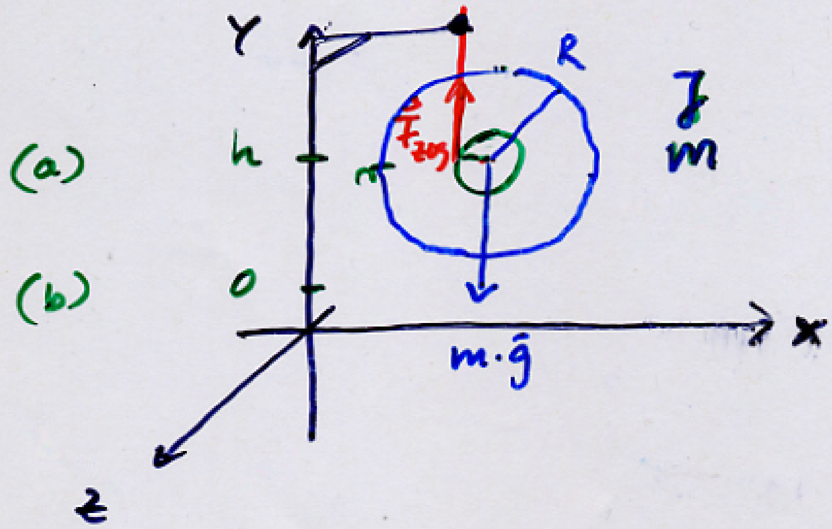
Zu: Anwendungen

[2] : Yoyo



Energiebetrachtung

(a) $$E_{tot} = m \cdot g \cdot h$$

(b) $$= \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2$$

$$\overset{v = \omega \cdot r}{\Rightarrow} \quad v = \sqrt{\frac{2gh}{1 + J/_{r^2 \cdot m}}}$$

Kräftebetrachtung

$$\Sigma \vec{M}_i = \vec{r} \times \vec{F}_{zug} = J \vec{\alpha}$$

$$\Sigma \vec{F}_i = m\vec{g} + \vec{F}_{zug} = m \cdot \vec{a}$$

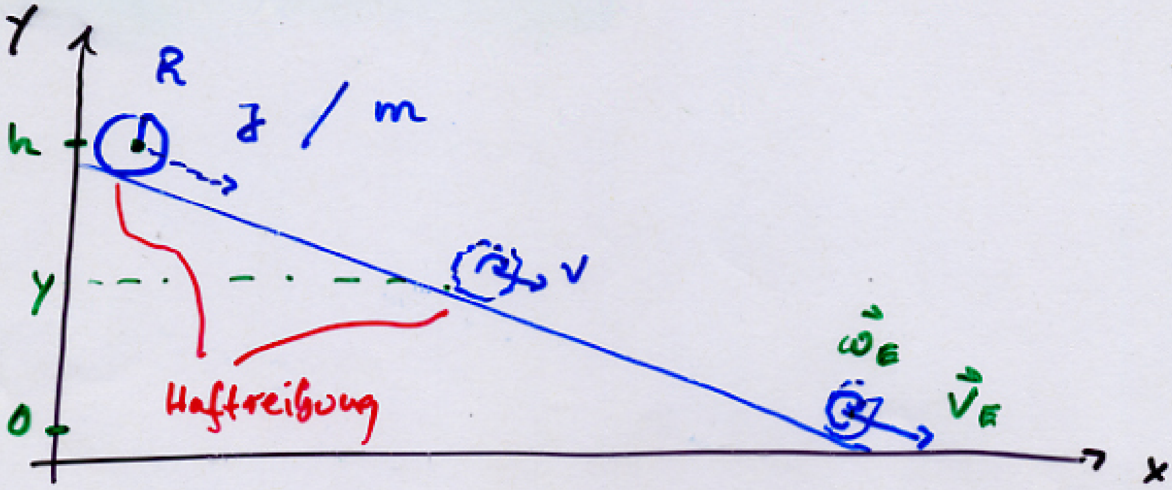
y-Richtung: $$-mg + F_{zug} = -ma$$

z " " : $$-r \cdot F_{zug} = -J\alpha$$

Auch : $$\alpha = \frac{a}{r}$$

$$\Rightarrow F_{zug} = \frac{J \cdot a}{r^2} \quad \Rightarrow \quad a = \frac{g}{1 + J/_{m r^2}}$$

