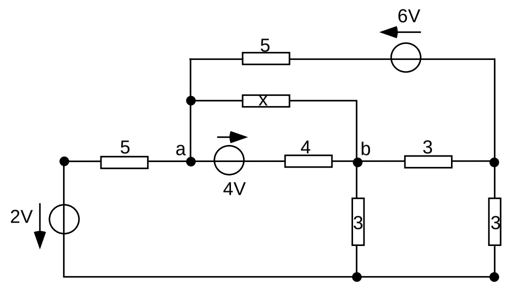
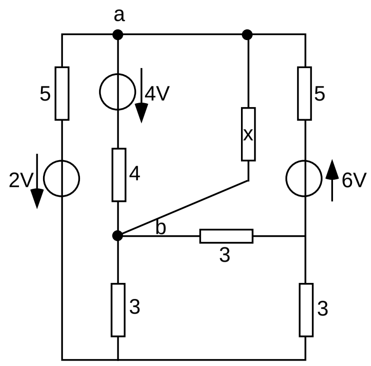
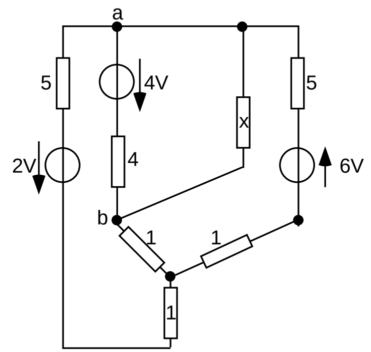
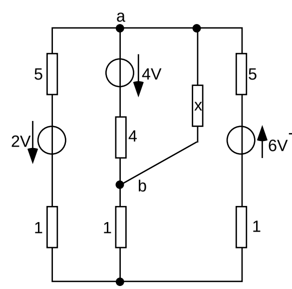
Lösung 1a)

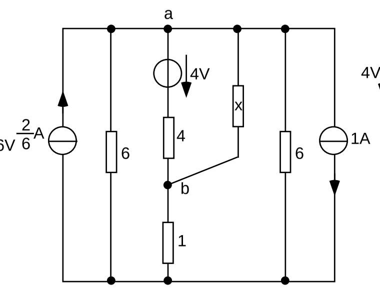


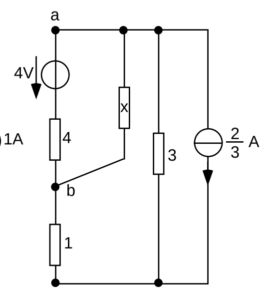


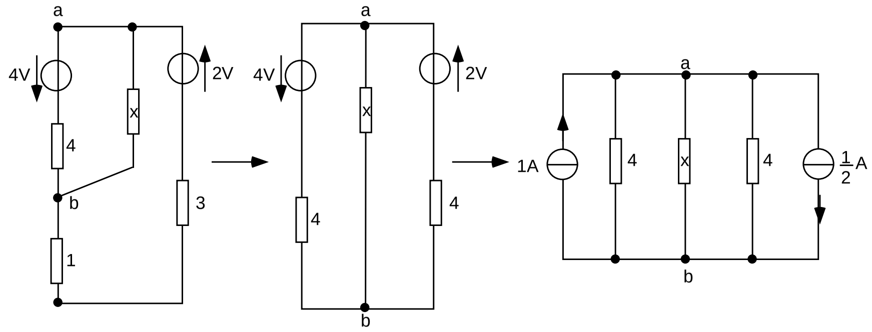
Dreieck-Stern

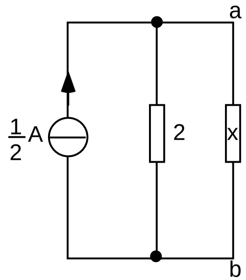


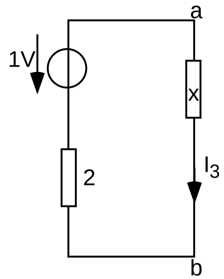












1b)

