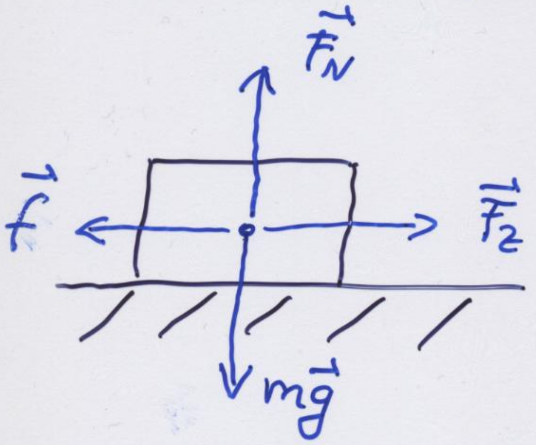
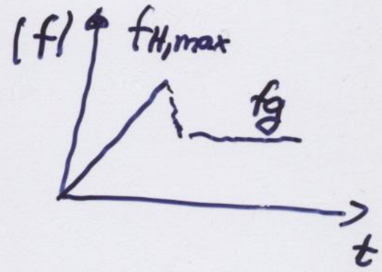
2.2.3 Reibung



$|\vec{f}| \sim |\vec{F}_N|$



a) Haftreibung:

$$\vec{F}_N + \vec{F}_Z + m\vec{g} + \vec{f}_H = \vec{0} \quad (|\vec{F}| < |f_{H,max}|)$$

b) Gleitreibung:

$$\vec{F}_N + \vec{F}_Z + m\vec{g} + \vec{f}_G = m\vec{a} \quad (|\vec{F}| \geq |\vec{f}_G|)$$

Sonderfall: $|\vec{a}| = 0 \quad (v = \text{konst.})$

$$\vec{F}_N + \vec{F}_Z + m\vec{g} + \vec{f}_G = \vec{0}$$

$$\vec{F}_Z = -\vec{f}_G$$

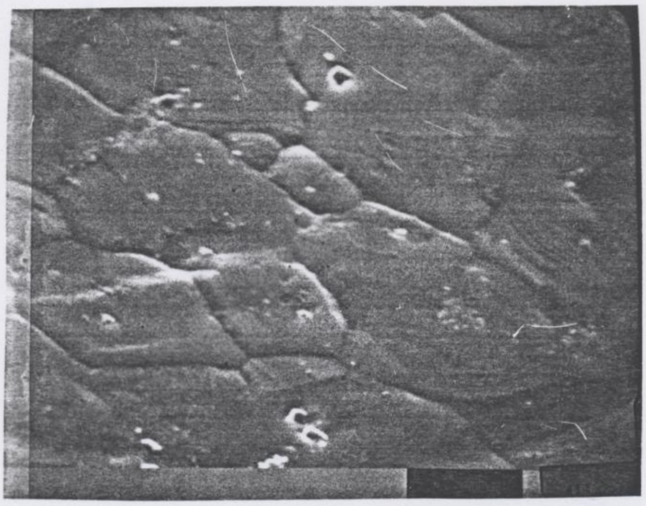
Gleiten mit konst. Geschwindigkeit

Electron microscope photography of inox samples

→| 4 μm |←

surface of cathode

(not cleaned)



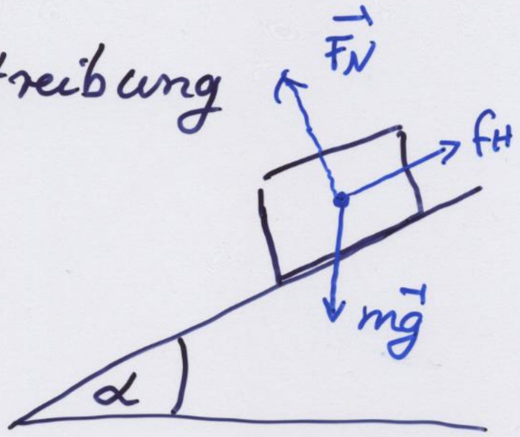
→| 4 μm |←

surface of anode

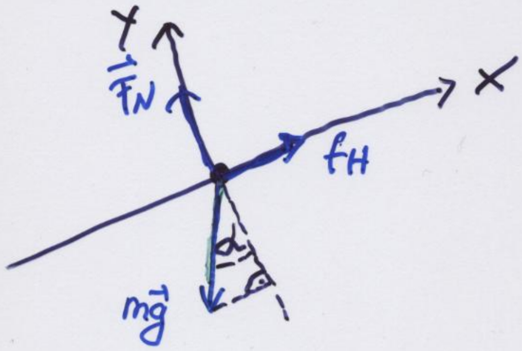
(not cleaned)

