2.3.2 Elastische und Inelastische Stöße

I. **Elastische S.:** Kinetische Energie vor und nach Stoß unverändert.

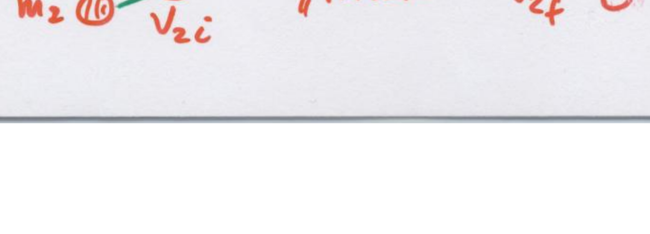
$$E_{tot} = E_{ki} + E_{pi}$$
$$= E_{kf} + E_{pf} \quad ; \quad E_{ki} = E_{kf}$$

II. **Inelast. Stoß:** Ein Teil der kin. Energie in „Innere Energie" Q transformiert

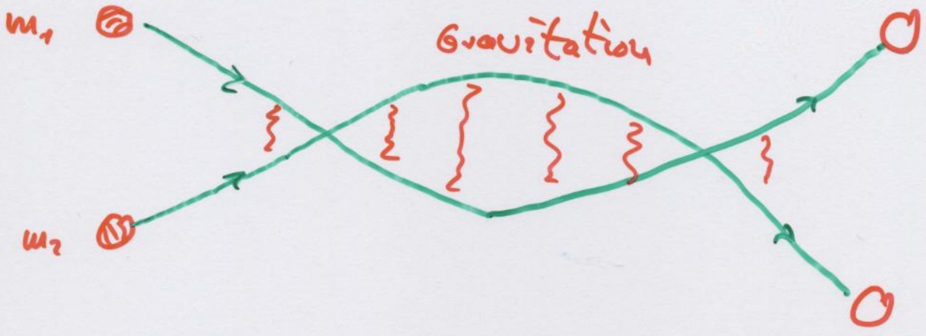
$$E_{kf} = E_{ki} - Q$$

Stöße finden durch Wechselwirkungen zweier Objekte statt.

Bs.

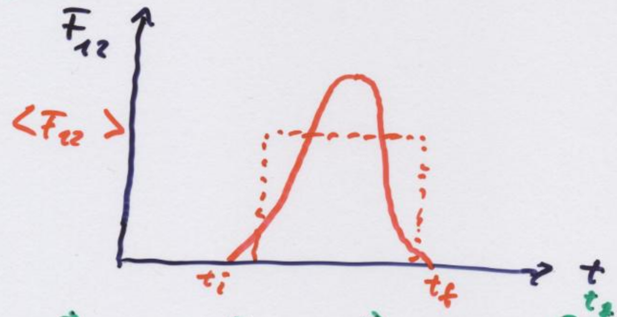


B9. Gravitation



1] Allgemeine Betrachtung:

Kraftstoß:



$$\Delta \vec{p}_1 = \vec{p}_{1f} - \vec{p}_{1i} = \int_{t_i}^{t_f} \vec{F}_{21}\, dt$$

$$\Delta \vec{p}_2 = \vec{p}_{2f} - \vec{p}_{2i} = \int_{t_i}^{t_f} \vec{F}_{12}\, dt$$

N_3: $\vec{F}_{21} = -\vec{F}_{12}$: $\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0$

also: $\vec{P}_{CM} = \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} = \text{const}$

Impulserhaltungssatz

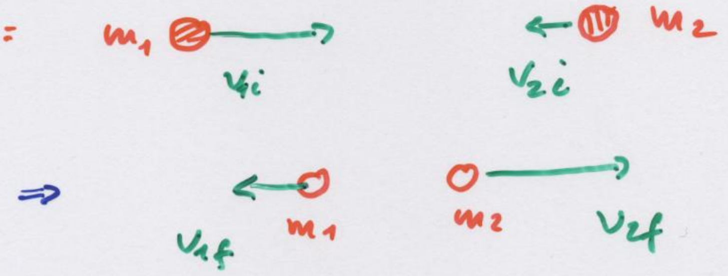
2] Elastischer Stoß

$$\vec{P}_{cM} = \text{const.} = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}$$
$$= m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$E_k = \text{const.} = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2$$
$$= \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Illustration:

<u>1d</u> :



$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$
$$\tfrac{1}{2} m_1 (v_{1i}^2 - v_{1f}^2) = \tfrac{1}{2} m_2 (v_{2f}^2 - v_{2i}^2)$$

$$\Rightarrow \quad v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

Ergebnis: $v_{1f} = v_{1i} \cdot \dfrac{m_1 - m_2}{m_1 + m_2} + v_{2i} \dfrac{2 m_2}{m_1 + m_2}$

$$v_{2f} = v_{1i} \frac{2 m_1}{m_1 + m_2} + v_{2i} \frac{m_2 - m_1}{m_1 + m_2}$$

Spezielle Beispiele :

- $m_1 = m_2 \Rightarrow v_{1f} = v_{2i}$

  $v_{2f} = v_{1i}$

- $m_1 = m_2$ und $v_{2i} = 0$

  $\Rightarrow v_{1f} = 0\ ,\ v_{2f} = v_{1i}$

- $m_2 = \infty\ ,\ v_{2i} = 0$

  $\Rightarrow v_{1f} = -v_{1i}$

  $v_{2f} = v_{2i} = 0$

- $m_1 = \infty\ ,\ v_{2i} = 0$

  $\Rightarrow v_{1f} = v_{1i}$

  $v_{2f} = 2\, v_{1i}$

3] Inelastische Stöße

$\vec{P}_{CM} = \text{const}$. $\quad E_{CMS} \neq \text{const}!$

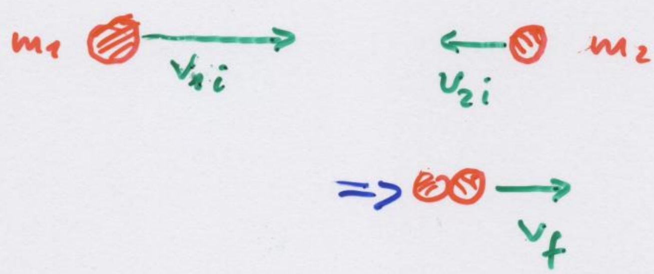
$$E_{tot} = E_{k1i} + E_{k2i}$$
$$= \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$
$$= E_{k1f} + E_{k2f} + Q$$

Innere Energie

$$= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + Q$$

Ein Stoß ist inelastisch, wenn $Q > 0$

Zur Illustration: 1d, total inelastisch:



- Impulserhaltung:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f = P_{CM} = \text{const}$$

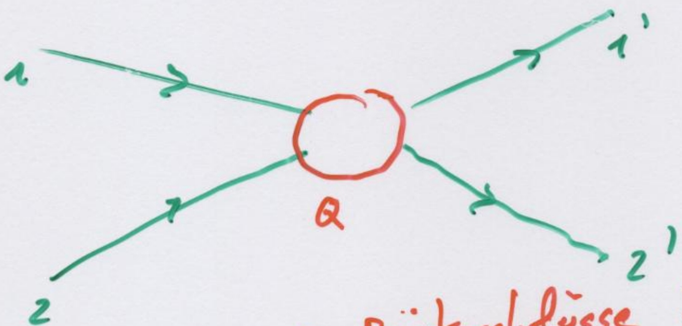
$$\Rightarrow v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = v_{CM}$$

- Energiebilanz:

$$E_{tot} = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$
$$= \frac{1}{2}(m_1 + m_2)\, v_f^2 + Q$$

$$\Rightarrow Q = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} \left(v_{1i} - v_{2i} \right)^2$$

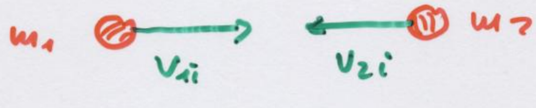
Generelle Anwendung:



→ Rückschlüsse über Vorgang bei Streuung

Beispiele:

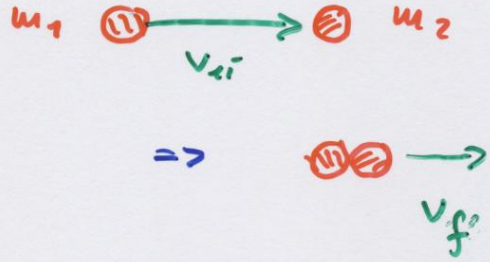
- $m_1 = m_2$, $v_{2i} = -v_{1i}$



$\Rightarrow$ 

$v_f = 0$ $\qquad E_{tot} = 2 \cdot \frac{1}{2} m_1 v_{1i}^2 = Q$

• $m_1 = m_2$, $v_{2i} = 0$



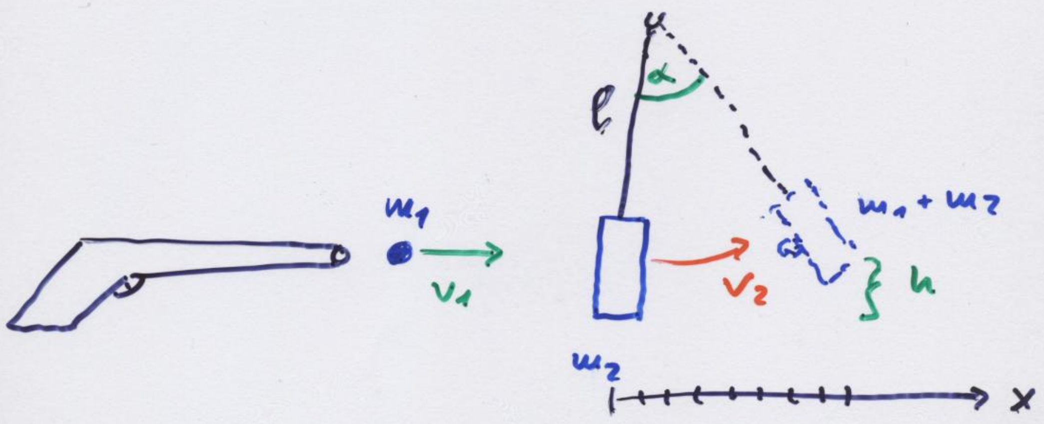
$$v_f = \frac{1}{2} v_{1i} \quad ; \quad E_{tot} = \frac{1}{2} m_1 v_1^2$$

$$= \frac{1}{2}(m_1 + m_2) v_f^2 + Q$$

$$= \frac{1}{4} m_1 v_{1i}^2 + Q$$

$$Q = \frac{1}{4} m_1 v_{1i}^2$$

Anwendung : Ballistisches Pendel



$$m_1 v_1 = (m_1 + m_2)\, v_2$$

Impuls der Kugel

Impuls von Pendel und Kugel

$$\frac{1}{2}(m_1 + m_2)\, v_2^2 = (m_1 + m_2)\, g \cdot h$$

$$\Rightarrow v_2 = \sqrt{2gh}$$

$$= \sqrt{2gl \cdot (1 - \cos\alpha)}$$

$$\Rightarrow V_1 = \sqrt{2gl(1-\cos\alpha)}\ \frac{m_2 + m_1}{m_1}$$

Auch: $\tan\alpha \approx \alpha \approx \frac{x}{l}$

$x = 8\ \text{cm} \pm 1\ \text{cm}$

$l = 88\ \text{cm}$

$m_1 = 0{,}5\ \text{g}$

$m_2 = 355\ \text{g}$

$g = 9{,}81\ \frac{\text{m}}{\text{s}^2}$

$V_1 = 189\ \frac{\text{m}}{\text{s}}$

$\pm 12\ \frac{\text{m}}{\text{s}}$

Berechnung mit Kettenregel