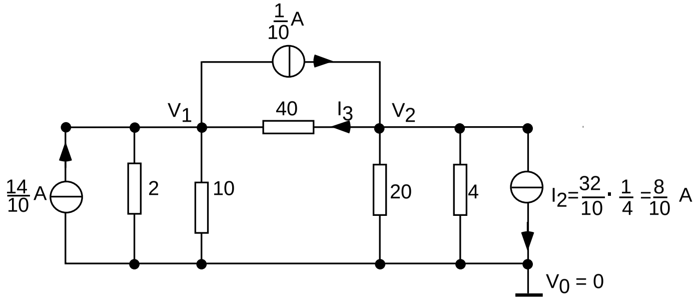
$U = R \cdot I$

$\frac{U}{I} = 2 + x = 4 \Rightarrow x = 2$

$\boxed{R_3 = 2\Omega}$

Lösung 2

1. Umwandlung U $\rightarrow$ I



Knotenpotentialverfahren

1. Alle Spannugsquellen in Stromquellen wandeln
2. ein Punkt auf 0 Potential $\rightarrow V_0$
3. Knoten bezeichnen
4. Matrix aufstellen

$$\begin{pmatrix} \frac{1}{2}+\frac{1}{10}+\frac{1}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{1}{40}+\frac{1}{20}+\frac{1}{4} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=\begin{pmatrix} \frac{14}{10}-\frac{1}{10} \\ \frac{1}{10}-\frac{8}{10} \end{pmatrix}$$

$$\begin{pmatrix} \frac{25}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{13}{40} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=\begin{pmatrix} \frac{13}{10} \\ -\frac{7}{10} \end{pmatrix}$$

Gauss-Verfahren:

$$\left.\begin{array}{cc|c} \frac{25}{40} & -\frac{1}{40} & \frac{13}{10} \\ -\frac{1}{40} & \frac{13}{40} & -\frac{7}{10} \end{array}\right.\begin{array}{l} |\cdot 40 \\ |\cdot 40 \end{array} \rightarrow \begin{array}{cc|c} 25 & -1 & 52 \\ -1 & 13 & -28 \end{array}\ |\cdot 25\ \uparrow + \rightarrow \begin{array}{cc|c} 0 & 324 & -648 \\ -1 & 13 & -28 \end{array}\ |:324$$

$$\begin{array}{cc|c} 0 & 1 & -2 \\ -1 & 13 & -28 \end{array}\ |\cdot 13\ \downarrow -$$

$$\begin{array}{cc|c} 0 & 1 & -2 \\ -1 & 0 & -2 \end{array}$$

$$\begin{array}{cc|c} 0 & 1 & -2 \\ 1 & 0 & 2 \end{array} \rightarrow$$

$V_1 = 2$

$V_2 = -2$

$U_3 = V_2 - V_1 = -2-2 = -4$

$\frac{U_3}{R_3} = I_3 = -\frac{4}{40} = -\frac{1}{10}$

$I_3 = -\frac{1}{10}$A

Lösung 3)

$Z_L = j\omega L = j2\Omega$

$Z_C = \frac{1}{j\omega C} = -j\Omega$

$Z_{R1} = Z_{R2} = 1\Omega$

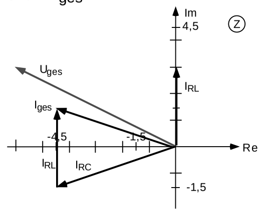
$Z_{RL} = 1 + j2$

$Z_{RC} = 1 - j$

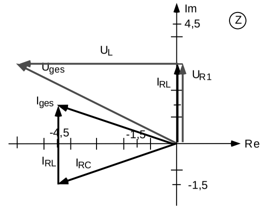
$I_{RL} = \frac{U}{Z_{RL}} = \frac{-6+j3}{1+j2} = \frac{-6+j3+12j+6}{5} = \frac{15j}{5} = 3j$

$I_{RC} = \frac{U}{Z_{RC}} = \frac{-6+j3}{1-j} = \frac{-6+j3-6j-3}{2} = \frac{-9-j3}{2} = -4{,}5 - j1{,}5$

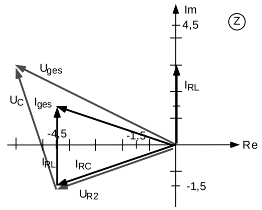
Die Teilströme einzeichnen und eine Vektoraddition durchführen. Daraus ergibt sich I_{ges}.



