Rotationsenergie

$$E_{kin}^{rot} = \frac{1}{2}\sum_i m_i \vec{v}_i^{\,2}$$

$$= \frac{1}{2}\sum_i m_i (\vec{\omega} \times \vec{r}_i)^2$$

oder $$= \frac{1}{2}\int (\vec{\omega} \times \vec{r})^2 dm \quad , dm = \rho\, dV$$

$$= \frac{1}{2}\int \begin{pmatrix} \omega_y z - \omega_z y \\ \omega_z x - \omega_x z \\ \omega_x y - \omega_y x \end{pmatrix}^2 dm$$

$$= \frac{1}{2}\int \Big[(\omega_y z - \omega_z y)^2 + (\omega_z x - \omega_x z)^2 + (\omega_x y - \omega_y x)^2\Big] dm$$

$$= \frac{1}{2}\int \Big[z^2\omega_y^2 - 2yz\,\omega_y\omega_z + y^2\omega_z^2 + x^2\omega_z^2 - 2xz\,\omega_x\omega_z + z^2\omega_x^2 + y^2\omega_x^2 - 2xy\,\omega_x\omega_y + x^2\omega_y^2\Big] dm$$

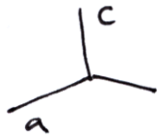
s.o. $$= \frac{1}{2}\Big[\Theta_{xx}\omega_x^2 + \Theta_{yy}\omega_y^2 + \Theta_{zz}\omega_z^2 - 2\Theta_{xy}\omega_x\omega_y - 2\Theta_{yz}\omega_y\omega_z - 2\Theta_{xz}\omega_x\omega_z\Big]$$

$$= \frac{1}{2}\vec{\omega}\cdot\underline{\underline{\Theta}}\,\vec{\omega}$$

$$\boxed{E_{kin}^{rot} = \frac{1}{2}\vec{\omega}\cdot\vec{L}}$$

Die Flächen konstanter Energie E_{kin}^{rot}, dargestellt im $\vec{\omega}$-Raum, sind <u>Ellipsoide</u>

besonders klar, falls $\underline{\underline{\Theta}}$ diagonal



$$E_{kin}^{rot} = \frac{1}{2}\vec{\omega}\,\underline{\underline{\Theta}}\,\vec{\omega}$$

$$= \frac{1}{2}\left(\Theta_a \omega_a^2 + \Theta_b \omega_b^2 + \Theta_c \omega_c^2\right)$$

$$= \frac{1}{2}\left(\frac{L_a^2}{\Theta_a} + \frac{L_b^2}{\Theta_b} + \frac{L_c^2}{\Theta_c}\right)$$