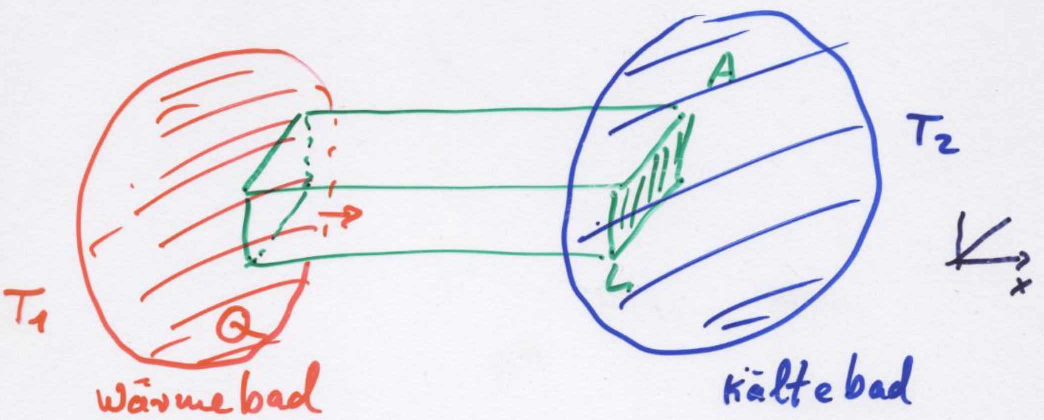
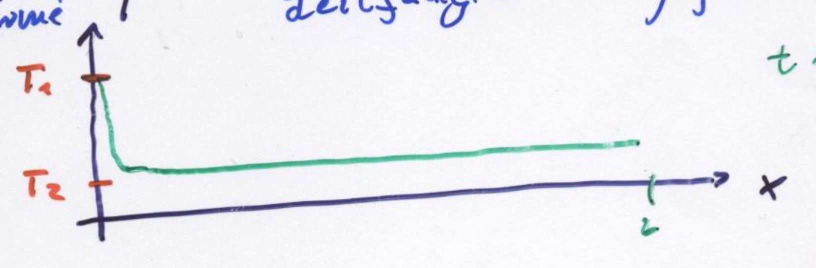
b] Wärmeleitung



