6) Energiesatz der Mechanik

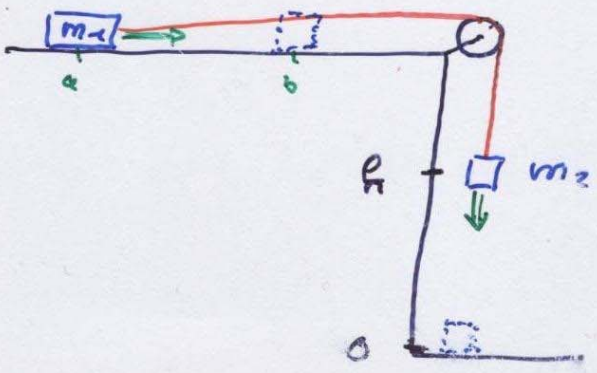
In einem konservativen Kraftfeld:

$$E_{tot} = E_K(\vec{a}) + E_p(\vec{a})$$
$$= E_K(\vec{b}) + E_p(\vec{b}) = \text{const}$$

$$\hat{=} \quad \Delta E_{tot} = \Delta E_K + \Delta E_p = 0$$

Allgemein: In einem geschlossenen System ist die Summe aller Energieformen konstant $\Leftrightarrow$ Energie ist erhalten

b) Demo: Beschleunigung eines Gleiters



$$E_{tot\,a} = E_{pa} + E_{ka}$$
$$= m_2 g \cdot h + 0$$

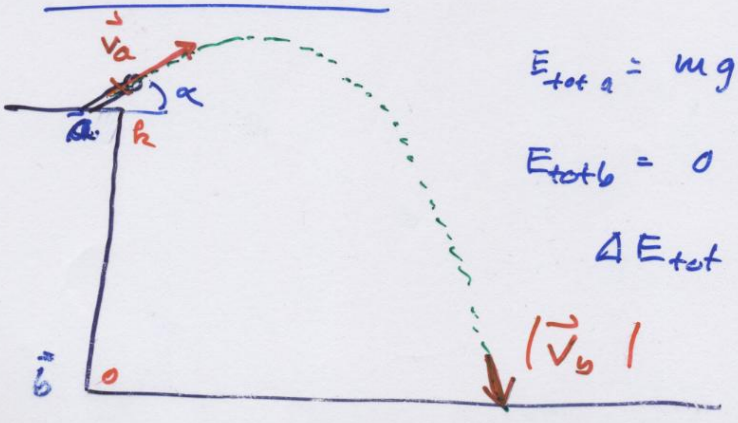
$$E_{tot\,b} = E_{pb} + E_{kb}$$
$$= 0 + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$\Delta E_{tot} = E_{tot\,b} - E_{tot\,a}$$
$$= \frac{1}{2}(m_1 + m_2) v^2 - m_2 g h$$
$$= 0$$

$$\Rightarrow \quad v = \sqrt{\frac{2 m_2}{m_1 + m_2} g \cdot h}$$

Bsp: $h = 100\ cm$
$m_1 = 100 \times m_2$
$\Big\} \; v = 0{,}4 \ \frac{m}{s}$

c) Kanonenschuß



$$E_{tot\,a} = mgh + \frac{1}{2} m v_a^2$$

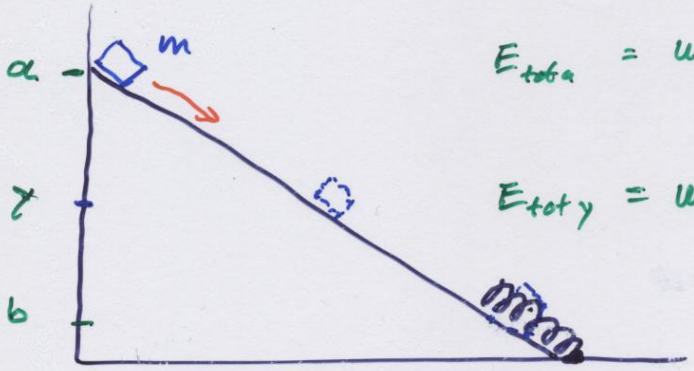
$$E_{tot\,b} = 0 + \frac{1}{2} m v_b^2$$