[3] Rollen



$$E_{tot} = m \cdot g \cdot h$$

$$= \frac{1}{2} m v^2 + m g h + \frac{1}{2} J \omega^2$$

mit $\omega = \frac{v}{R}$:

$$E_{tot} = \frac{1}{2} m v_E^2 + \frac{1}{2} J \omega_E^2$$

$$= \frac{1}{2} \left(m + \frac{J}{R^2} \right) v_E^2$$

$$\Rightarrow v_E = \sqrt{\frac{2gh}{1 + J/mR^2}}$$; $J = \varkappa m R^2$

Rotationssymm.-Objekte mit Radius R

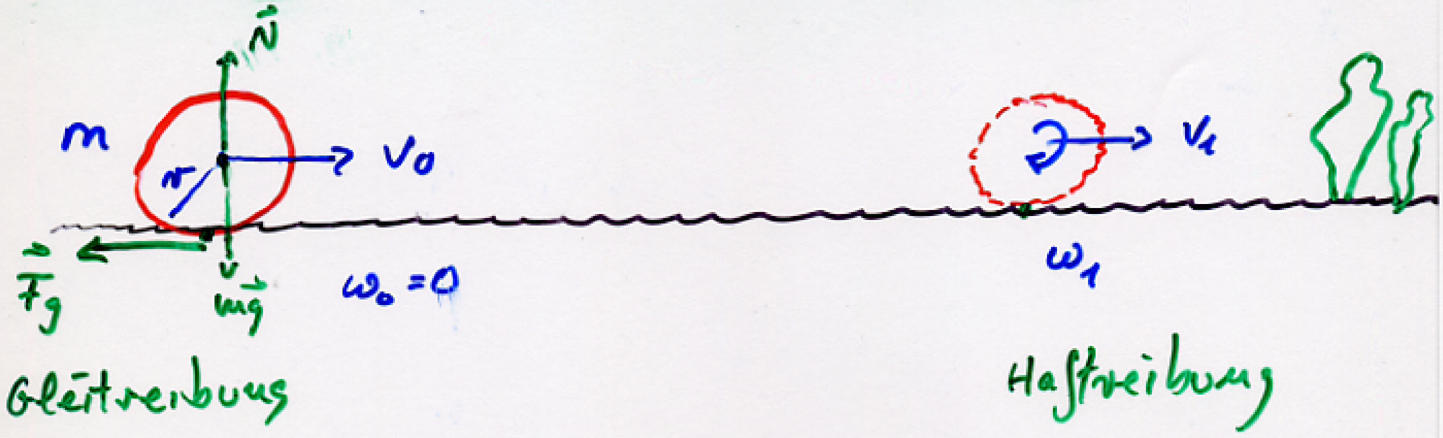
$$\Rightarrow v_E = \sqrt{\frac{2gh}{1 + \varkappa}}$$

Bsp.: Kugel: $\varkappa = \frac{2}{5}$ $\Rightarrow v_E = \sqrt{\frac{5}{7}} \cdot \sqrt{2gh}$

Zylinder: $1/2$ $\Rightarrow v_E = \sqrt{\frac{2}{3}} \sqrt{2gh}$

Hohlzyl.: 1 $\Rightarrow v_E = \sqrt{1/2} \sqrt{2gh}$

4] Kegeln / Bowling



(a) $$E_{tot} = \frac{1}{2} m v_0^2$$

$$= \frac{1}{2} m v_1^2 + \frac{1}{2} J \omega_1^2 + E_{int}$$

b) $$F_g = m a = m \cdot \frac{v_1 - v_0}{\Delta t}$$

$$M = r \cdot F_g = J \cdot \alpha = J \cdot \frac{\omega_1 - \omega_2}{\Delta t}$$