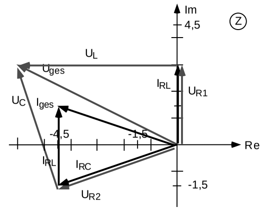
Durch R_1 fließt I_{RL}. Somit ergibt sich U_{R1}. Mit U_{ges} und U_{R1} kann U_L konstruiert werden.



Durch R_2 fließt I_{RC}. Somit ergibt sich U_{R2}. Mit U_{ges} und U_{R2} kann U_C konstruiert werden.



Vollständiges Zeigerdiagramm:



Zur Ergänzung hier die Rechnung (war nicht gefragt!) :

$U_L = I_{RL} \cdot Z_L = \frac{-6+j3}{1+j2} \cdot j2 = \frac{(-j12-6)(1-j2)}{1+4} = \frac{-j12-6-24+j12}{5} = \frac{-30}{5} = -6$

$U_{R_1} = I_{RL} \cdot Z_{R_1} = \frac{-6+j3}{1+j2} \cdot 1 = \frac{(-6+j3)(1-j2)}{5} = \frac{-6+j3+12j+6}{5} = \frac{j15}{5} = j3$

$U_C = I_{RC} \cdot Z_C = \frac{-6+j3}{1-j} \cdot (-j) = \frac{+j6+3}{1-j} = \frac{(j6+3)(1+j)}{2} = \frac{j6+3-6+3j}{2} = \frac{-3+j9}{2} = -1{,}5 + j4{,}5$

$U_{R_2} = I_{RC} \cdot Z_{R_2} = \frac{-6+j3}{1-j} \cdot 1 = \frac{(-6+j3)(1+j)}{2} = \frac{-6+j3-j6-3}{2} = \frac{-9-j3}{2} = -4{,}5 - j1{,}5$

3d)

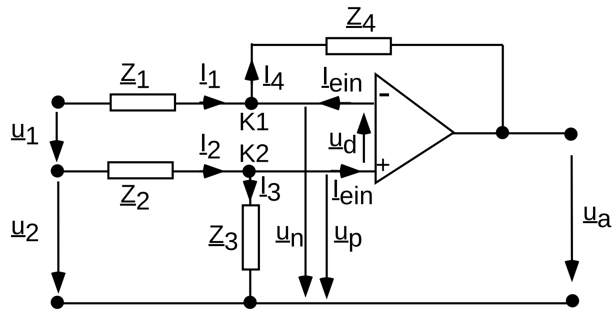
- Für große Frequenzen fällt die Amplitude mit 20 dB pro Dekade bis zu einer charakteristischen Frequenz. Ab dort fällt die Amplitude mit 40 dB pro Dekade.

- Für kleine Frequenzen steigt die Amplitude mit 20 dB pro Dekade. Da der OP nur eine maximale Ausgangspannung (=Versorgungsspannung) liefern kann, steigt die Verstärkung im realen Fall nicht linear bis unendlich, sondern läuft in die Sättigung.

Lösung zur LEN Klausur WS 2002/03

Lösung 4

a)



Idealer OP
$\Rightarrow \underline{u}_d = 0 \rightarrow \underline{u}_n = \underline{u}_p$
$I_{ein} = 0$

Knotenpotentialverfahren

K_1: $\quad I_1 - I_4 = 0$

$$\frac{(U_1 + U_2) - U_n}{Z_1} - \frac{U_n - U_a}{Z_4} = 0$$

K_2: $\quad I_2 - I_3 = 0$

$$\frac{U_2 - U_p}{Z_2} - \frac{U_p}{Z_3} = 0$$

$U_n = U_p$

K_1^*: $\quad \frac{U_1}{Z_1} + \frac{U_2}{Z_1} - \frac{U_n}{Z_1} - \frac{U_n}{Z_4} + \frac{U_a}{Z_4} = 0$

$$\left(\frac{U_1}{Z_1} + \frac{U_2}{Z_1} + \frac{U_a}{Z_4}\right) = U_n\left(\frac{1}{Z_1} + \frac{1}{Z_4}\right)$$

$$\left(\frac{U_1}{Z_1} + \frac{U_2}{Z_1} + \frac{U_a}{Z_4}\right)\left(\frac{1}{\frac{1}{Z_1} + \frac{1}{Z_4}}\right) = U_n$$

