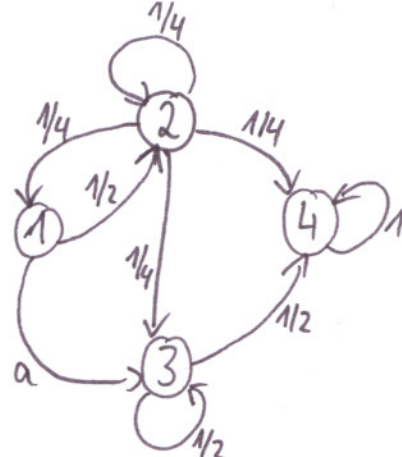


Aufgabe 37

a)
$$\underline{P} = \begin{bmatrix} 0 & \frac{1}{2} & a & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} = 1 \\ = 1 \\ = 1 \\ = 1 \end{matrix} \rightarrow a = \frac{1}{2}$$



b) ges.: $\vec{p}^T(3)$

$\vec{p}^T(3) = \vec{p}^T(0) \cdot \underline{P}^3$ mit $\vec{p}^T(0) = (1, 0, 0, 0)$ (Anfangszustand ist 1!)

$\underline{P}^3 = \underline{P}^2 \cdot \underline{P}$

$$\underline{P}^2 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{16} & \frac{3}{16} & \frac{5}{16} & \frac{7}{16} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{P}^3 = \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{16} & \frac{3}{16} & \frac{5}{16} & \frac{7}{16} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{32} & \frac{3}{32} & \frac{9}{32} & \frac{19}{32} \\ \frac{3}{64} & \frac{5}{64} & \frac{15}{64} & \frac{41}{64} \\ 0 & 0 & \frac{1}{8} & \frac{7}{8} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \vec{p}^T(3) = (1, 0, 0, 0) \cdot \underline{P}^3 = \left(\frac{1}{32}, \frac{3}{32}, \frac{9}{32}, \frac{19}{32} \right)$

$\Rightarrow$ Zustand 4 $\left(\frac{19}{32} \right)$ ist nach drei Schritten am wahrscheinlichsten!

c) ges.: ~~Mittel~~ Dauer bis zur Absorption in 4

allg.: $m_i = 1 + \sum_{k=1}^N p_{ik} \cdot m_k \rightarrow \text{hier: } N=4, i=1,2,3,4$

$m_1 = 1 + p_{11} \cdot m_1 + p_{12} \cdot m_2 + p_{13} \cdot m_3 + p_{14} \cdot m_4 = 1 + \frac{1}{2} m_2 + \frac{1}{2} m_3 \quad (1)$

$m_2 = 1 + p_{21} \cdot m_1 + p_{22} \cdot m_2 + p_{23} \cdot m_3 + p_{24} \cdot m_4 = 1 + \frac{1}{4} m_1 + \frac{1}{4} m_2 + \frac{1}{4} m_3 + \frac{1}{4} m_4 \quad (2)$

$m_3 = 1 + p_{31} \cdot m_1 + p_{32} \cdot m_2 + p_{33} \cdot m_3 + p_{34} \cdot m_4 = 1 + \frac{1}{2} m_3 + \frac{1}{2} m_4 \quad (3)$

$m_4 = 0$ "absorbierender Zustand" (4)

$$\underline{\text{in (3)}}: m_3 = 2$$

$$\underline{\text{in (2)}}: m_2 = 1 + \frac{1}{4} m_1 + \frac{1}{4} m_2 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 0$$

$$\Leftrightarrow \frac{3}{4} m_2 = \frac{3}{2} + \frac{1}{4} m_1$$

$$\Leftrightarrow m_2 = 2 + \frac{1}{3} m_1$$

$$\underline{\text{in (1)}}: m_1 = 1 + \frac{1}{2} \cdot \left(2 + \frac{1}{3} m_1\right) + \frac{1}{2} \cdot 2$$

$$= 1 + 1 + \frac{1}{6} m_1 + 1$$

$$= 3 + \frac{1}{6} m_1$$

$$\Leftrightarrow \frac{5}{6} m_1 = 3$$

$$\Leftrightarrow m_1 = \frac{18}{5} = \underline{\underline{3,6}}$$