3.d)



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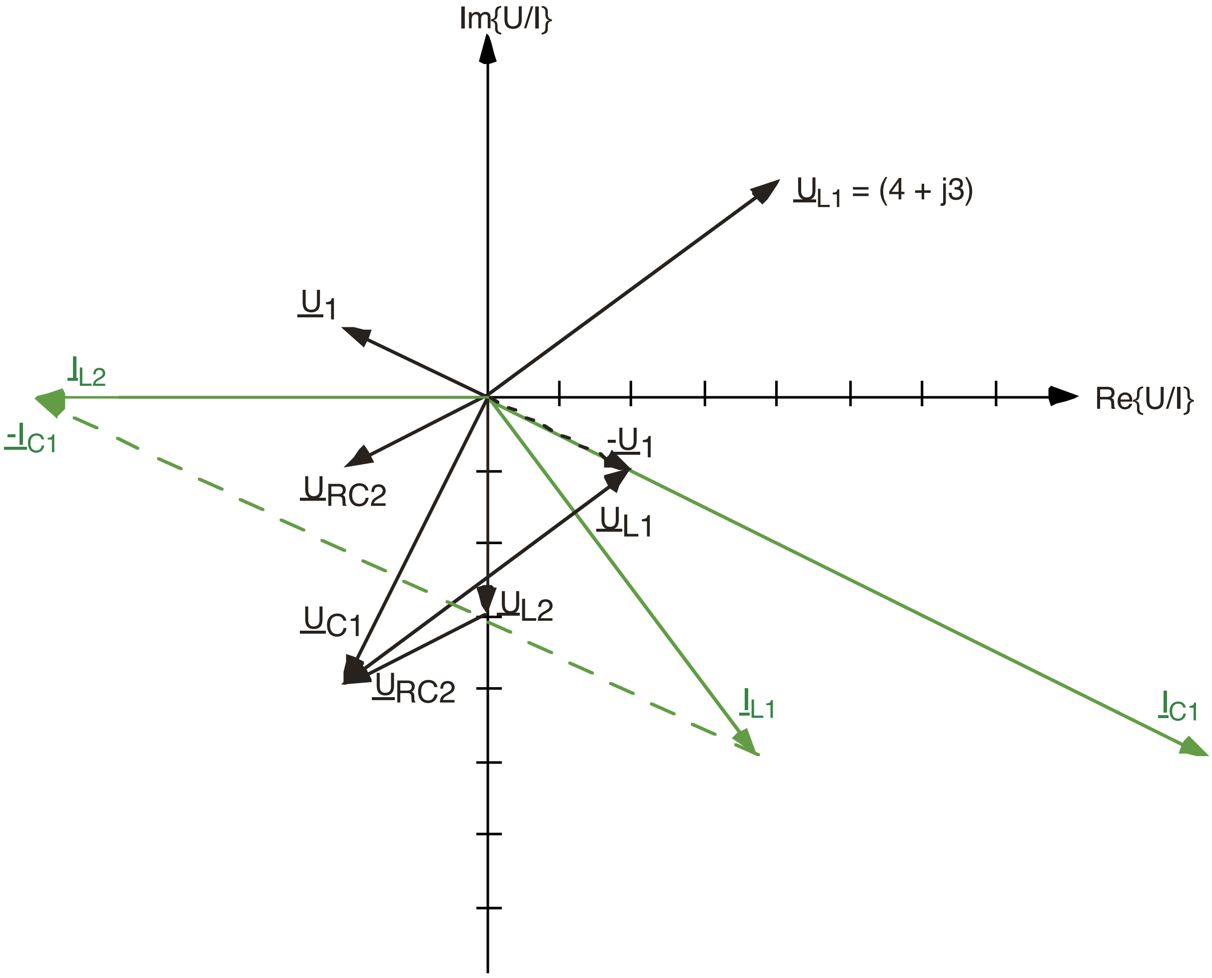
Lösung Aufgabe 4a-c)



Lösung Aufgabe 4a)

$\underline{U}_1 + \underline{U}_{L1} + \underline{U}_{C1} = 0 \Rightarrow \underline{U}_{L1} = 4 + j3$

$\underline{U}_1 + \underline{U}_{L1} + \underline{U}_{L2} + \underline{U}_{RC2} = 0 \Rightarrow \underline{U}_{RC2} = -2 - j1$

Lösung Aufgabe 4b)

$\underline{I}_{L1}$ muss $\underline{U}_{L1}$ um 90^o nachlaufen.
$\underline{I}_{C1}$ muss $\underline{U}_{C1}$ um 90^o voraneilen.