$$\Delta E_{tot} = 0$$

$$\Rightarrow v_b = \sqrt{2gh + v_a^2}$$

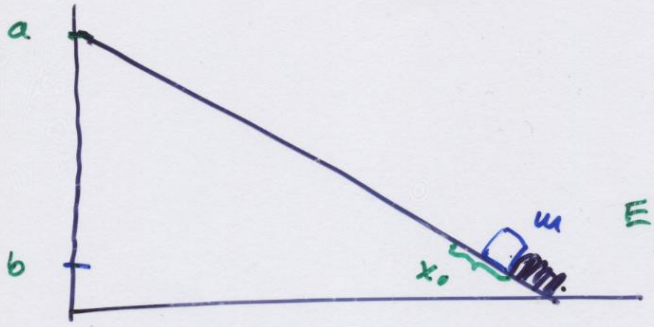
unabhängig vom Winkel

d) Zusammendrücken einer Feder



$E_{tot\,a} = mga$

$E_{tot\,y} = mgy + \frac{1}{2}mv_y^2$



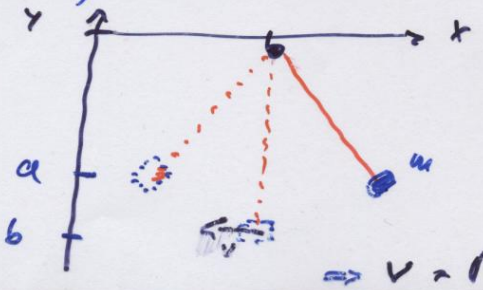
$E_{tot\,b} = mgb + \frac{1}{2}kx_0^2$

$\Delta E_{tot} = 0 \rightarrow$

$$mga = mgb + \frac{1}{2}kx_0^2$$

$$\Rightarrow x_0 = \sqrt{\frac{2mg(a-b)}{k}}$$

e) schwingendes Pendel

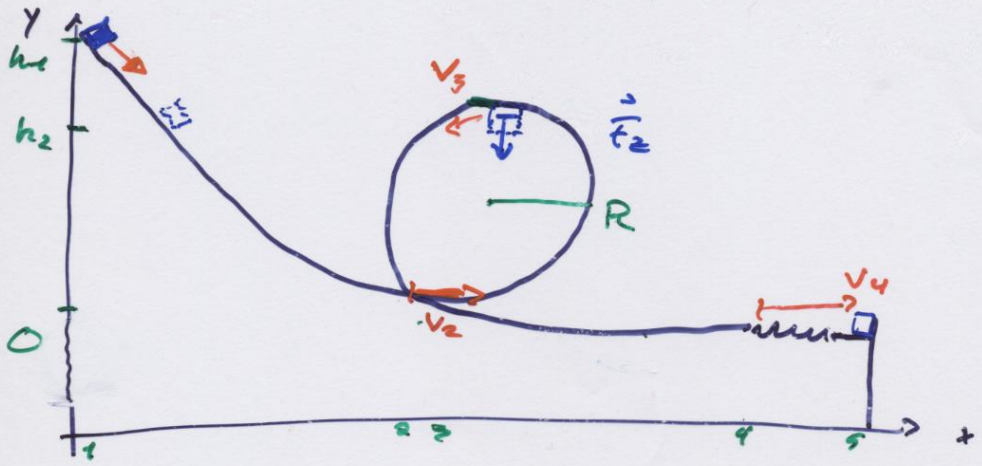


$E_{tot\,a} = mga$

$E_{tot\,b} = mgb + \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{2g(a-b)}$

f1 Achterbahn mit Looping



$$E_{tot} = mgh_1 + 0$$

$$= 0 + \frac{1}{2} m v_2^2$$

$$= mgh_2 + \frac{1}{2} m v_3^2$$

$$= 0 + \frac{1}{2} m v_4^2$$

$$= \int_{x_4}^{x_5} f_G \, dx$$

$$v_2 = \sqrt{2gh_1}$$

$$v_3 = \sqrt{2g(h_1 - h_2)}$$

am Ort 3

$$\vec{F}_N + m\vec{g} = m\vec{a}_Z$$

$$-F_N - mg = -m a_z$$

$$\Rightarrow F_N = m \left(\frac{v_3^2}{R} - g \right)$$

Zahlenbeispiel:

$m = 60\,kg$

$h_1 = 60\,m$

$h_2 = 40\,m$

$R = 20\,m$

$$\Rightarrow v_3 = \sqrt{2 \cdot 9{,}81 \frac{m}{s^2} (60m - 40m)} = 19{,}8 \frac{m}{s}$$

$$\Rightarrow F_N = 60\,kg \left(-9{,}81 \frac{m}{s^2} + \frac{\left(19{,}8 \frac{m}{s}\right)^2}{20\,m} \right)$$

$$= 588\,N$$

[$\hat{=} \approx 1g$ Beschleunigung]