K_2^*: $\quad \frac{U_2}{Z_2} = U_n\left(\frac{1}{Z_2} + \frac{1}{Z_3}\right)$

$$\frac{U_2}{Z_2}\left(\frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3}}\right) = U_n$$

$$K_1^* = K_2^* \qquad \frac{U_2}{Z_2}\left(\frac{1}{\left(\frac{1}{Z_2}+\frac{1}{Z_3}\right)}\right) = \left(\frac{U_1}{Z_1}+\frac{U_2}{Z_1}+\frac{U_a}{Z_4}\right)\left(\frac{1}{\frac{1}{Z_1}+\frac{1}{Z_4}}\right)$$

$$\frac{U_2}{Z_2}\frac{\frac{1}{Z_1}+\frac{1}{Z_4}}{\frac{1}{Z_2}+\frac{1}{Z_3}} - \frac{U_2}{Z_1} - \frac{U_1}{Z_1} = \frac{U_a}{Z_4} \qquad \Big|\cdot Z_4 \quad \text{mit} \quad Z_4 \neq 0$$

$Z_4\left(\frac{\frac{1}{Z_1}+\frac{1}{Z_4}}{1+\frac{Z_2}{Z_3}} - \frac{1}{Z_1}\right)\cdot U_2 - \frac{Z_4}{Z_1}\cdot U_1 = U_a$	$\frac{Z_1Z_3 - Z_2Z_4}{Z_1(Z_2+Z_3)}\cdot U_2 - \frac{Z_4}{Z_1}\cdot U_1 = U_a$

b)

$U_a(U_1) \Rightarrow$ Faktor vor U_2 muss 0 sein

$$\Rightarrow \frac{Z_1Z_3 - Z_2Z_4}{Z_1(Z_2+Z_3)} \overset{!}{=} 0$$

$$Z_1Z_3 - Z_2Z_4 \overset{!}{=} 0$$

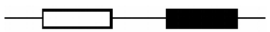
$$Z_3 = Z_2\cdot\frac{Z_4}{Z_1} = j\omega L_2\cdot\frac{\frac{1}{j\omega C_4}}{R_1+j\omega L_1}$$

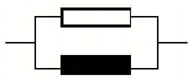
$$= j\omega L_2\cdot\frac{1}{j\omega C_4}\,\frac{1}{(R_1+j\omega L_1)}$$

$$= \frac{L_2}{C_4}\cdot\frac{1}{R_1+j\omega L_1}$$

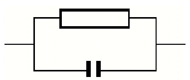
$$\boxed{Z_3 = \frac{1}{\frac{C_4R_1}{L_2}+j\omega\frac{C_4L_1}{L_2}}}$$

Mögliche Kombinationen aus 2 Bauteilen:

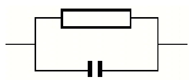
1.  $\rightarrow Z = R + j\omega L$ geht nicht

2.  $\rightarrow Z = \frac{1}{\frac{1}{R}+\frac{1}{j\omega L}}$ geht nicht

3.  $\rightarrow Z = R + \frac{1}{j\omega C}$ geht nicht

4.  $\rightarrow Z = \frac{1}{\frac{1}{R}+j\omega C}$ geht!

Vgl. mit $Z_3 \Rightarrow$



$$\frac{1}{\frac{1}{R_3}+j\omega C_3} = \frac{1}{\frac{C_4R_1}{L_2}+j\omega\frac{C_4L_1}{L_2}}$$

Koeffizientenvergleich:

$$\frac{1}{R_3} = \frac{C_4R_1}{L_2} \quad \text{und} \quad C_3 = \frac{C_4L_1}{L_2}$$

Werte einsetzen:

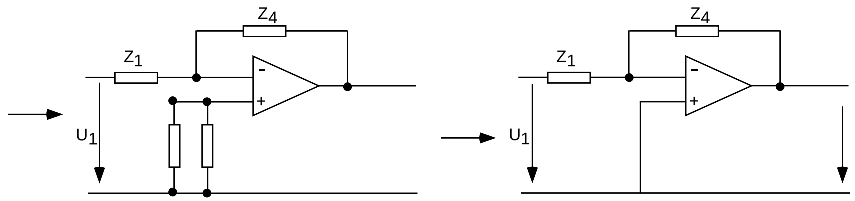
$$\frac{1}{R_3} = \frac{1\mu F\cdot 1k\Omega}{5mH} = 0{,}2S \qquad C_3 = \frac{1\mu F\cdot 10mH}{5mH} = 2\mu F$$

$R_3 = 5\Omega \qquad C_3 = 2\mu F$

Parallelschaltung aus Kondensator und Widerstand

c)

U_2=0



$$\frac{U_a}{U_1} = -\frac{Z_4}{Z_1} \quad \text{(oder aus a))}$$

$$= -\frac{1}{j\omega C_4}\cdot\frac{1}{R_1+j\omega L_1} \qquad \text{nicht ausmultiplizieren!}$$

$$= -\frac{1}{j\omega C_4}\cdot\frac{1}{R_1}\cdot\frac{1}{1+j\omega\frac{L_1}{R_1}}$$

$$= -\frac{1}{j\omega C_4R_1}\cdot\frac{1}{1+j\omega\frac{L_1}{R_1}}$$

$$\omega_0 = \frac{1}{C_4R_1} \rightarrow \Omega = \frac{\omega}{\omega_0}$$

1) Nomierung auf:

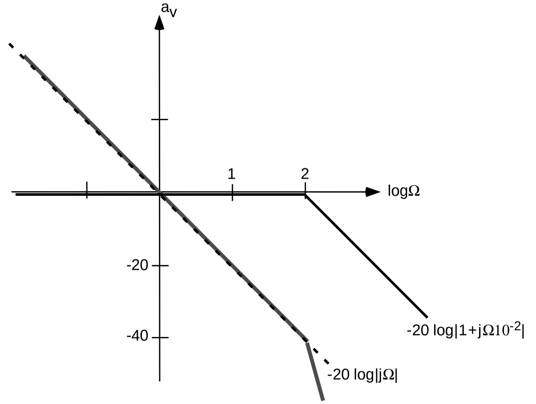
$$= -\frac{1}{j\Omega}\cdot\frac{1}{1+j\Omega\frac{1}{C_4R_1}\cdot\frac{L_1}{R_1}} \qquad \text{NR:} \quad \frac{L_1}{C_4\cdot R_1^2} = \frac{10mH}{1\mu F\cdot 10^6\Omega^2} = 10^{-2}$$

$$= -\frac{1}{j\Omega}\cdot\frac{1}{1+j\Omega 10^{-2}}$$

$$a_V = 20\log\left|\frac{1}{j\Omega}\cdot\frac{1}{1+\Omega 10^{-2}}\right|$$

$$a_V = 20\log\left|\frac{1}{j\Omega}\right| + 20\log\left|\frac{1}{1+\Omega 10^{-2}}\right|$$

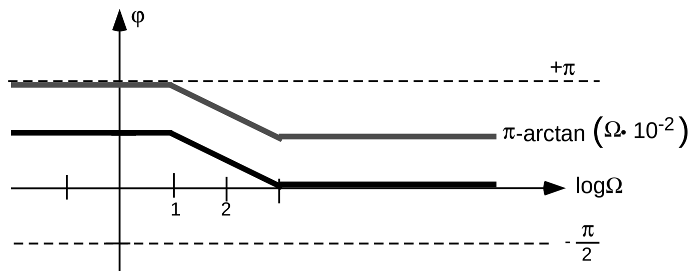
$$= -20\log|j\Omega| - 20\log|1+j\Omega 10^{-2}|$$



φ: $-1 \rightarrow \varphi = \pm k\cdot\pi = \pi$

$$\frac{1}{j\Omega} \rightarrow \varphi = \arg\left(\frac{1}{j\Omega}\right) = -\arg(j\Omega) = -\frac{\pi}{2}$$

$$\frac{1}{1+j\Omega 10^{-2}} \rightarrow \varphi = \arg\left(\frac{1}{1+j\Omega 10^{-2}}\right) = -\arg\left(1+j\Omega 10^{-2}\right) = -\arctan\left(\frac{\Omega 10^{-2}}{1}\right)$$



