Aufgabe 1

a) Energieerhaltung $E_{el} = E_{pot}$

(1/2) $\frac{1}{2} D l_0^2 = mgh = mg\,2l_0$

(1/2) $\Leftrightarrow \quad D = \frac{4mg}{l_0} = \frac{4 \cdot 80\,kg \cdot 10\,\frac{m}{s^2}}{20\,m}$

(1/2) $\underline{\underline{D = 160\,\frac{N}{m}}}$

b) $E_{pot} = mg(2l_0 - z)$ (1/2)

$\forall z \;;\; 0 \le z \le 2l_0$

$E_{el} = 0 \quad , \; z \le l_0$ (1/2)

$E_{el} = \frac{1}{2} D (z - l_0)^2 \quad , \; z > l_0$ (1/2)

$E_{kin} + E_{pot} = \text{Konst.} = E_{pot,max} \quad , \; z \le l_0$

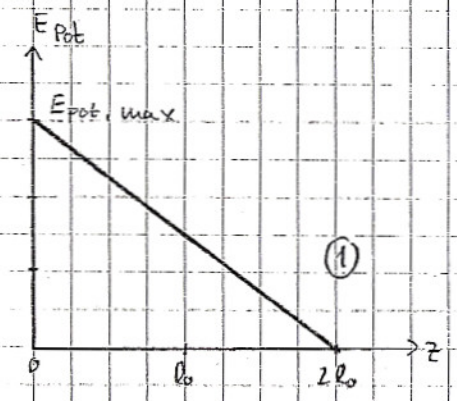
$E_{kin} + mg(2l_0 - z) = mg\,2l_0$

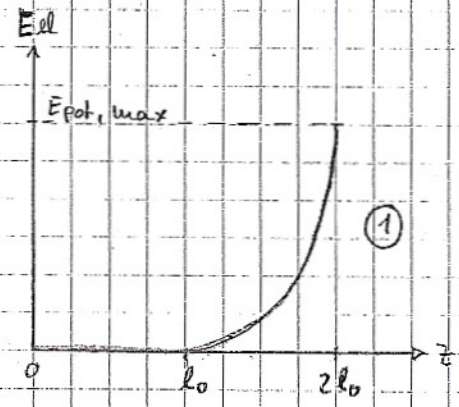
(1) $\Leftrightarrow \quad E_{kin} = mgz \qquad z < l_0$

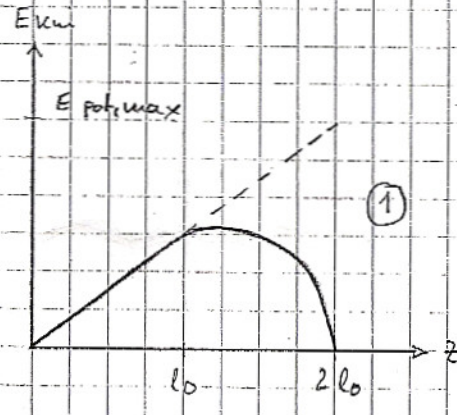
$E_{kin} + E_{pot} + E_{el} = \text{Konst.} = E_{pot,max} \quad , \; z > l_0$

$E_{kin} + mg(2l_0 - z) + \frac{1}{2} D (z - l_0)^2 = mg\,2l_0$

(1) $\Leftrightarrow \quad E_{kin} = mgz - \frac{1}{2} D (z - l_0)^2 \quad , \; z > l_0$







Aufgabe 2

a) $p_0 = 1\,\text{bar} = 10^5 \frac{N}{m^2}$

$\rho = 1{,}25 \cdot 10^{-3} \frac{g}{cm^3} = 1{,}25 \cdot 10^{-3} \frac{10^{-3}\,kg}{(10^{-2})^3\,m^3} = 1{,}25 \frac{kg}{m^3}$

$p + \rho \cdot g \cdot h + \frac{1}{2} \rho v^2 = \text{konst.}$

1 = im Schuppen

2 = oberhalb des Schuppens

(1) $p_1 + \rho g h_1 + \underbrace{\frac{1}{2} \rho v_1^2}_{=0} = p_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$

$p_1 = p_2 + \frac{1}{2} \rho v_2^2$

$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} = \sqrt{\frac{2 \cdot 0{,}01 \cdot 10^5 \frac{N}{m^2}}{1{,}25 \frac{kg}{m^3}}}$

(1/2) $v_2 = \sqrt{\frac{2000}{1{,}25} \frac{m^2}{s^2}} = \sqrt{1600 \frac{m^2}{s^2}}$

(1/2) $\underline{v_2 = 40 \frac{m}{s}}$

NR: 2000 : 125 = 16
125
750

b) $F = A \cdot \Delta p = 10\,m^2 \cdot 0{,}01 \cdot 10^5 \frac{N}{m^2}$

(1) $F = 10^4\,N$

(1/2) c) $\Delta p = p_1 - p_2 = \frac{1}{2} \rho v_2^2 = \frac{1}{2}\, 1{,}25 \frac{kg}{m^3}\, 2500 \frac{m^2}{s^2}$

