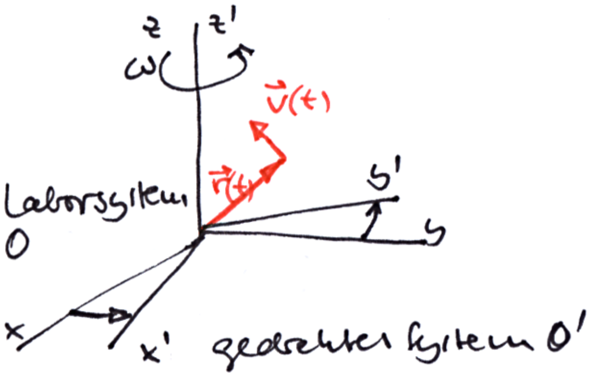
Kreisel

Betrachtung im rotierenden Bezugssystem (verkürzt!)

Änderungen von $\vec{L}$ im Inertialsystem: $\frac{d\vec{L}}{dt} = \vec{M}$

im Kreiselsystem?



Transformation eines Vektors

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}'}{dt} + \vec{\omega} \times \vec{r} \qquad (\vec{v} = \vec{\omega} \times \vec{r})$$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \vec{\omega} \times \vec{r}'$$

analog:

$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}'}{dt} + \vec{\omega} \times \vec{L}'$$

$$\rightarrow \quad \vec{M} = \frac{d}{dt}\left(\underline{\underline{\Theta}}'\vec{\omega}\right) + \vec{\omega} \times \left(\underline{\underline{\Theta}}'\vec{\omega}\right)$$

komponentenweise:

$$M_a = \frac{dL_a}{dt} + \left[\begin{pmatrix} \omega_a \\ \omega_b \\ \omega_c \end{pmatrix} \times \begin{pmatrix} \Theta_a \omega_a \\ \Theta_b \omega_b \\ \Theta_c \omega_c \end{pmatrix}\right]_a$$

$$= \Theta_a \frac{d\omega_a}{dt} + \omega_b \Theta_c \omega_c - \omega_c \Theta_b \omega_b$$

$$M_b \; \ldots \quad M_c \; \ldots$$

$$\boxed{\begin{aligned} M_a &= \Theta_a \dot{\omega}_a + (\Theta_c - \Theta_b)\,\omega_b \omega_c \\ M_b &= \Theta_b \dot{\omega}_b + (\Theta_a - \Theta_c)\,\omega_a \omega_c \\ M_c &= \Theta_c \dot{\omega}_c + (\Theta_b - \Theta_a)\,\omega_a \omega_b \end{aligned}}$$

Eulersche Kreiselgleichungen

drehmomentfreier symmetrischer Kreisel:

$$\vec{M} = 0 \quad , \quad \Theta_a = \Theta_b$$

$$\begin{cases} \Theta_a \dot{\omega}_a = (\Theta_a - \Theta_c)\,\omega_b \omega_c \\ \Theta_b \dot{\omega}_b = (\Theta_c - \Theta_a)\,\omega_a \omega_c \\ \Theta_c \dot{\omega}_c = 0 \end{cases}$$

$$\begin{cases} \dot{\omega}_a = \left(1 - \frac{\Theta_c}{\Theta_a}\right) \omega_b \omega_c \\ \dot{\omega}_b = \left(\frac{\Theta_c}{\Theta_a} - 1\right) \omega_c \omega_a \\ \dot{\omega}_c = 0 \quad \rightarrow \quad \omega_c = \text{const} ! \quad ✓ \end{cases}$$

mit Abkürzung $\Omega \equiv \left(\frac{\Theta_c}{\Theta_a} - 1\right)\omega_c$

$$\boxed{\begin{aligned} \dot{\omega}_a &= -\Omega\, \omega_b \\ \dot{\omega}_b &= \Omega\, \omega_a \\ \dot{\omega}_c &= 0 \end{aligned}}$$

Bewegungsgleichung

Ansatz mit

$$\boxed{\begin{aligned} \omega_a &= A \cos \Omega t \\ \omega_b &= A \sin \Omega t \\ \omega_c &= c \end{aligned}}$$

Lösung!

einsetzen:

$$-A\Omega \sin \Omega t = -\Omega A \sin \Omega t \quad ✓$$

$$A\Omega \cos \Omega t = \Omega A \cos \Omega t \quad ✓$$