2) Normierung auf $\omega_0 = \frac{R_1}{L_1} \rightarrow \Omega = \frac{\omega}{\omega_0}$

$$= -\frac{1}{j\Omega\frac{R_1^2}{L_1}\cdot C_4}\cdot\frac{1}{1+j\Omega}$$

$$= -\frac{1}{j\Omega 10^2}\cdot\frac{1}{1+j\Omega}$$

NR:

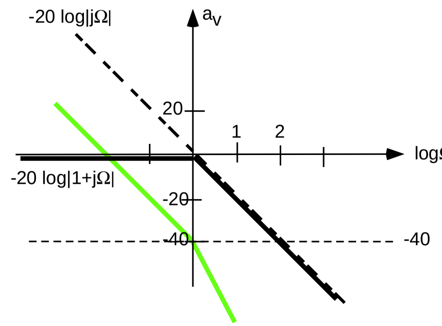
$$C_4\cdot\frac{R_1^2}{L} = 1\mu F\cdot\frac{(1k\Omega)^2}{10mH} = 100 = 10^2$$

$$a_V = 20\log\left|\frac{1}{j\Omega 10^2}\cdot\frac{1}{1+j\Omega}\right|$$

$$= -20\log\left|j\Omega 10^2\right| - 20\log|1+j\Omega|$$

$$= -20\log|j\Omega| - 20\log\left|10^2\right| - 20\log|1+j\Omega|$$

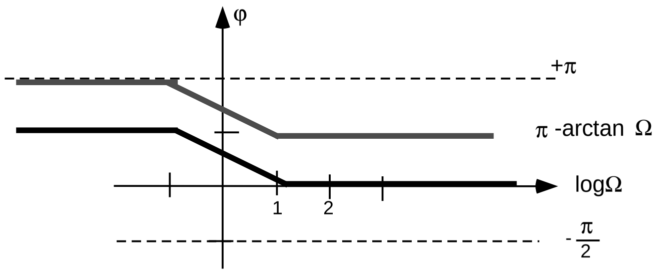
$$= -20\log|j\Omega| - 40 - 20\log|1+j\Omega|$$



φ: $-1 \rightarrow \varphi = \pm k\pi = \pi$

$$\frac{1}{j\Omega 10^2} \rightarrow \varphi = \arg\left(\frac{1}{j\Omega 10^{-2}}\right) = -\arg\left(j\Omega 10^{-2}\right) = -\frac{\pi}{2}$$

$$\frac{1}{1+j\Omega} \rightarrow \varphi = -\arctan\frac{\Omega}{1}$$



5)

Schalter offen:

$$Z_{ges} = R_1 + j\omega L$$

$$I_o = \frac{U}{Z_{ges}}$$

$$I_o = \frac{1}{R_1 + j\omega L} \cdot U$$

Schalter geschlossen:

$$Z_{ges} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}}$$

$$I_g = \frac{U}{Z_{ges}}$$

$$I_g = \frac{1}{R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}}} \cdot U$$

$$|I_o| = |I_g|$$

$\downarrow$

Quadrate vergleichen

$$|I_o|^2 = |I_g|^2$$

$$\frac{1}{|R_1 + j\omega L|^2} \cdot |U|^2 = \frac{1}{\left|R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}}\right|^2} |U|^2$$

$$|R_1 + j\omega L|^2 = \left|R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}}\right|^2$$

$$R_1^2 + \omega^2 L^2 = \left|R_1 + \frac{jR_2\omega L}{R_2 + j\omega L}\right|^2 = \left|\frac{R_1R_2 + j\omega L R_1 + jR_2\omega L}{R_2 + j\omega L}\right|^2 = \left|\frac{R_1R_2 + j\omega L(R_1 + R_2)}{R_2 + j\omega L}\right|^2$$

