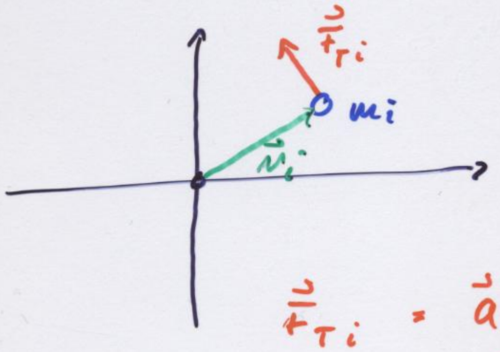
Betrachte Massepunkt m_i



$$\vec{F}_{Ti} = \vec{a}_T \cdot m_i$$

$$\Rightarrow \quad a_T = \frac{F_{Ti}}{m_i} = \alpha \cdot r_i$$

Drehmoment: $M_i = r_i F_i$

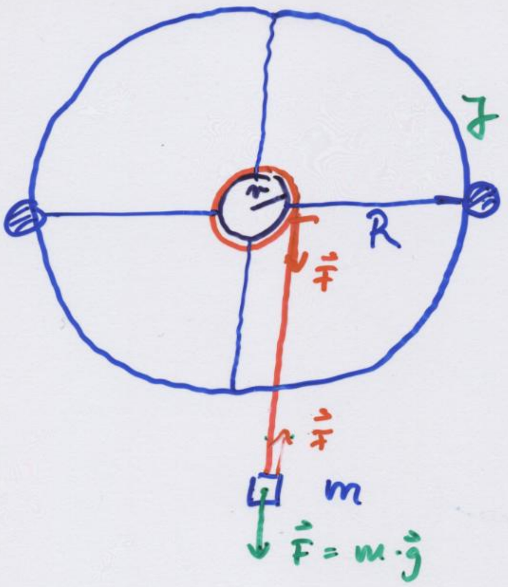
$$= \underbrace{m_i r_i \; r_i}_{J_i} \alpha$$

[engl: torque]

$$= J_i \; \alpha$$

Allgemein: $\vec{M} = J \cdot \vec{\alpha}$

Demo



$$\left[\begin{array}{l} \vec{M} = \vec{r} \times \vec{F} = J \cdot \vec{\alpha} \\ m \cdot \vec{g} - \vec{F} = m \cdot \vec{a} \end{array}\right.$$

In Komponenten: $M = r \cdot F = J \cdot \alpha$

$$mg - F = m \cdot a$$

$$\Rightarrow \quad mg - \frac{J}{r^2} a = m \cdot a$$

$$\left[\alpha = \frac{a}{r} \right]$$

$$\Rightarrow \quad a = \frac{g}{1 + J/mr^2}$$

Überprüfung des Ergebnisses :

$m \to \infty \Rightarrow a \to g$ ✓

$m \to 0 \Rightarrow a \to 0$ ✓

$J \to \infty \Rightarrow a \to 0$ ✓

$J \to 0 \Rightarrow a \to g$ ✓

Bewegungsgleichung :

$$\alpha = \frac{g/r}{1 + J/mr^2}$$

für $J \gg mr^2$: $\alpha \approx \frac{m \cdot g \cdot r}{J}$

$$\theta(t) = \frac{1}{2} \alpha t^2 = \frac{1}{2} \frac{mgr}{J} t^2$$

Demo : für $m = 0{,}5$ kg :

2 Umdrehungen $\hat{=}$ 8 s

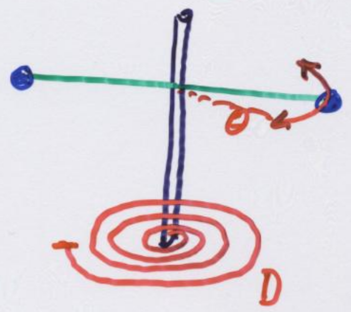
für $m = 1$ kg :

4 Umdrehungen $\hat{=}$ 7,8 s

$(\theta \sim m)$

$\sim \vec{M}$

Anwendung: Drehschwingungen



Federkraft $\Rightarrow$ $M = D \cdot \theta$

$$= J\,\alpha$$

$$= J \cdot \frac{d^2\theta}{dt^2}$$

$$J \cdot \frac{d^2\theta}{dt^2} = -D \cdot \theta$$

Ansatz: $\theta(t) = \theta_0 \cos(\omega t + \phi)$

$$\Rightarrow \quad \omega = \sqrt{\frac{D}{J}}$$

$$T = 2\pi \sqrt{\frac{J}{D}}$$

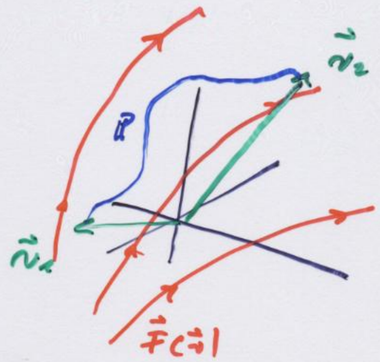
Bsp: $J = 2 m r^2$

$$\Rightarrow \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{25}{46} \approx \frac{1}{2}$$

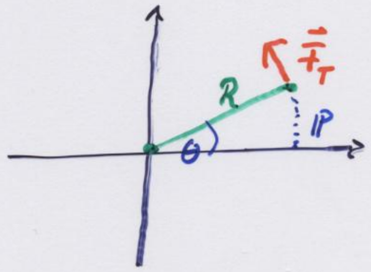
4] Arbeit, Energie

Lineare Dynamik:

$$A = \int_{\mathbb{P}\,\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{s}$$



