Karlsruhe Institute of Technology (KIT)
Institut für Biomedizinische Technik

Prof. Dr. rer. nat. O. Dössel
Kaiserstr. 12 / Geb 30.33
Tel.: 0721 / 608 - 2650

Dipl.-Ing. W. Schulze
Kaiserstr. 12 / Geb 30.33
Tel.: 0721 / 608 - 42751

Bachelorprüfung: Lineare elektrische Netze
02. April 2012

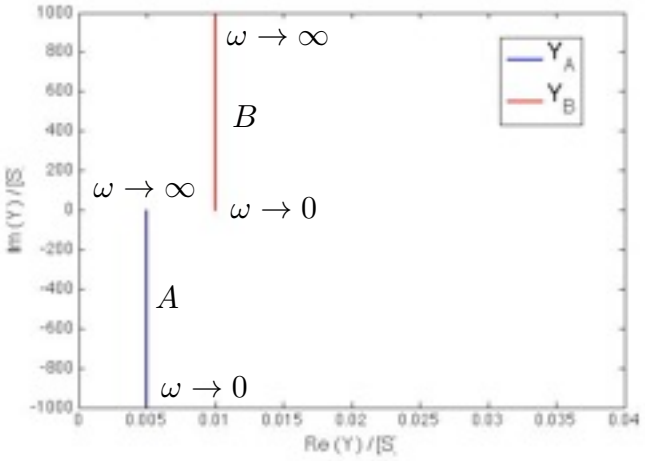
Lösung

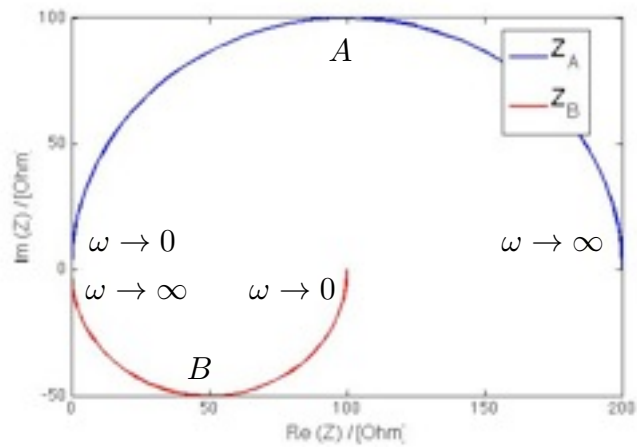
Aufgabe 1 — Ortskurve (21 Punkte)

a) $\underline{Y}_A(\omega) = \frac{1}{2R} + \frac{1}{j\omega L} = \frac{1}{2R} - j\frac{1}{\omega L}$

$$\underline{Z}_A(\omega) = \frac{1}{\frac{1}{2R} + \frac{1}{j\omega L}} = \frac{2R \cdot j\omega L}{2R + j\omega L}$$

b) , e)





c) $\underline{Y}_B(\omega) = \frac{1}{R} + j\omega C$

$$\underline{Z}_B(\omega) = \frac{1}{\frac{1}{R} + j\omega C} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

d) $\underline{Z}_B(\omega = 0) = R$

$\lim_{\omega \to \infty} \underline{Z}_B(\omega) = 0$

e) siehe oben.

f) $\underline{Z}_B(\omega) = \frac{1}{\frac{1}{R} + j\omega C} = \frac{\frac{1}{R} - j\omega C}{\left(\frac{1}{R}\right)^2 + (\omega C)^2} = \frac{R}{1 + \omega^2 R^2 C^2} - j\frac{\omega C}{\left(\frac{1}{R}\right)^2 + (\omega C)^2} = \frac{R}{1 + \omega^2 R^2 C^2} - j\frac{\omega R^2 C}{1 + \omega^2 R^2 C^2}$

g) $\underline{Z}_B(\omega) = \frac{\frac{1}{R} - j\omega C}{\left(\frac{1}{R}\right)^2 + (\omega C)^2} = \frac{\frac{1}{R}}{N} - j\frac{\omega C}{N}$

$$\left(\frac{\frac{1}{R}}{N} - \frac{R}{2}\right)^2 + \left(\frac{\omega C}{N}\right)^2 = \frac{R^2}{4}$$

$$\Leftrightarrow \frac{1}{(RN)^2} - 2\frac{R}{2RN} + \frac{R^2}{4} + \left(\frac{\omega C}{N}\right)^2 = \frac{R^2}{4}$$