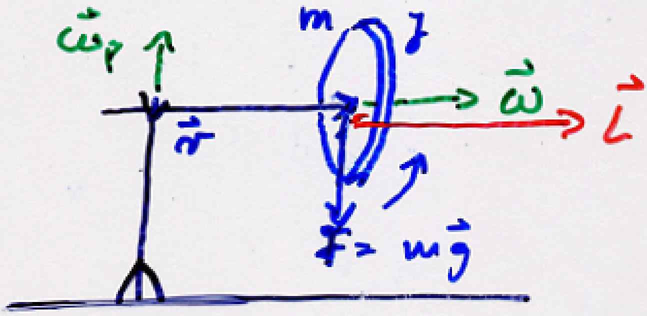
$$\Rightarrow M = m \cdot \frac{v_1 - v_0}{\Delta t} \cdot r = J \cdot \frac{v_1}{r \cdot \Delta t}$$

$$\Rightarrow v_1 = \frac{v_0}{1 + J/_{m r^2}}$$

Für Vollkugel: $J = \frac{2}{5} m r^2$

$$\boxed{v_1 = v_0 \cdot \frac{5}{7}}$$

5] Kreisel



Drehmoment auf Kreisel: $\vec{M} = \vec{r} \times \vec{F}$

$$= \frac{d\vec{L}}{dt}$$

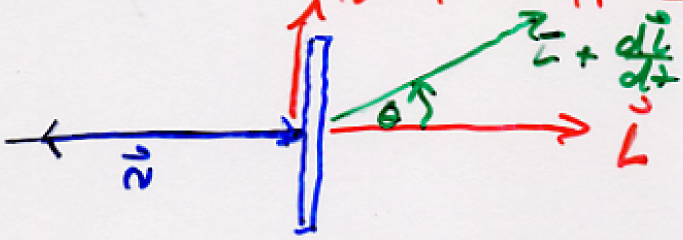
$$\left[\frac{d\vec{L}}{dt} = \frac{d(\vec{r} \times \vec{p})}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \right.$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times \vec{F}$$

$$\left. = 0 + \vec{r} \times \vec{F} \right]$$

Kreisel von oben:

$\vec{r} \times \vec{F} = \vec{M} = d\vec{L}/dt$



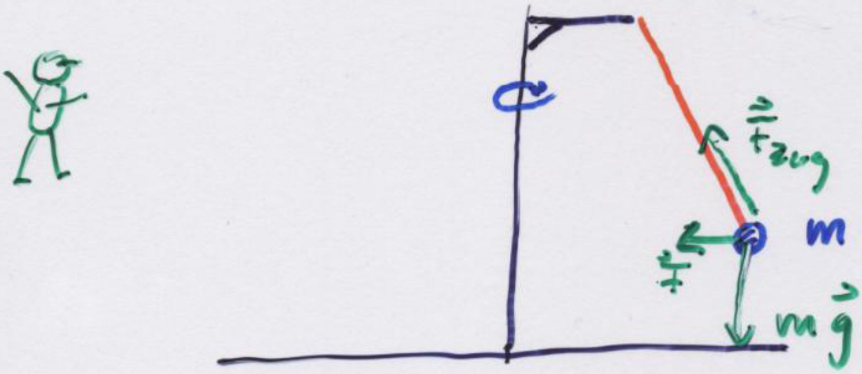
$$\omega_p = \frac{dL/dt}{L} = \frac{r \cdot F}{J \cdot \omega}$$ Präzessionsfrequenz