Beispiele

a) Haftreibung



Klotz haftet: $\vec{F_N} + m\vec{g} + \vec{f_H} = \vec{0}$



x: $-mg \cdot \sin\alpha + f_H = 0$

y: $-mg \cdot \cos\alpha + F_N = 0$

$\Rightarrow \quad f_H = mg \cdot \sin\alpha$

$F_N = mg \cdot \cos\alpha$

$\underline{f_H = \mu_H \cdot F_N}$ $\quad$ μ_H: Haftreibungs-koeff.

$$\mu_H \cdot mg \cdot \cos\alpha = mg \cdot \sin\alpha$$

$$\Rightarrow \quad \mu_H = \tan\alpha$$

α: maximal möglicher Winkel ohne Gleiten

Demo

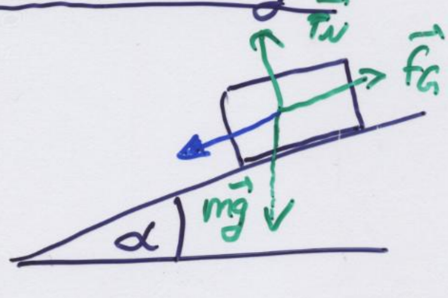
1. Holz auf Holz

   $\alpha \approx 30°$

2. Schmirgelpapier auf Holz

   $\alpha \approx 35°$

b) Gleitreibung



Klotz gleitet

$$\vec{F_N} + \vec{F_G} + m\vec{g} = m\vec{a} \qquad \vec{a} = \begin{pmatrix} a_x \\ 0 \end{pmatrix}$$

$$x: \quad -mg \cdot \sin\alpha + f_G = m \cdot a_x$$

$$y: \quad -mg \cdot \cos\alpha + F_N = 0$$

$$f_G = \mu_G \cdot F_N$$

μ_G: Gleitreibungs-koeff.

$$\Rightarrow$$

$$-mg \cdot \sin\alpha + \mu_G \cdot mg \cdot \cos\alpha = m \cdot a_x$$

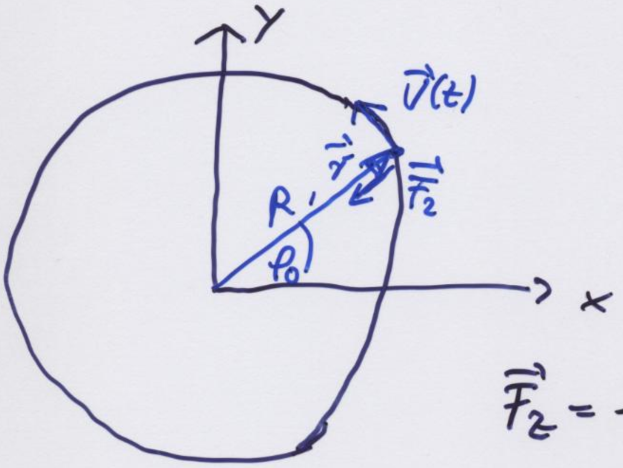
$$\Leftrightarrow a_x = -g \left(\sin\alpha - \mu_G \cdot \cos\alpha \right)$$

$a_x = 0$ wenn $\mu_G = \tan\alpha$

$a_x > 0$ $\mu_G > \tan\alpha$

$a_x < 0$ $\mu_G < \tan\alpha$

2.2.4 Drehbewegungen – Rotationsdynamik



Zentripetalkraft:

$$\vec{F_z} = m \cdot \vec{a_z}$$

$$\Rightarrow \vec{a_z} = \frac{\vec{F_z}}{m}$$

$$\vec{F_z} = -m\omega^2 R \cdot \vec{e_r}$$

$$\omega = \frac{d\varphi}{dt}$$ Winkelgeschwindigkeit

Gleichförmige Kreisbewegung:

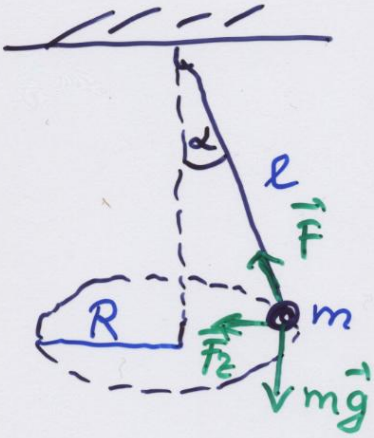
$$\vec{a_z}(t) = -\omega^2 R\,\vec{e_r} = -\omega^2 R \cdot \begin{pmatrix} \cos(\omega t + \phi_0) \\ \sin(\omega t + \phi_0) \end{pmatrix}$$

$$\vec{r}(t) = R \cdot \begin{pmatrix} \cos\left(\sqrt{\frac{F_z}{mR}} \cdot t + \phi_0\right) \\ \sin\left(\sqrt{\frac{F_z}{mR}} \cdot t + \phi_0\right) \end{pmatrix}$$

$$\vec{v}(t) = \omega \cdot R \cdot \begin{pmatrix} -\sin(\omega t + \phi_0) \\ \cos(\omega t + \phi_0) \end{pmatrix} = \omega R \cdot \vec{e_\varphi}$$

Beispiele

a) Drehpendel



$$m\vec{g} + \vec{F} = m\vec{a_z}$$

$$y: \quad F \cdot \cos\alpha = mg$$

$$x: \quad F \cdot \sin\alpha = m \cdot a_z$$

$$a_z = g \cdot \tan\alpha = \omega^2 \cdot R = \omega^2 l \cdot \sin\alpha$$

$$\Rightarrow \omega = \sqrt{\frac{g}{l \cdot \cos\alpha}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l \cdot \cos\alpha}{g}}$$

$$\alpha = 90^\circ \quad \Rightarrow T = 0$$

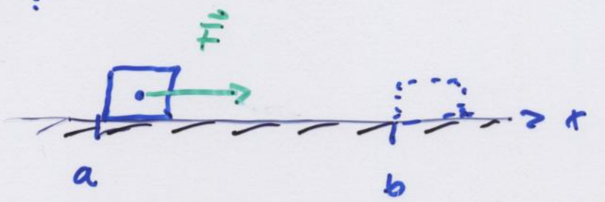
$$\alpha = 0^\circ \quad \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$

Periode des klass. Pendels

2.2.5 Arbeit und Energie

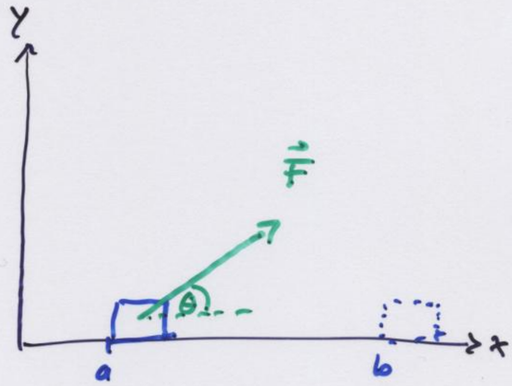
„Arbeit = Kraft × Weg"

Einfachster Fall:



$$A = F \cdot (b - a)$$

Allgemeiner:



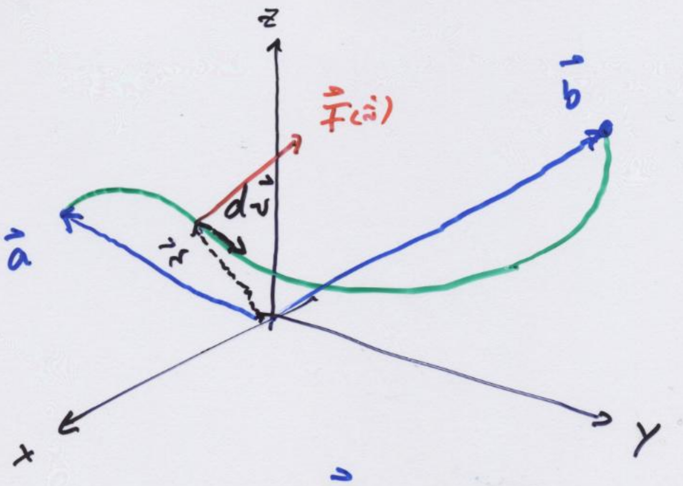
$$A = \vec{F} \cdot (\vec{b} - \vec{a})$$

$$= F_x \cdot (b_x - a_x)$$

$$= |\vec{F}| \cdot |\vec{b} - \vec{a}| \cdot \cos\theta$$

Allgemein: $A_{\vec{a} \to \vec{b}} = \sum_i \Delta A_i = \sum_i \vec{F}_i \cdot \Delta \vec{r}_i$

$$\longrightarrow \int_{\vec{a},\, P}^{\vec{b}} \vec{F}(\vec{r})\, d\vec{r}$$



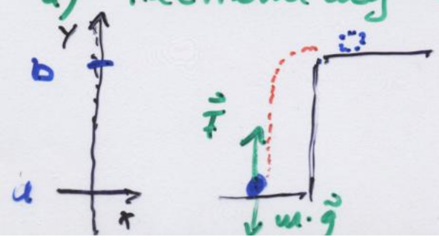
$$A_{\vec{a}\to\vec{b}} = \int_{\vec{a},\,P}^{\vec{b}} (F_x\,dx + F_y\,dy + F_z\,dz)$$

$$[A] = kg\,\frac{m}{s^2}\cdot m$$
$$= N\,m$$
$$= \text{Joule}$$
$$= W\,s$$

Beispiele

1) Arbeit in 1d

a) Klettern auf eine Stufe (v = cons)



$$\vec{F} + m\vec{g} = 0$$
(v = const)

$$A = \vec{F} \cdot (\vec{b} - \vec{a})$$

$$= \begin{pmatrix} 0 \\ mg \end{pmatrix} \begin{pmatrix} 0 \\ b-a \end{pmatrix}$$

$$= m \cdot g \, (b-a) \qquad \hat{=} \; m \cdot g \cdot h$$

Arbeit eines Studenten:

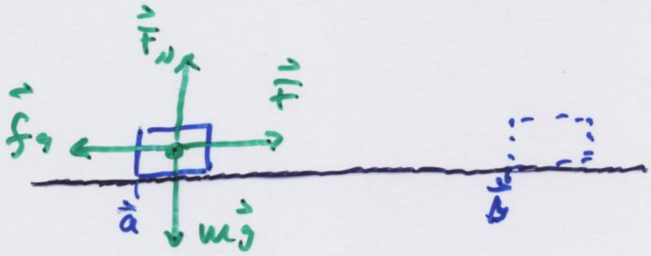
$h = b - a = 0{,}5\,m$

$g = 9{,}8 \, \frac{m}{s^2}$

$m = 80 \, kg$

$A = 392 \, kg \, \frac{m^2}{s^2} = 392 \, Ws$

b) Schieben mit Reibung $\quad (v = const)$



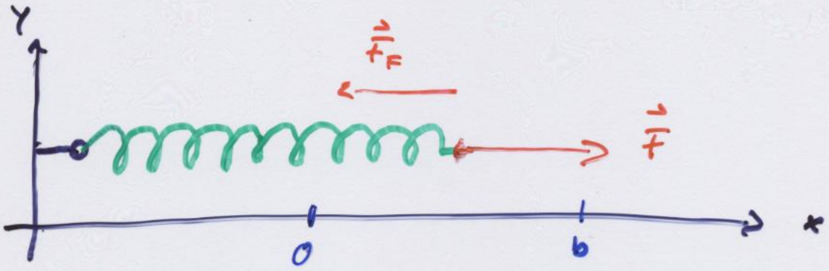
$$\vec{F}_N + m\vec{g} + \vec{f}_g + \vec{F} = 0$$