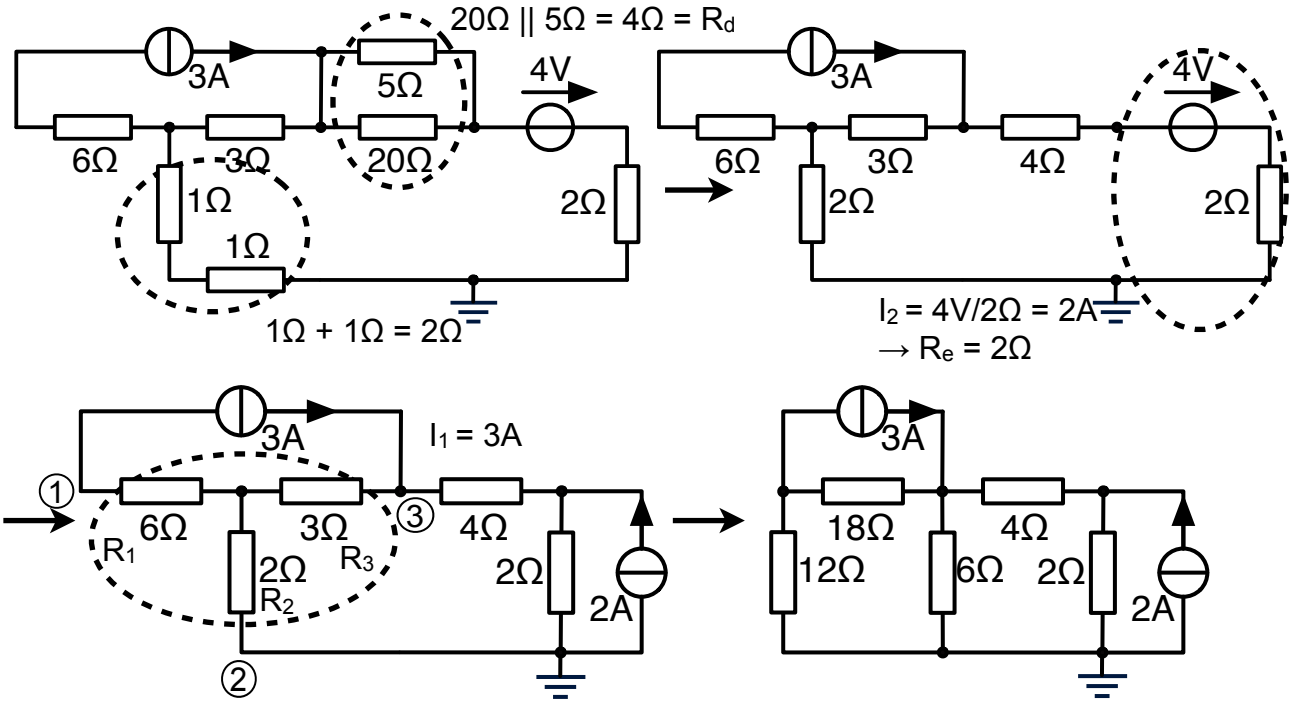
$$\Leftrightarrow \frac{1}{(RN)^2} - \frac{1}{N} + \left(\frac{\omega C}{N}\right)^2 = 0$$

$$\Leftrightarrow \frac{1}{R^2} + (\omega C)^2 = N$$

q.e.d.

Aufgabe 2 — Netzwerk (14 Punkte)

a)



Stern-Dreiecks-Transformation:

$R_{12} = 6\Omega + 2\Omega + (6\Omega \cdot 2\Omega)/3\Omega = 12\Omega = R_a$

$R_{23} = 3\Omega + 2\Omega + (3\Omega \cdot 2\Omega)/6\Omega = 6\Omega = R_c$

$R_{31} = 6\Omega + 3\Omega + (6\Omega \cdot 3\Omega)/2\Omega = 18\Omega = R_b$

$R_d = 4\Omega$ $R_e = 2\Omega$ $I_1 = 3A$ $I_2 = 2A$

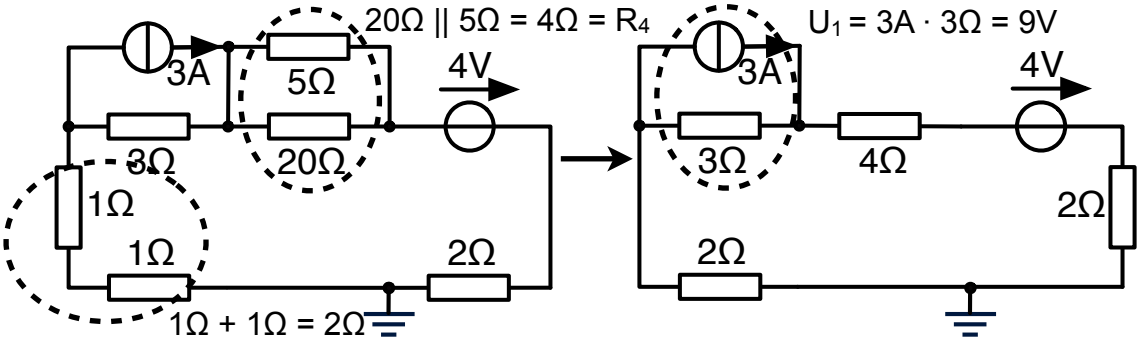
b)

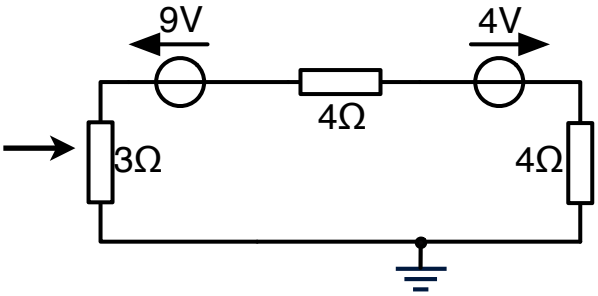
$$(\frac{1}{R_a} + \frac{1}{R_b})V_1 - \frac{1}{R_b}V_2 = -I_1$$

$$-\frac{1}{R_b}V_1 + (\frac{1}{R_b} + \frac{1}{R_c} + \frac{1}{R_d})V_2 - \frac{1}{R_d}V_3 = I_1$$

$$-\frac{1}{R_d}V_2 + (\frac{1}{R_d} + \frac{1}{R_e})V_3 = I_2$$

c)





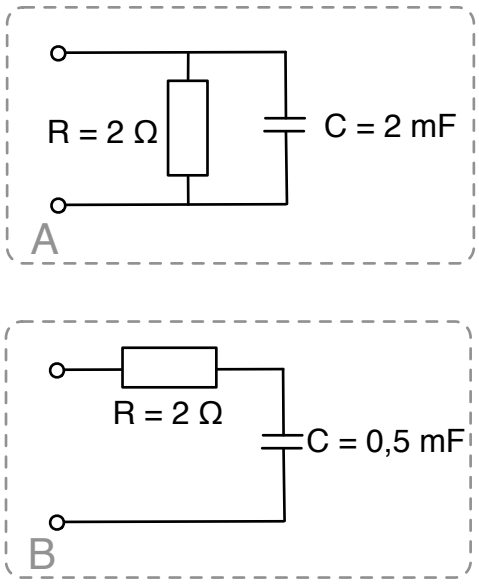
$P = U \cdot I = U^2/R = 25/11W = 2{,}27W$

Aufgabe 3 Zeigerdiagramm (15 Punkte)

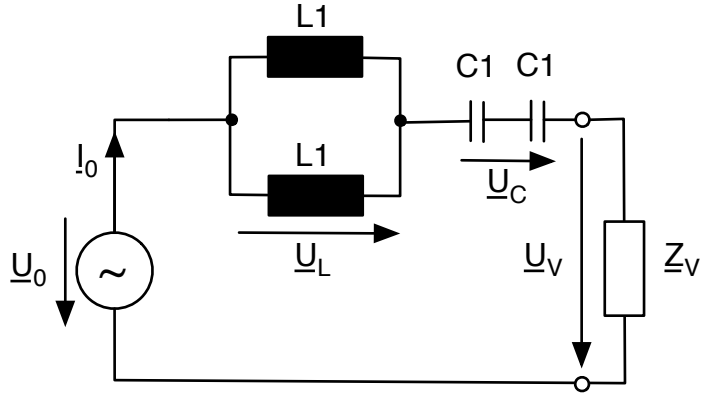
a) $\underline{I}_{res}(\underline{U}_S) = \left(\frac{1}{2\Omega} + j\frac{2}{\Omega}\right)\underline{U}_S = \left(\frac{1}{2}\,\mathrm{S} + \mathrm{j}2\,\mathrm{S}\right)\underline{U}_S$