ω_p kleiner, je größer J, ω

2.4.3 Rotierende Bezugssysteme

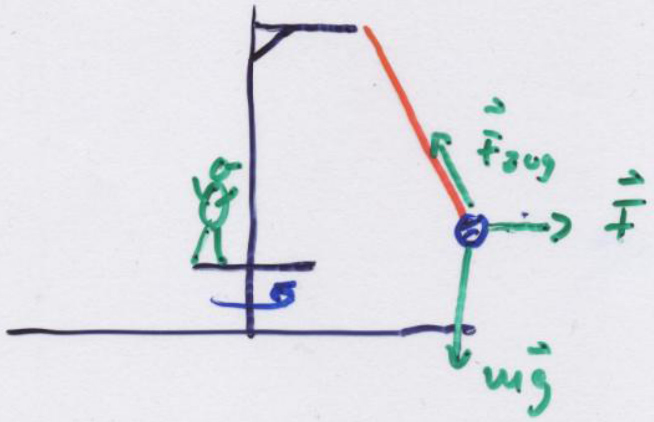
Trägheitskräfte

1] Zentrifugalkraft



$\vec{F} = m\vec{a}_z$

Zentripetalkraft



$\vec{F}_{zug} + m\vec{g} + \vec{F} = 0$

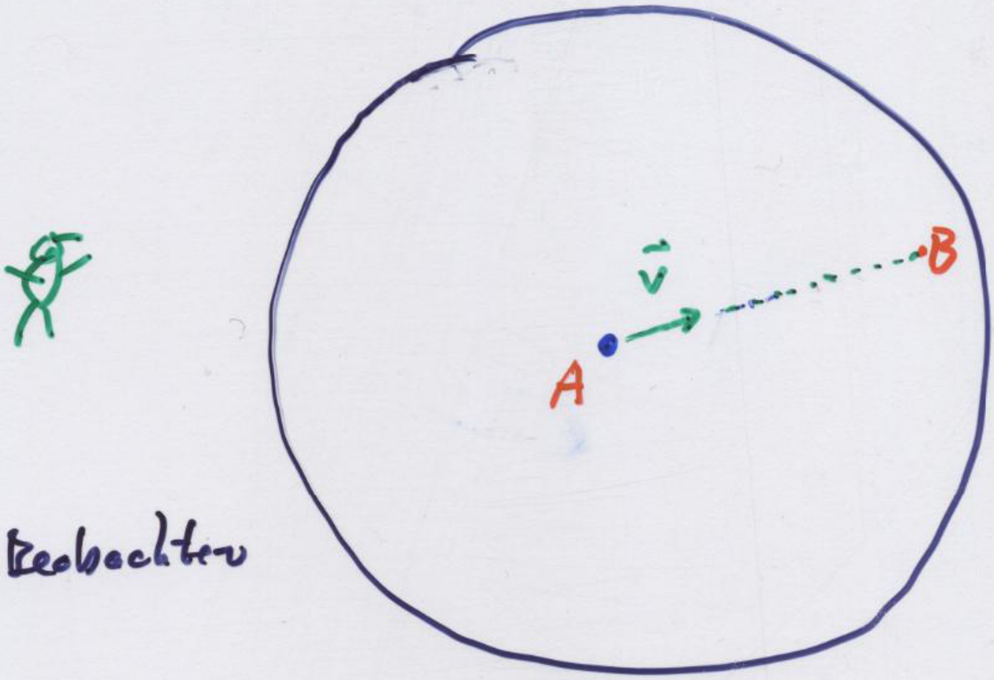
Im drehenden System.

$\vec{F}$ Zentrifugalkraft

Zentrifugalkraft ist Trägheitskraft bzw Scheinkraft.

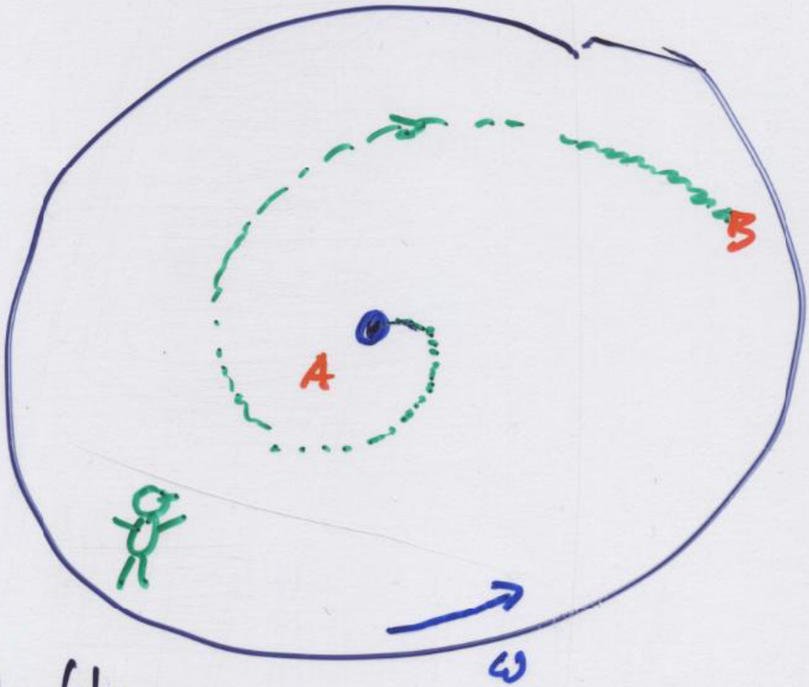
2] Corioliskraft

a)



