X-Ray Imaging

Photoelectric-effect:

Probability of photoelectric interactions:

$\frac{\rho Z^3}{E^3}$

ρ: mass density
Z: atomic number
E: photon energy

Anti-scatter grid

Grid Ratio (GR): $GR = \frac{h}{D+t}$

h: height of lead strips
D: distance between lead strips (interstrip distance)
t: thickness of interstrip material

Lambert-Beer Law: $I(x) = I_0 e^{-\mu x}$

$I(x)$: Intensity of the x-Ray after passing through material of thickness x
I_0: Initial intensity of the x-Ray
μ: linear attenuation (weakening) coefficient
x: thickness of the material the beam passes through

$A \hat{=} n_p$

A: tube current
n_p: number of photons

CT Technology

Hounsfield Unit (UN): $HU = 1000 \cdot \left(\frac{\mu - \mu_{water}}{\mu_{water}}\right)$

μ: Linear weakening coefficient of the tissue
μ_{water}: Linear weakening coefficient of water
HU: density (-1000 → air ; 1000 → Metal)

Spiral mode pitch: $p = \frac{d}{s}$

d: table distance travel per 360° gantry rotation
s: slice thickness

Forward projection:

$$p_\theta(r) = -\ln\frac{I_\theta(r)}{I_0} = \int_{L_{r,\theta}} \mu(r\cdot\cos(\theta) - s\cdot\sin(\theta), r\cdot\sin(\theta) + s\cdot\cos(\theta))\,ds$$

$$p_\theta(r,\theta) = R\{f(x,y)\} = \int_{-\infty}^{\infty} f(r\cdot\cos(\theta) - s\cdot\sin(\theta), r\cdot\sin(\theta) + s\cdot\cos(\theta))\,ds$$

$p_\theta(r)$: Projection of $\mu(x,y)$ along the angle θ
$p_\theta(r,\theta)$: Radon transformation
$\mu(x,y)$: X-Ray absorption at point (x,y)
r: radius
θ: angle

Backward projection:

$$b(x,y) = B\{p(r,\theta)\} = \int_0^{2\pi} p(r,\theta)\,d\theta = \int_0^{2\pi} p(x\cdot\cos(\theta) + y\cdot\sin(\theta), \theta)\,d\theta$$

Diskret:

$$b(x_i,y_j) = B\{p(r_n,\theta_m)\} = \sum_{m=1}^{M} p(x_i\cdot\cos(\theta_m) + y_j\cdot\sin(\theta_m), \theta_m)\,\Delta\theta$$

Ultrasound Imaging

Acoustic Impedance (Z): $Z = \rho c$

ρ: Density of the medium
c: speed of sound in the medium

Reflection Coefficient (R) for interfaces: $R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$

Z_1, Z_2: Acoustic impedances of two media

Near Field Depth (NFD) or Fresnel Zone: $L = \frac{d^2}{4\lambda} = \frac{d^2 f}{4c}$

L: Distance from transducer to the end of the near field
d: Diameter of the transducer
λ: Wavelength of the transducer
f: Frequency of the ultrasound wave
c: Speed of sound in the medium

$f = \frac{c}{\lambda} \rightarrow c = f\cdot\lambda$

λ: Wavelength
f: frequency
c: speed

Noise and Resolution in Imaging

μ: $mean = \frac{\Sigma \text{Werte}}{\text{Anzahl der Werte}}$

σ: $std = \sqrt{\frac{\Sigma((\text{Wert} - \text{Mean})^2)}{\text{Anzahl der Werte} - 1}}$

$contrast = \frac{\text{Mean(Object)} - \text{Mean(Background)}}{\text{Mean(Background)}}$

Signal-Noise-Ratio: $SNR = \frac{\text{Mean(Object)}}{\text{std (Background)}}$

$SNR = \frac{N}{\sqrt{N}} = \sqrt{N}$ N = Average number of Photons per Pixel

Contrast-Noise-Ratio: $CNR = \frac{\text{Mean(Object)} - \text{Mean(Background)}}{\text{std(Object)}}$

Fourier Transform in Imaging

Fourier Transform (FT): $F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x}\,dx$

$$F(k_x,k_y) = \int_{-\infty}^{\infty} f(x,y) e^{-2\pi i(k_x x + k_y y)}\,dx\,dy$$

Inverse Fourier Transform (IFT): $f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x}\,dk$

F(k): Fourier transform of f(x)
f(x): Spatial domain function
k: Spatial frequency