$\underline{U}_{res}(\underline{I}_0) = (2\Omega - j2\Omega)\cdot\underline{I}_0$

b)

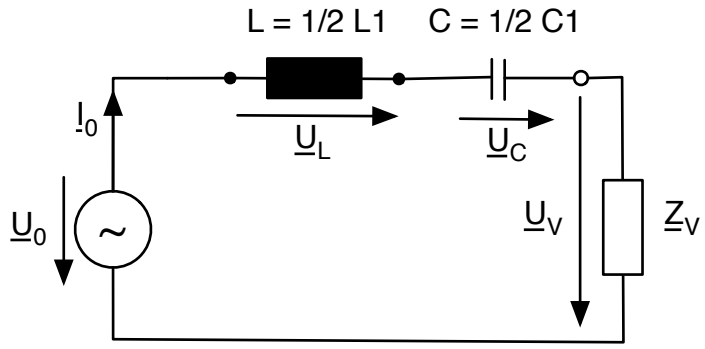


c) Schaltung vereinfachen:



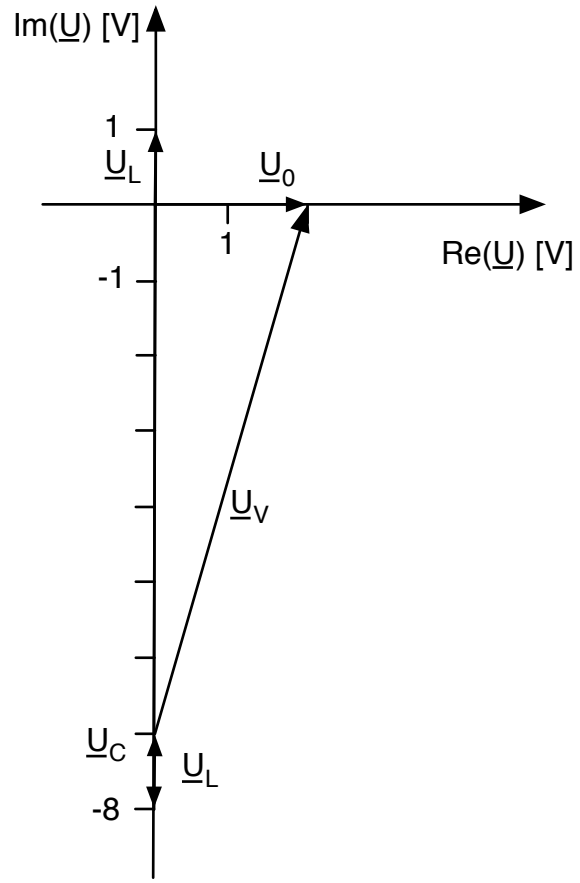
$$\frac{1}{\frac{1}{j\omega \mathrm{L1}} + \frac{1}{j\omega \mathrm{L1}}} = j\frac{1}{2}\omega \mathrm{L1}$$

$$\frac{1}{j\omega \mathrm{C1}} + \frac{1}{j\omega \mathrm{C1}} = \frac{1}{j\omega\frac{1}{2}\mathrm{C1}}$$



$$\underline{U}_L = j\omega L \underline{I}_0 = j\omega \frac{1}{2}\mathrm{L1}\,\underline{\mathrm{I}}_0 = \mathrm{j}\frac{2\pi}{2\pi}\mathrm{kHz\,mH\,A} = \mathrm{j1\,V}$$

$$\underline{U}_C = \frac{1}{j\omega\,\frac{1}{2}\,\mathrm{C1}}\underline{I}_0 = -j\frac{1\mathrm{A}}{2\pi 1\,\mathrm{kHz}\,\frac{1}{2}\,\frac{1}{8\pi}\mathrm{mF}} = -j8\,\mathrm{V}$$



$$\underline{U}_V = (2 + j7)\,\mathrm{V}$$

d) $\underline{Z}_V = \dfrac{\underline{U}_V}{\underline{I}_0} = (2 + j7)\dfrac{\mathrm{V}}{\mathrm{A}} = (2 + j7)\,\Omega$