Ruhender Beobachter

b)

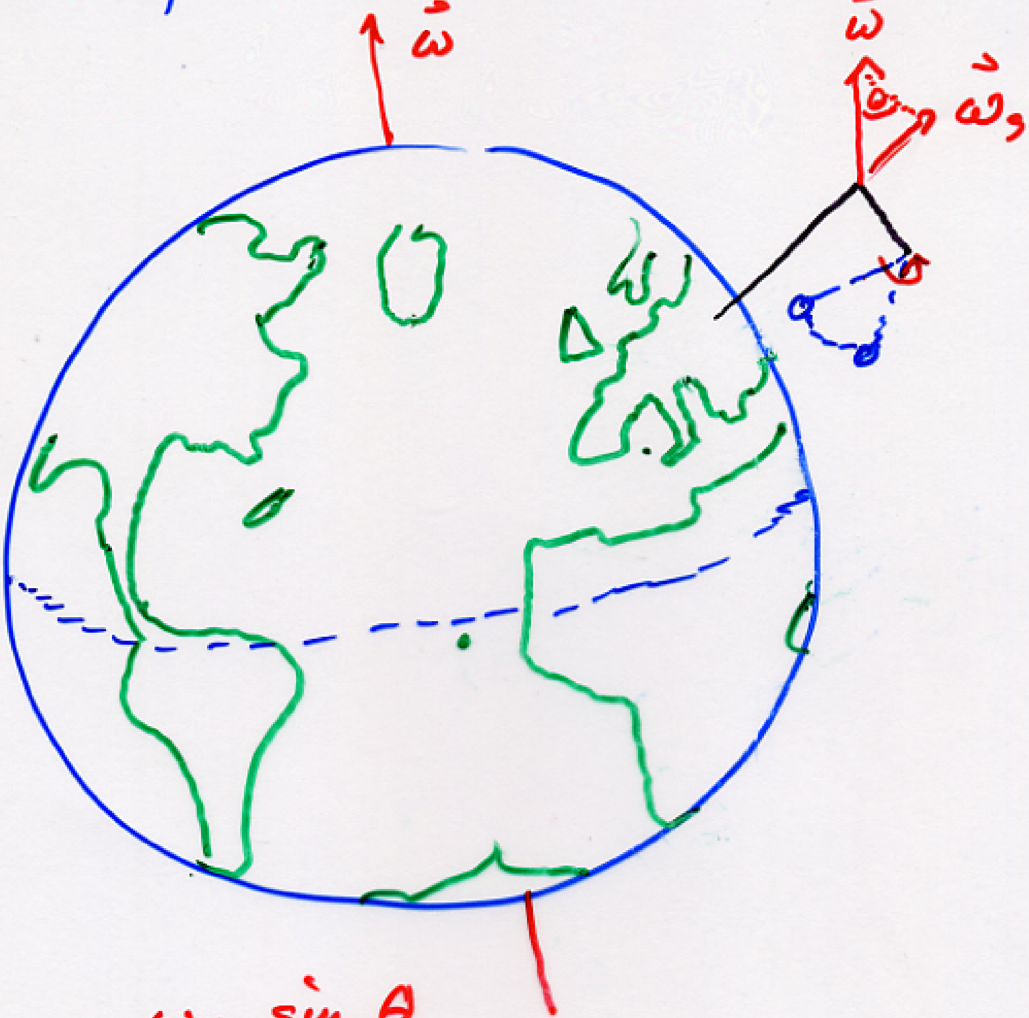


Drehender Beobachter

$$\vec{F}_c = 2m(\vec{v} \times \vec{\omega})$$

Bedeutung für
- Wetter (Stürme)
- Foucault Pendel

Anwendung: Foucault-Pendel

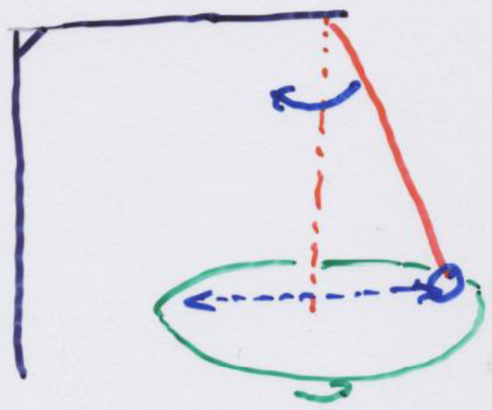


$\omega_s = \omega \cdot \sin\theta$

In Ka: $T_s = 28\ h$!