$$A = \vec{F} \cdot (\vec{b} - \vec{a})$$

$$= f_g \, (b-a)$$

$$= \mu_g \cdot m \cdot g \, (b-a)$$

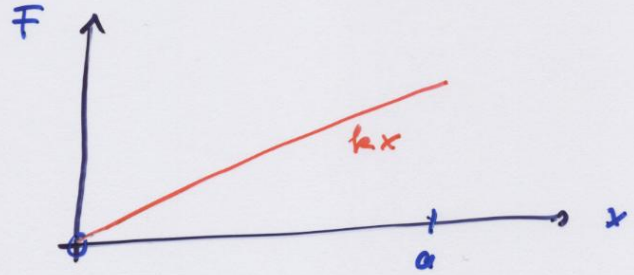
c) Dehnung einer Feder ($v = const$)

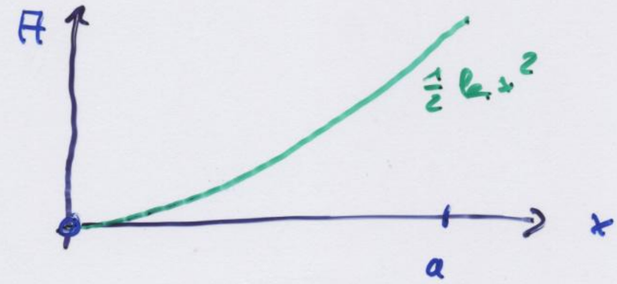


$$\vec{F} + \vec{F}_F = 0$$

$$F - k \cdot x = 0$$

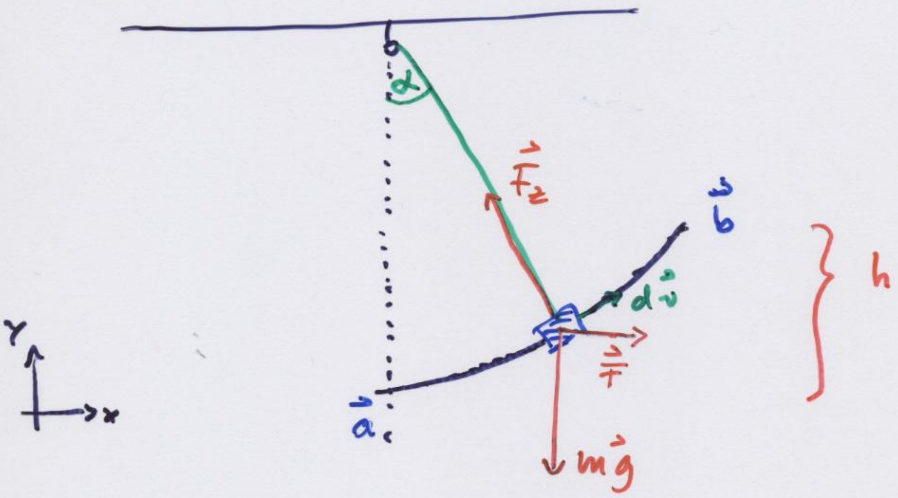
$$A = \int_0^b k x \, dx = \frac{1}{2} k b^2$$





2. Arbeit in 2d, 3d

Beispiel: Schiebe Schaukel an ($v = \text{const}$)



$$\vec{F}_z + \vec{F} + m\vec{g} = 0$$

$$x: \quad F - F_z \sin\alpha = 0$$

$$y: \quad F_z \cos\alpha - mg = 0$$

$$\Rightarrow F = F_z \sin\alpha$$
$$= mg \tan\alpha$$

Arbeit: $A = \int_{\vec{a}}^{\vec{b}} \vec{F}(\vec{r})\, d\vec{r}$

$$= \int_{\vec{a}}^{\vec{b}} m \cdot g \cdot \tan\alpha \, dx$$

substituiere $\tan\alpha = \dfrac{dy}{dx}$

$$A = \int_{\vec{a}}^{\vec{b}} mg \frac{dy}{dx}\, dx = \int_{a_y}^{b_y} mg\, dy$$

$$= mg\,(b_y - a_y)$$

$$= mgh$$