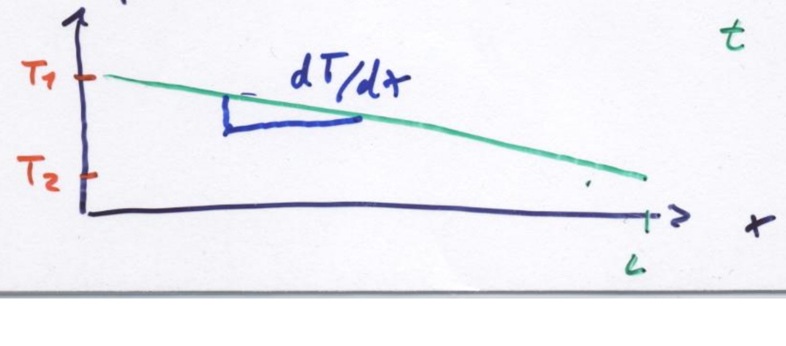
$T_1 > T_2$

$$\frac{dQ}{dt} = -\lambda \cdot A \cdot \frac{dT}{dx}$$

Transportierte Wärme — Therm. Leitfähigkeit — Temperaturgefälle



$t \approx 0$

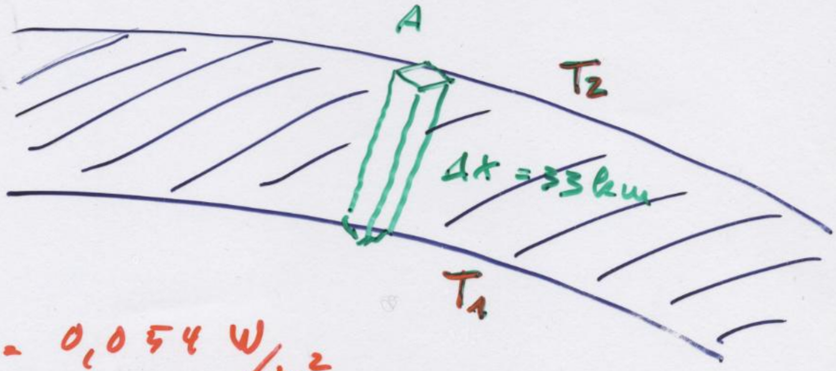


$t = t_\infty$

Bs. Silber $\lambda = 420 \frac{W}{m\,K}$

Silber	420
Alum.	220
Gestein	2,5
Wasser	0,6
Luft	0,026

Anwendung: Wärmefluß durch Erdkruste



$\frac{dQ}{dt} = 0{,}054\ W/m^2$

$T_2 = 10\,°C$

$T_1 = ?$

$$\frac{dQ}{dt} = -\lambda \cdot A \cdot \frac{\Delta T}{\Delta x}$$

$$\Rightarrow |\Delta T| = \frac{\Delta x \cdot \frac{dQ}{dt}}{\lambda \cdot A}$$

$\left(A = 1m^2, \ \frac{dQ}{dt} = 0{,}054 \, W, \right.$

$\left. \Delta x = 33 \, km, \ \lambda = 2{,}5 \, \frac{W}{mK} \right) \Rightarrow$

$\Delta T = 713 \, K$

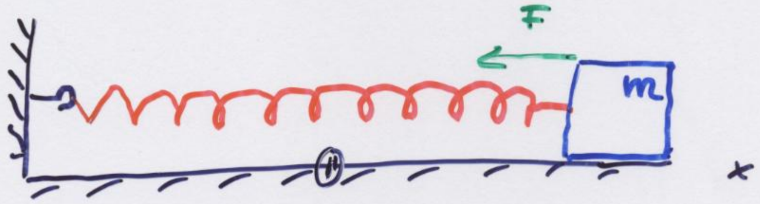
$\Rightarrow T_1 = 723°C$

6 Schwingungen und Wellen

6.1. Schwingungen

1] Federschwingungen

a] Ungedämpft



$$F = -k \cdot x$$

$$= m \frac{d^2x}{dt^2}$$

(1) $\Rightarrow x(t) = A \cos(\omega t + \phi)$

Eingesetzt:

$(F =)\ -k \cdot x(t) = -A\,k \cos(\ )$

$= -m\omega^2 A \cos(\ )$

gilt für alle t

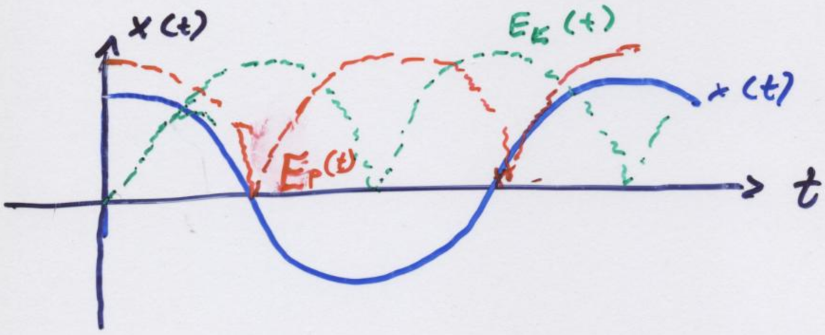
$$\Rightarrow \quad \omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A \cdot \cos\left(\sqrt{\frac{k}{m}}\, t + \phi\right)$$

Anfangsbedingungen:

$t = 0: \quad x(t=0) = A$

$$\Rightarrow \quad \phi = 0$$



Bewegung von m

(3) Energiebilanz

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$

$$= \frac{1}{2} m \frac{k}{m} A^2 \cdot \sin^2 \sqrt{\frac{k}{m}}\, t$$

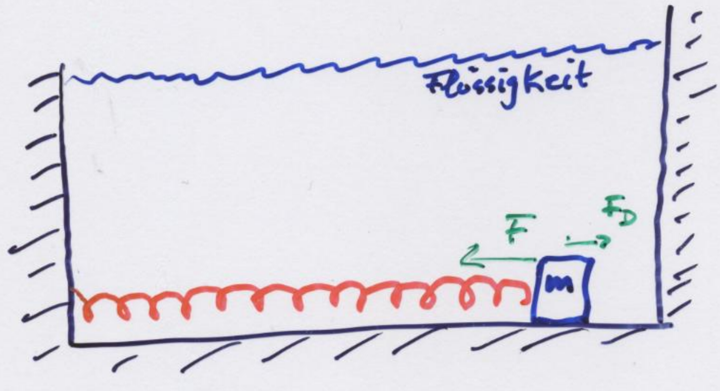
$$E_p = \int_0^x k \cdot x' \, dx' = \frac{1}{2} k x^2$$