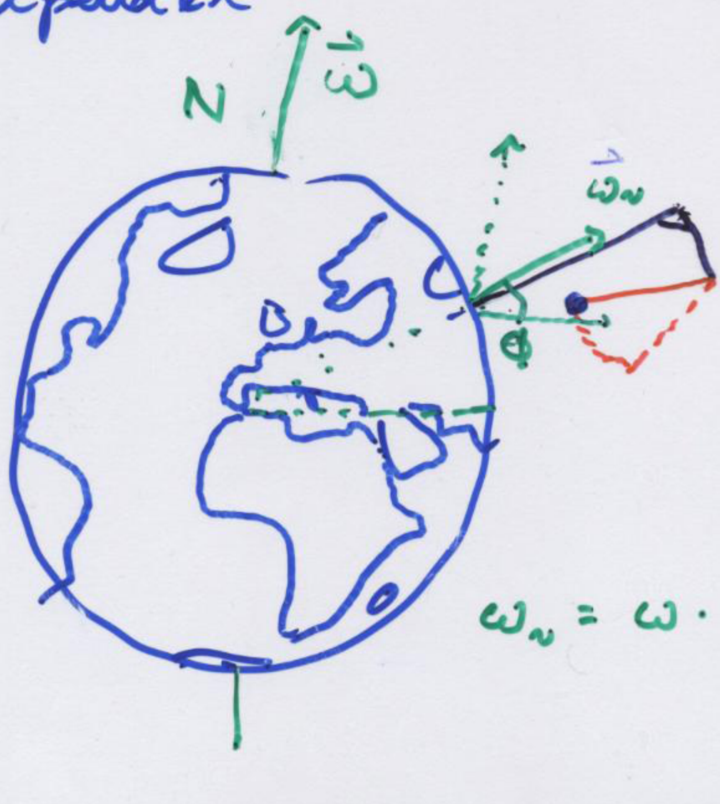
Demonstration

a) Bahn eines Pendels auf rotierender Scheibe

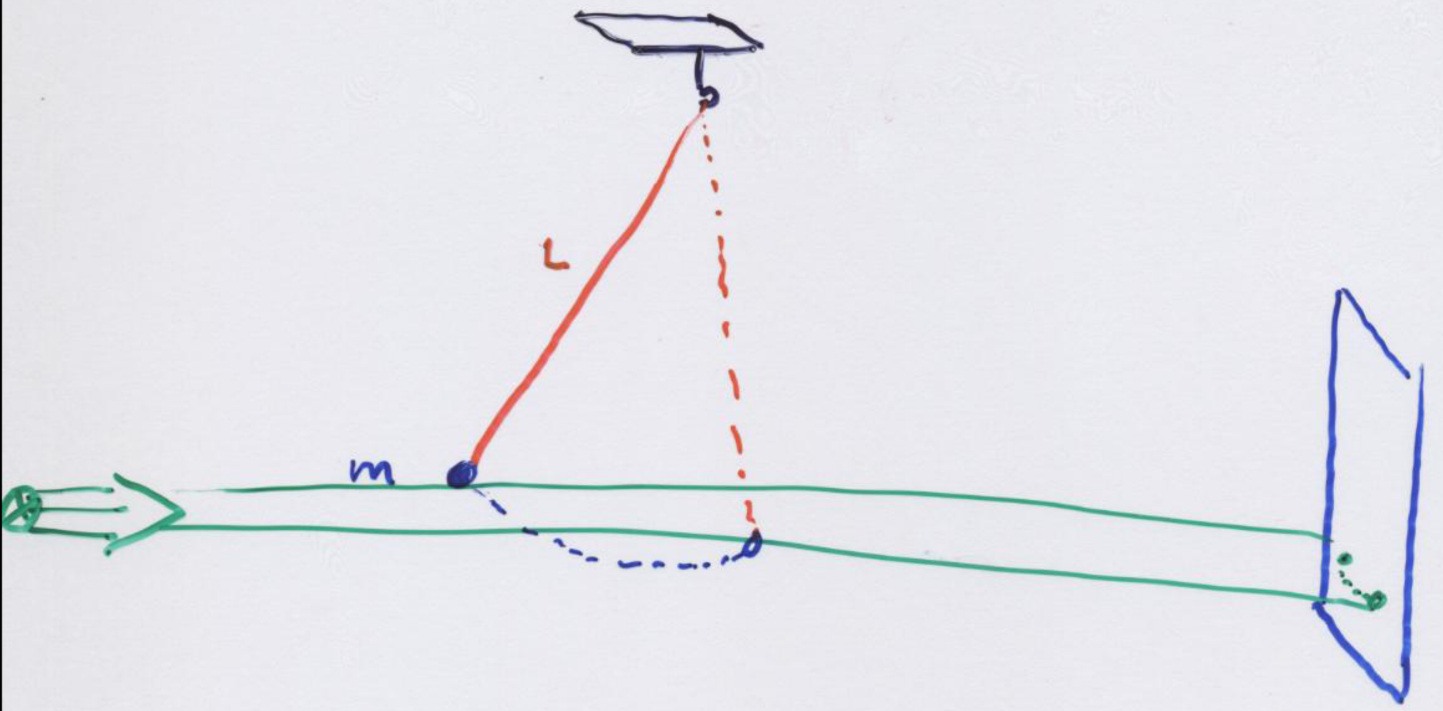


von oben

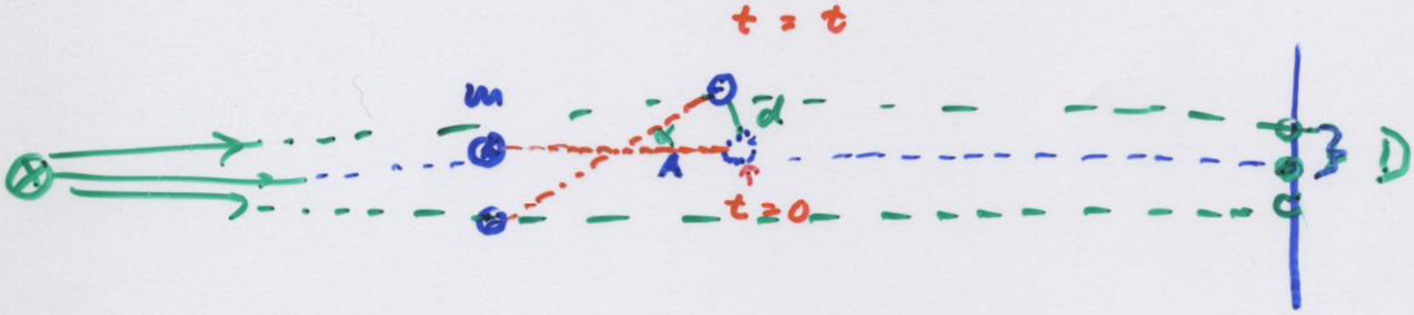
b) Foucaultpendel



$$\omega_N = \omega \cdot \sin \phi$$



von oben :



Messe $\alpha(t)$ aus $\alpha = \arctan \frac{d}{A}$

$$= \arctan \frac{D/\text{Abbildung v.}}{A}$$

$$\approx \frac{D}{A \cdot \text{Abb}} \cdot \frac{360°}{2\pi}$$

$A = 50$ cm
$D = 2$ Striche $\times$ 8 cm
Abb $= 62{,}5$
$t = 22$ Perioden $= 103$ s

$\Big\} \; \alpha = 0{,}29°$

$$\alpha\,(103\,s) = 0{,}29^\circ$$

$$\alpha\,(1\,\text{Tag}) = 0{,}29^\circ \times \frac{86\,400\,s}{103\,s}$$

$$= 246^\circ$$

$$\Rightarrow \sin\phi = \frac{\omega_v}{\omega} = \frac{246^\circ/\text{Tag}}{360^\circ/\text{Tag}}$$

$$\boxed{\Rightarrow \phi = 43^\circ}$$