(1/2) $\Delta p = 1562{,}5 \frac{N}{m^2}$

NR: 1250 · 1,25
1250
2500
6250
1562,50

(1/2) $F_w = A \cdot \Delta p = 1\,m^2 \cdot 1562{,}5 \frac{N}{m^2}$

$F_w = 1562{,}5\,N$

(1/2) $F = mg = 90\,kg \cdot 10 \frac{m}{s^2} = 900\,N < F_w$

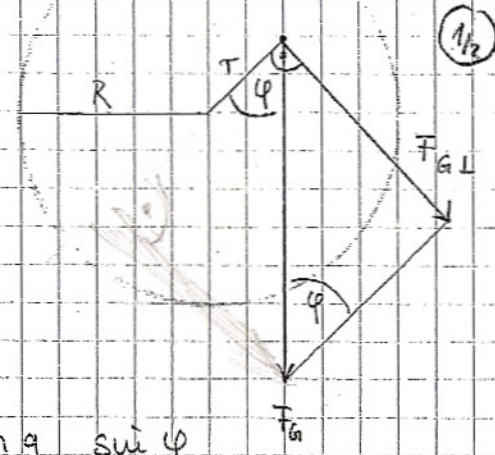
⇒ die Dachplatte hebt ab

Aufgabe 3

a) Trägheitsmoment

(1) $\Theta = \underbrace{\frac{1}{2} m R^2}_{\text{Kreis-scheibe}} + \underbrace{m r^2}_{\text{Steiner'scher Satz}}$

$\left[\Theta = \int r^2 \, dm = 2\pi s \int r^2 \, dr \right.$
$\left. = 2\pi s \frac{1}{4} r^4 = \frac{1}{2} m r^2 \right]$



(1/2)

(1) Drehmoment $\vec{M} = \vec{r} \times \vec{F}_G \Rightarrow M = r \underbrace{m g \sin\varphi}_{F_{GL}}$

b) $M = -\dot{L}$

(1/2) $mgr \sin\varphi = -\Theta \ddot{\varphi}$

(1/2) $\sin\varphi \propto \varphi$ für kleine Winkel

(1/2) $\rightsquigarrow \ddot{\varphi} + \frac{mgr}{\Theta} \varphi = 0$ DGL

(A) Lsg-Ansatz: $\varphi = \varphi_0 \cos\omega_0 t$, wenn $\omega_0 = \sqrt{\frac{mgr}{\Theta}}$

$\ddot{\varphi} = -\omega_0^2 \varphi_0 \cos\omega_0 t$

(1) c) $T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{\Theta}{mgr}} = 2\pi \sqrt{\frac{\frac{1}{2}R^2 + r^2}{gr}}$

(1/2) $r = 0 \rightsquigarrow T \to \infty$

(1/2) $r = R \rightsquigarrow T = 2\pi \sqrt{\frac{3R}{2g}}$

T_{min}: $T' = 2\pi \frac{1}{2} \left(\frac{\frac{1}{2}R^2 + r^2}{gr} \right)^{-1/2} \left(2r \frac{1}{gr} - \frac{1}{gr^2} \left(\frac{1}{2}R^2 + r^2 \right) \right) \overset{!}{=} 0$

(1/2) $\frac{\pi}{g} \underbrace{(\ldots)}^{-1/2} \left(2 - \frac{R^2}{2r^2} - 1 \right) = \frac{\pi}{g} (\ldots)^{-1/2} \left(1 - \frac{R^2}{2r^2} \right) \overset{!}{=} 0$

Wurzel im Nenner $> 0 \; \forall r$!

$\Rightarrow T' = 0$, wenn $\left(1 - \frac{R^2}{2r^2} \right) = 0$

(1/2) $r = \frac{1}{\sqrt{2}} R$

$\left[\rightsquigarrow T_{min} = 2\pi \sqrt{\frac{\sqrt{2} R}{g}} \right]$

Aufgabe 4

a) $p \cdot V = \text{konst.} \rightsquigarrow \frac{p}{p_0} = \frac{V_0}{V}$

$V_0 = \pi r^2 H = \pi r^2 \, 5m$; $V = \pi r^2 (H - h_1) = \pi r^2 \, 1m = \frac{1}{5} V_0$

(1) $\rightsquigarrow$ $p = 5 p_0 = 5 \text{ bar}$

(1/2) Bernoulli: $p + \rho g h = p_0 + \frac{1}{2} \rho v^2$

$v^2 = \frac{2}{\rho} ((p - p_0) + \rho g h) = 2gh + \frac{2(p - p_0)}{\rho}$

(1/2) $v^2 = 2 \cdot 10 \frac{m}{s^2} \, 4m + 2 \cdot 4 \cdot 10^5 \frac{kg}{m s^2} \, (10^3 \frac{kg}{m^3})^{-1} = 880 \frac{m^2}{s^2}$

(1/2) $v \approx 29 \frac{m}{s}$

b) schiefer Wurf

(1/2) $y = v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2$

(1/2) $v_y = v_0 \sin\alpha - g t$ $\rightsquigarrow$ t_{max} für $v_y = 0$

(1/2) $\rightsquigarrow t_{max} = \frac{v_0 \sin\alpha}{g}$

(1/2) $\rightsquigarrow y_{max} = \frac{1}{g} (v_0 \sin\alpha)^2 - \frac{1}{2g} (v_0 \sin\alpha)^2 = \frac{v_0^2 \sin^2\alpha}{2g}$

$y_{max} = \frac{880 \, m^2}{s^2 \, 20 \frac{m}{s^2} \cdot 2}$

(1/2) $y_{max} = 22 \text{ m}$ (22,5 m)

(1/2) c) $x = v_0 \cos\alpha \, t$

(1) $x_{max} = v_0 \cos\alpha \, 2 t_{max} = \frac{2 v_0^2 \sin\alpha \cos\alpha}{g}$

$x_{max} = \frac{2 \cdot 880 \frac{m^2}{s^2}}{2 \cdot 10 \frac{m}{s^2}}$

(1/2) $x_{max} = 88 \text{ m}$ (90 m)