Aufgabe 4 Bodediagramm (26 Punkte)

a) Schaltung A: nichtinvertierender Spannungsverstärker;
Schaltung B: passiver Tiefpass oder RC-Tiefpass;
Schaltung C: passiver Hochpass oder RC-Hochpass;

b)

$$\frac{\underline{u}_{a,A}}{\underline{u}_{e,A}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{\underline{u}_{a,B}}{\underline{u}_{e,B}} = \frac{\frac{1}{j\omega C_1}}{R_3 + \frac{1}{j\omega C_1}} = \frac{1}{1 + j\omega C_1 R_3}$$

$$\frac{\underline{u}_{a,C}}{\underline{u}_{e,C}} = \frac{R_4}{R_4 + \frac{1}{j\omega C_2}} = \frac{j\omega C_2 R_4}{1 + j\omega C_2 R_4}$$

c)

$$\frac{a_v}{\mathrm{dB}} = 20\log\left|\frac{\underline{u}_a}{\underline{u}_e}\right| \qquad \varphi = \arctan\left(\frac{Im\left(\frac{\underline{u}_a}{\underline{u}_e}\right)}{Re\left(\frac{\underline{u}_a}{\underline{u}_e}\right)}\right)$$

$$\frac{a_{v,A}}{\mathrm{dB}} = 20\log\left|\frac{R_1 + R_2}{R_2}\right| \qquad \varphi_A = \arctan\left|\frac{0}{\frac{R_1+R_2}{R_2}}\right| = 0$$

$$\frac{a_{v,B}}{\mathrm{dB}} = 20\log\left|\frac{1}{1 + j\omega C_1 R_3}\right| = 20\log|1| - 20\log|1 + j\omega C_1 R_3| =$$
$$= -20\log|1 + j\omega C_1 R_3|$$

$$\varphi_B = 0 - \arctan\left(\frac{\omega C_1 R_3}{1}\right) = -\arctan(\omega C_1 R_3)$$

$$\frac{a_{v,C}}{\mathrm{dB}} = 20\log\left|\frac{j\omega C_2 R_4}{1 + j\omega C_2 R_4}\right| = 20\log|j\omega C_2 R_4| - 20\log|1 + j\omega C_2 R_4| =$$
$$= 20\log(\omega C_2 R_4) - 20\log|1 + j\omega C_2 R_4|$$

$$\varphi_B = \arctan\left(\frac{\omega C_2 R_4}{0}\right) - \arctan\left(\frac{\omega C_2 R_4}{1}\right) = 90^\circ - \arctan(\omega C_2 R_4)$$

d) Normierung Schaltung C:

$$\Omega = \frac{\omega}{\omega_c} \quad \omega_c = \frac{1}{R_4 C_2} \Rightarrow f_c = \frac{1}{2\pi R_4 C_2} = 1kHz \quad \Rightarrow \frac{\underline{u}_{a,C}}{\underline{u}_{e,C}} = \frac{j\Omega}{1+j\Omega}$$