$$= \frac{1}{2} k A^2 \cos^2 \sqrt{\frac{k}{m}} t$$

$$E_{tot} = E_p + E_k$$

$$\underline{E_{tot} = \frac{1}{2} k A^2} = \text{const.}$$

b] Gedämpfte Schwingung



$$F = -kx$$

$$F_D = -bv$$

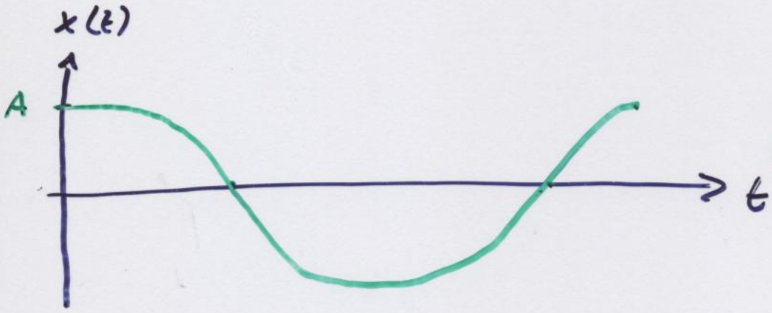
$$\Rightarrow -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$x(t) = A \cdot e^{-\frac{b}{2m}t} \cos \underbrace{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}_{\omega}\, t$$

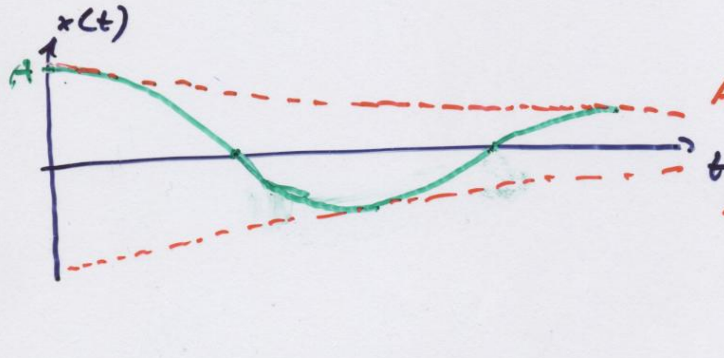
Energie : $E_{tot} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