$$R_1^2 + \omega^2 L^2 = \frac{R_1^2R_2^2 + \omega^2L^2(R_1 + R_2)^2}{R_2^2 + \omega^2 L^2}$$

$$R_1^2R_2^2 + \omega^2L^2R_2^2 + R_1^2\omega^2L^2 + \omega^4L^4 - R_1^2R_2^2 - \omega^2L^2\left(R_1^2 + 2R_1R_2 + R_2^2\right) = 0$$

$$\omega^4L^4 - \omega^2L^2 2R_1R_2 = 0$$

$$\omega^4L^4 = \omega^2L^2 2R_1R_2 \qquad \Big| : \omega^2L^2$$

$$\omega^2L^2 = 2R_1R_2$$

$$\boxed{R_2 = \frac{\omega^2L^2}{2R_1}}$$

Lösung zur LEN Klausur WS 2002/03

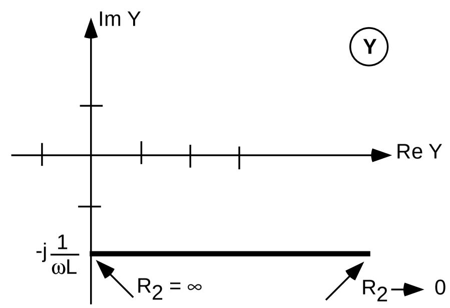
b)

Z_{ges}: $\quad R = \dfrac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}}$

<u>Parallelschaltung</u>

$$Y_p = \frac{1}{R_2} + \frac{1}{j\omega L}$$

$$= \frac{1}{R_2} - j\frac{1}{\omega L} \quad \rightarrow$$



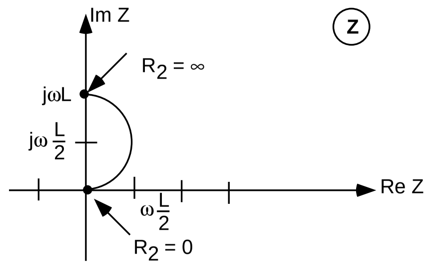
<u>Inversion am Einheitskreis</u>

$\rightarrow$ Gerade $\Rightarrow$ Kreis

$$Z_p = \frac{1}{\frac{1}{R_2} - j\frac{1}{\omega L}}$$

$$Y(R_2 \rightarrow \infty) = -j\frac{1}{\omega L} \Rightarrow Z(R_2 \rightarrow \infty) = j\omega L$$

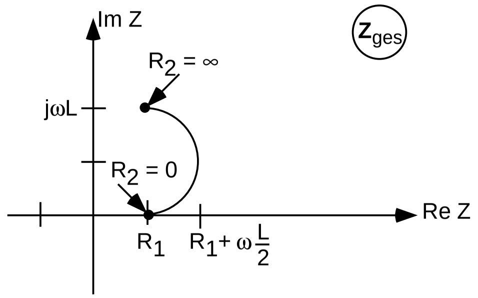
$$Y(R_2 \rightarrow 0) = \infty - j\frac{1}{\omega L} \Rightarrow Z(R_2 \rightarrow 0) = 0$$



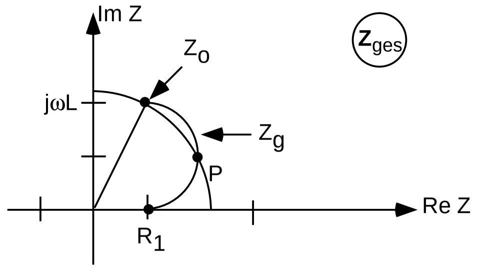
$$Y(R_2 = a) = \frac{1}{a} - j\frac{1}{\omega L} \Rightarrow Z(R_2 = a) = \frac{a\omega L}{\omega L - ja}$$

$$= \frac{a\omega L(\omega L + ja)}{\omega^2L^2 + a^2} \Rightarrow \text{positive reelle und positive imaginŠre Zahl}$$

$Z_{ges} = R_1 + Z_p \rightarrow$ verschieben der Kurve nach rechts



c)



$Z_o = R_1 + j\omega L$

Kreis um den Ursprung mit Radius $|R_1 + j\omega L|$ geschnitten mit $Z_g \Rightarrow P$