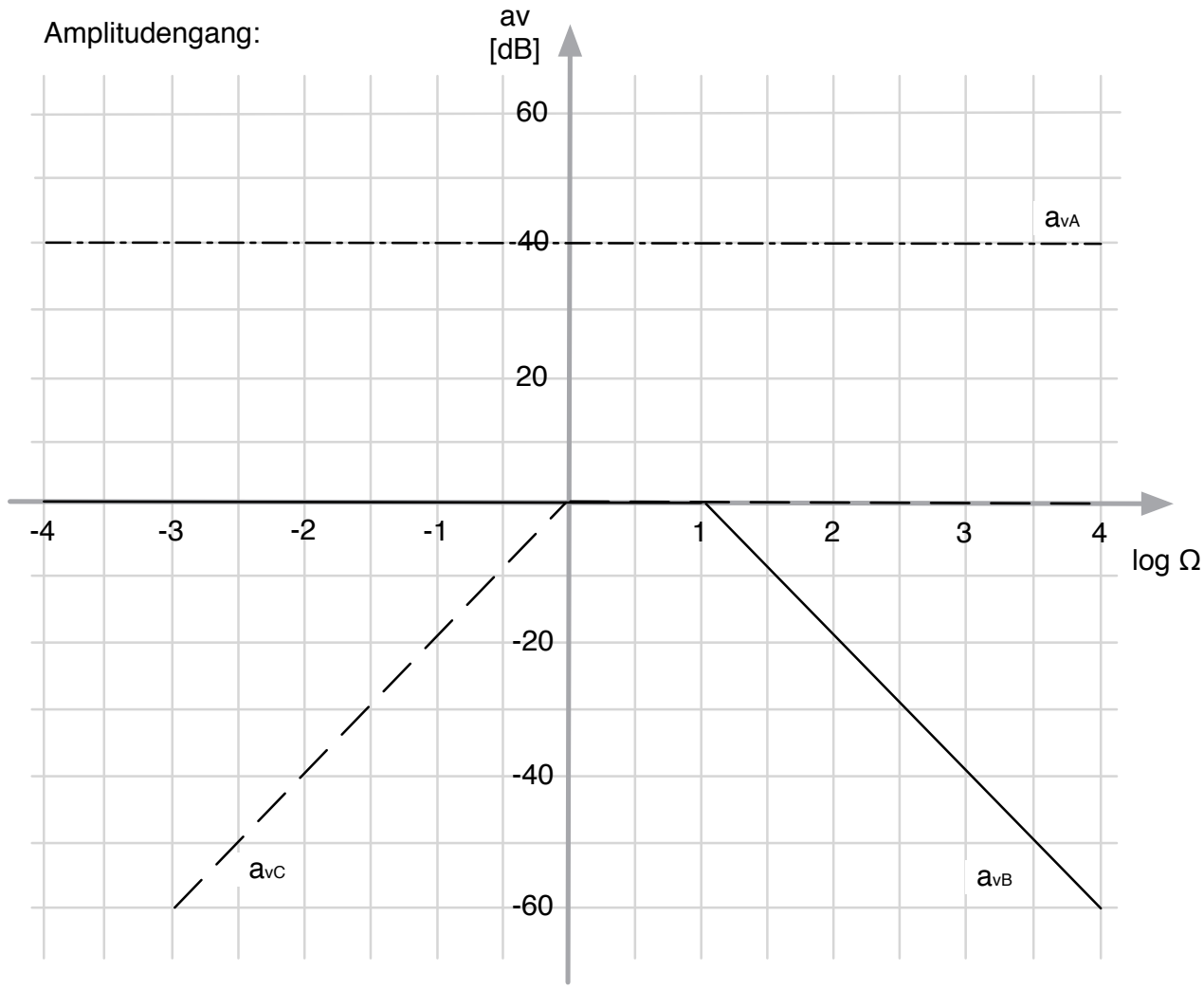
e) Normierung Schaltung B:

$$\Omega = \frac{\omega}{\omega_c} \quad \omega_B = \frac{1}{R_3 C_1} \Rightarrow f_B = \frac{1}{2\pi R_3 C_1} = 10kHz \Rightarrow \frac{\underline{u}_{a,B}}{\underline{u}_{c,B}} = \frac{1}{1+j10^{-1}\Omega}$$