$$= E_0 \cdot e^{-t/\tau_E} ,$$

$\tau_E = \frac{2m}{b}$ Lebensdauer

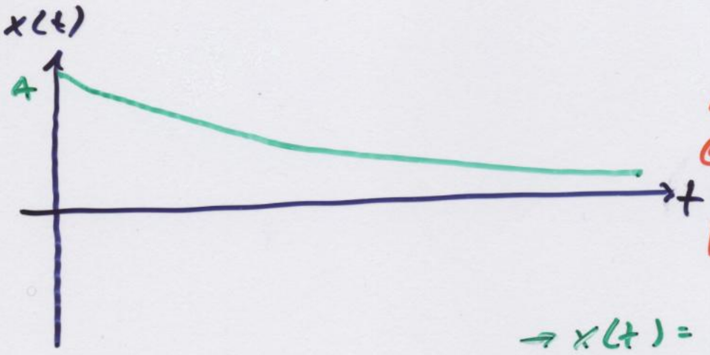


$b = 0$
keine Dämpfung



$A \cdot e^{-b/2m\, t}$

Dämpfung schwach
$b < \sqrt{4 m \cdot k}$



Aperiodischer Grenzfall

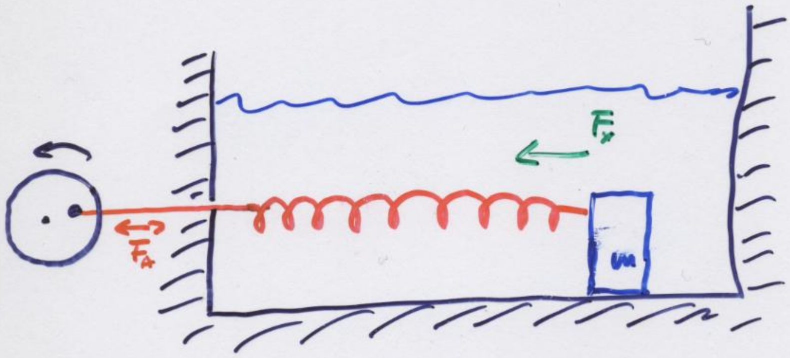
$b = \sqrt{4mk}$

$\rightarrow x(t) = A \cdot e^{-b/2m \, t}$

„Kriechfall"

$b > \sqrt{4mk}$

C] Erzwungene Schwingungen



$$\Sigma F = -b \frac{dx}{dt} - k \cdot x + F_0 \cos \omega t = m \frac{d^2 x}{dt^2}$$

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$$

$$\gamma = \frac{b}{2m} \quad \left(\frac{1}{\tau}\right)$$

$\omega_0 = \sqrt{\frac{k}{m}}$ Eigenfrequenz des ungedämpften Systems

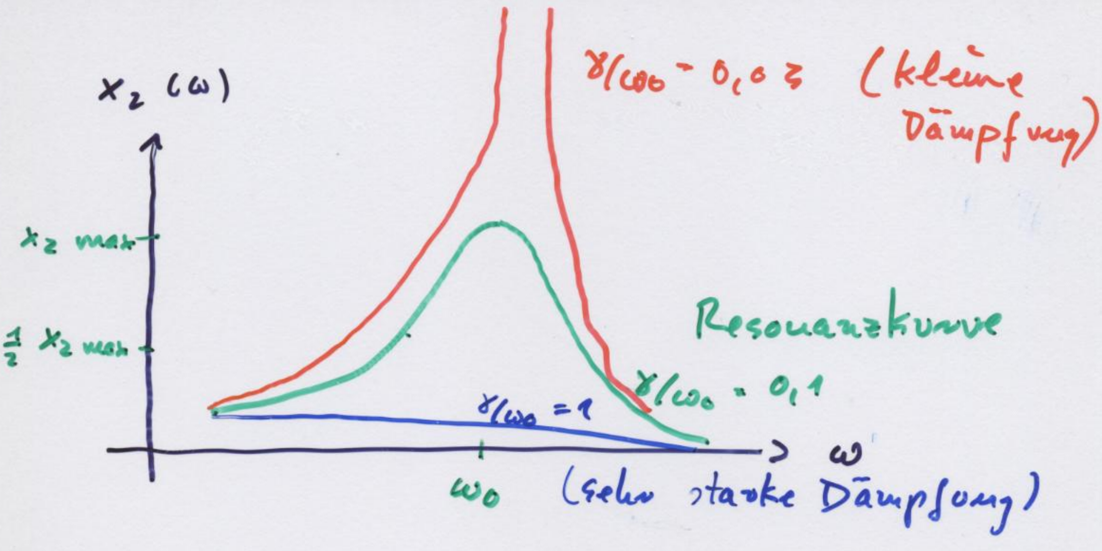
Lösung: Kombination aus Lösung der homogenen Diff.gl. ($\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$) und der inhomogenen

$$x(t) = x_1 \cdot e^{-\gamma t} \cdot \cos(\omega_1 t + \phi_1) + x_2 \cos(\omega t + \phi_2)$$

$$\omega_1 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega_0^2 - \gamma^2}$$

Für stationäre Lösung:

$$x_2 = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$$


x₂ (ω)
x₂ max
½ x₂ max
γ/ω₀ = 0,03 (kleine Dämpfung)
Resonanzkurve
γ/ω₀ = 0,1
γ/ω₀ = 1
ω
ω₀
(sehr starke Dämpfung)