Rotation:



$$A = \int F_T \cdot R\, d\theta$$

$$= \int \vec{M}\, d\vec{\theta}$$

Rotationsarbeit

Für beschleunigte Rotation:

$$\vec{M} = J \cdot \vec{\alpha} \neq 0$$

$$A = \int_{\theta_1}^{\theta_2} \vec{M}\, d\vec{\theta} = \int_{\theta_1}^{\theta_2} J \vec{\alpha}\, d\vec{\theta} = \int_{\theta_1}^{\theta_2} J \frac{d\vec{\omega}}{dt}\, d\vec{\theta}$$

$$\ldots = \int_{\omega_1}^{\omega_2} J \cdot \vec{\omega}\, d\vec{\omega}$$

$$= \frac{1}{2} J \omega_2^2 - \frac{1}{2} J \omega_1^2$$

$$= \Delta E_{rot}$$

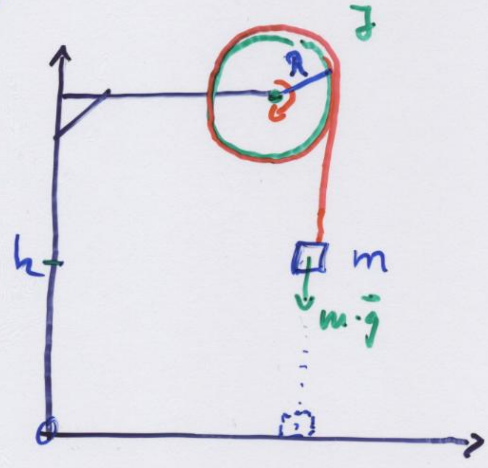
Allgemein: Energieerhaltungssatz:

$$E_{tot} = E_P + E_K + E_R \; (+ E_{inr})$$

$$= \text{const.}$$

Anwendung:

1)



a). $E_{tot} = m \cdot g \cdot h$

$$= 0 + \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2$$

mit $\omega = \frac{v}{R}$:

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} J \frac{v^2}{R^2}$$

$$\Rightarrow v = \sqrt{\frac{2 \cdot m \cdot g \cdot h}{m + J/R^2}}$$

Test: $m \to \infty$: $v \to \sqrt{2gh}$ ✓

Fallgeschwind.

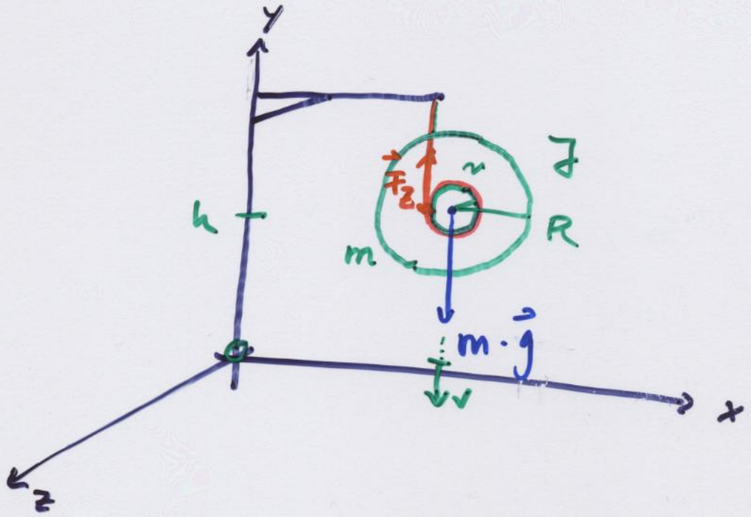
$m \to 0$: $v \to 0$ ✓

$J \to 0$: $v \to \sqrt{2gh}$ ✓

$J \to \infty$: $v \to 0$ ✓

$m \to 0,\ J \to 0$: $v \to \sqrt{2gh}$ ✓

2) Yoyo



Energien:

$$E_{tot} = m \cdot g \cdot h$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2$$

$$\Rightarrow v = \sqrt{\frac{2gh}{1 + J/r^2 m}} \quad ✓$$

Kräfte:

$$\sum \vec{M}_i = \vec{r} \times \vec{F}_z = J \cdot \vec{\alpha}$$

$$\sum \vec{F}_i = m\vec{g} + \vec{F}_z = m\,\vec{a}$$

y-Richtung: $\quad -mg + F_z = -ma$

z-Richtung $\quad -r \cdot F_z = -J \cdot \alpha$

$\Rightarrow F_z = \frac{J \cdot a}{r^2} \quad ; \quad a = g / 1 + J/mr^2$

Beispiele: Kugel: $\varkappa = \frac{2}{5} \Rightarrow v_E = \sqrt{\frac{5}{7}}\sqrt{2gh}$

Zylinder: $\varkappa = \frac{1}{2} \Rightarrow \sqrt{\frac{2}{3}}\sqrt{2gh}$

Hohlzylinder $\varkappa = 1 \Rightarrow \sqrt{\frac{1}{2}}\sqrt{2gh}$

Rutschender Massepunkt:

$\varkappa = 0 \rightarrow v_E = \sqrt{2gh}$

4] Bowling / Kegeln

Kugel rutscht

$[\omega_0 = 0]$

a) $E_{tot} = \frac{1}{2} m v_0^2$

$= \frac{1}{2} m v_1^2 + \frac{1}{2} J \omega_1^2 + E_{int}$

b) $F_g = ma = m \cdot \frac{v_1 - v_0}{\Delta t}$

$M = r \cdot F_g = J \cdot \alpha = J \cdot \frac{\omega_1 - \omega_0}{\Delta t}$