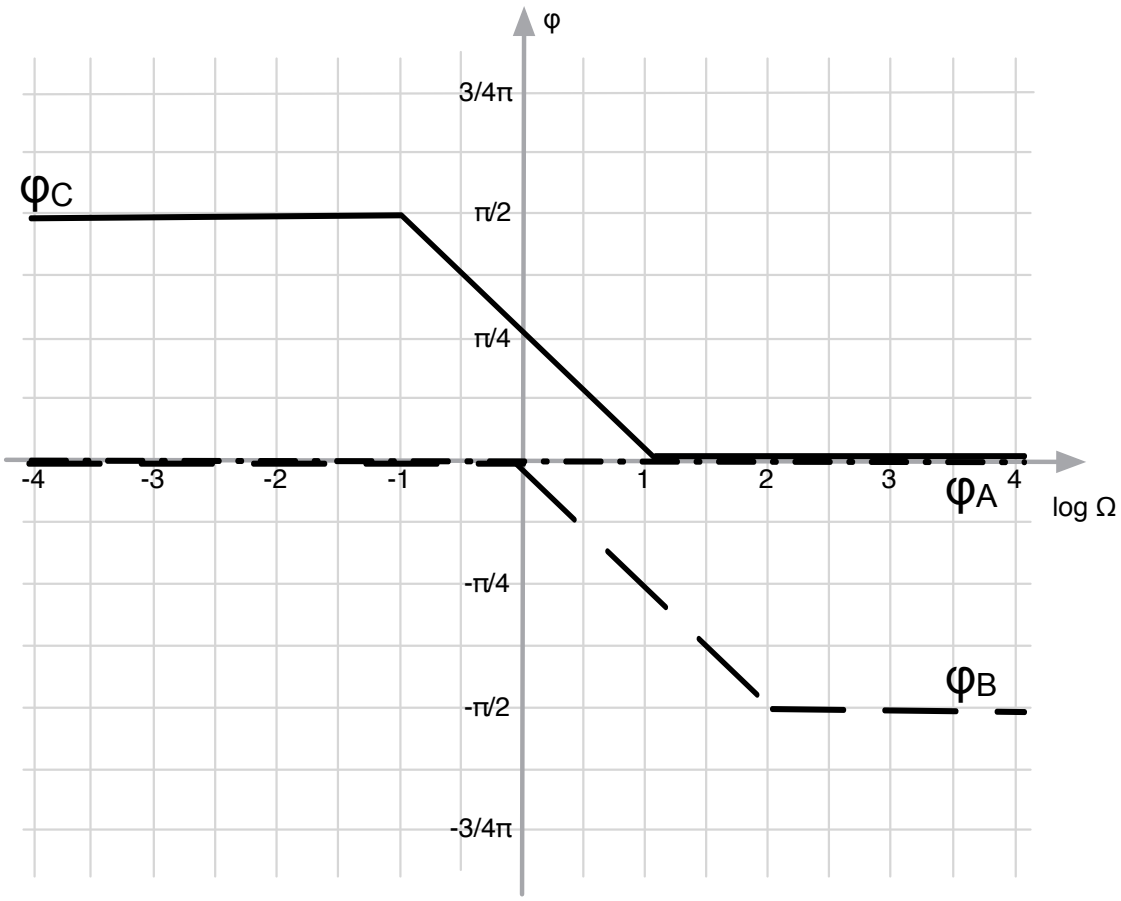
Verschiebung um 1 Dekade $\Rightarrow \omega_B = 10\omega_c$

f)

Amplitudengang:

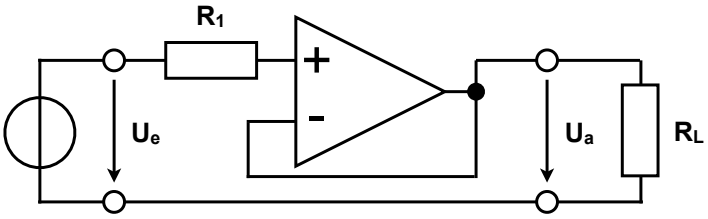


$$\frac{a_{v,A}}{\mathrm{dB}} = 20\log\left|\frac{100k\Omega + 1k\Omega}{1k\Omega}\right| \approx 40$$


φ
3/4π
φC
π/2
π/4
-4
-3
-2
-1
1
2
3
4
φA
log Ω
-π/4
φB
-π/2
-3/4π

Aufgabe 5 Operationsverstärker (14 Punkte)

a)



b) Durch die Betriebsspannung. Die Ausgangsspannung kann nicht größer sein als die Betriebsspannung.

c) Invertierender Addierer

d) Knotengleichung

$(U_{e1}-U_n)/R_1 + (U_{e2}-U_n)/R_2 + (U_a-U_n)/R_3 = 0 \qquad U_n \approx 0$

$\rightarrow -U_a = R_3/R_1 * U_{e1} + R_3/R_2 * U_{e2}$

e) $R_3/R_1 = 2$

$R_3/R_2 = 3$

$(R_3/R_2)/(R_3/R_1) = R_1/R_2 = 3/2$

f) $U_{e1} = - R_4 / R_5 * U_{e1}'$

Gesamübertragungsfunktion:

$U_a = 6*U_{e1} - 3*U_{e2}$

Daraus folgt die Bedingung:

$- R_4 / R_5 = -3$

$R_4 = 3* R_5$