Lösung Aufgabe 4c)

$\underline{I}_{L1} - \underline{I}_{C1} - \underline{I}_{L2} = 0$

Lösung Aufgabe 4d)

$\varphi_{ui} = -\frac{\pi}{2}$ bzw. -90^o

Der Strom durch eine Induktivität läuft immer der Spannung um 90^o hinterher.

Lösung Aufgabe 4e)

$\underline{U}_1 = (-2 + j1)$ $\quad\quad$ $\underline{U}_{RC2} = (-2 - j1)$

$\varphi_{URC2} = \pi + \arctan\left(\frac{-1}{-2}\right) = \pi + 0{,}4636 = 3{,}6052$

$\varphi_{U1} = \pi + \arctan\left(\frac{1}{-2}\right) = \pi - 0{,}4636 = 2{,}6779$

$\varphi_d = \varphi_{U1} - \varphi_{URC2} = 0{,}9273$ bzw. $53{,}1^o$

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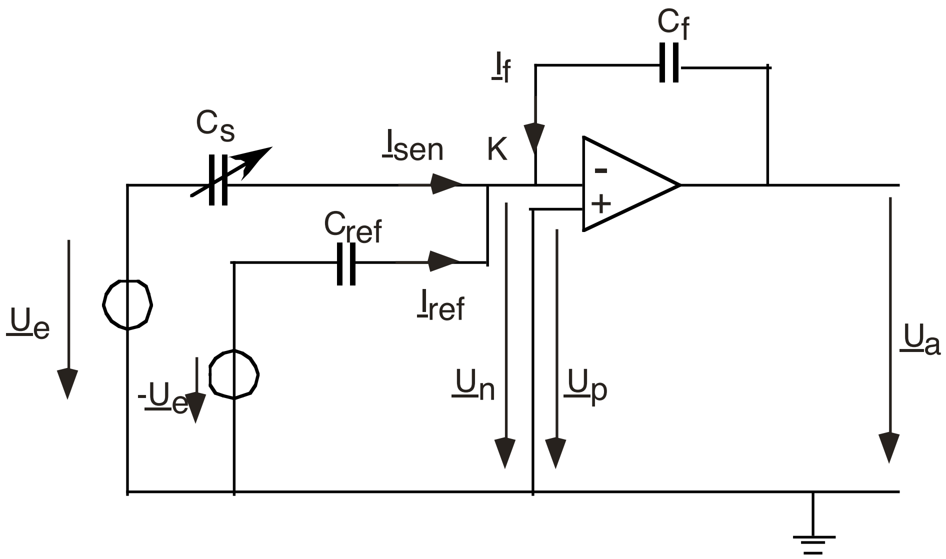
Lösung Aufgabe 5.a)

Ausgehend vom invertierenden Addierer:



Idealer Operationsverstärker: $\underline{U}_n=\underline{U}_p=0$

$$\underline{I}_{sen} + \underline{I}_{ref} + \underline{I}_f = 0$$

$$\underline{I}_{sen} = j\omega C_{sen}\underline{U}_e$$
$$\underline{I}_{ref} = -j\omega C_{ref}\underline{U}_e$$
$$\underline{I}_f = j\omega C_f\underline{U}_a$$

$$j\omega C_s\underline{U}_e - j\omega C_{ref}\underline{U}_e = -j\omega C_f\underline{U}_a$$
$$\Rightarrow -\underline{U}_a = \left(\frac{C_s}{C_f} - \frac{C_{ref}}{C_f}\right)\underline{U}_e$$

Lösung Aufgabe 5.b)

Bei 0 bar C_s = 100pF,

$$\Rightarrow -\underline{U}_a = \frac{100\text{pF} - C_{ref}}{C_f}\cdot\underline{U}_e \overset{!}{=} 0$$
$$\Rightarrow C_{ref} = 100\text{pF}$$

Damit $\underline{U}_a$ = 0 bei 0 bar wird, muß C_{ref} zu 100 pF gewählt werden.

Lösung Aufgabe 5.c)

$$\frac{C_{sen1bar}}{C_f} - \frac{C_{ref}}{C_f} = 1$$

$$C_{sen1bar} - C_{ref} = C_f$$

$$C_f = 110pF - 100pF = 10pF$$