A1) 4/4

$$F_1 = F_2 = F_{eff} \qquad s_1 + s_2 = s_{eff}$$

$$\frac{1}{D_{eff}} = \frac{s_{eff}}{F_{eff}} = \frac{s_1 + s_2}{F_1} = \frac{s_1}{F_1} + \frac{s_2}{F_2} = \frac{1}{D_1} + \frac{1}{D_2}$$

$$\Rightarrow D_{eff} = \frac{1}{\frac{1}{D_1} + \frac{1}{D_2}}$$

b)

$$F_{eff} = 2F_1 + F_2 \qquad s_{eff} = s_1 = s_2 \Rightarrow D_{eff} = \frac{F_{eff}}{s_{eff}} = \frac{2F_1}{s_1} + \frac{F_2}{s_2} = 2D_1 + D_2$$

A2) 5/5

$$m_R = 60\,kg - \boxed{1\,\frac{kg}{m} \cdot x_1}$$

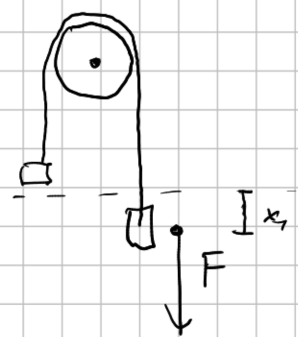
besser: $\frac{x_1}{L} \cdot m_{Seil}$, ist anschaulicher und man benutzt keine Zahlen!

($m$ wird in dieser Aufgabe ausschließlich als Einheit verwendet.)

$$m_L = 60\,kg + 1\,\frac{kg}{m} \cdot x_1$$

$$F = g \cdot (m_L - m_R) = g\, 2\frac{kg}{m}\, x_1$$

$$a = \frac{F}{m} = g\frac{2\,kg}{120\,kg} \cdot \frac{x_1}{m} = \frac{1}{60} g \frac{x_1}{m} = \ddot{x}_1$$




Warum setzt ihr Zahlen ein??

$$x_1 = Ae^{ct} + Be^{-ct}$$

$$\dot{x}_1 = cAe^{ct} - cBe^{-ct} = V_{10} = 0$$

$$\Rightarrow cA - cB = 0 \Leftrightarrow A = B$$

$$\ddot{x}_1 = c^2Ae^{ct} + c^2Ae^{-ct}$$

$$c^2(Ae^{ct} + Ae^{-ct}) = \frac{1}{60}\frac{g}{m}(Ae^{ct} + Ae^{-ct})$$

$$\Rightarrow c = \pm\sqrt{\frac{1}{60}\frac{g}{m}}$$

$$x_1 = A(e^{ct} + e^{-ct})$$

$$x_1(0) = x_{10} = 2A \Rightarrow A = \frac{x_{10}}{2}$$

$$x_1(t) = \frac{x_{10}}{2}\left(e^{\sqrt{\frac{1}{60}\frac{g}{m}}t} + e^{-\sqrt{\frac{1}{60}\frac{g}{m}}t}\right)$$

A3 5/5


$$r(t) = \begin{bmatrix} V_0 \cos(\alpha) t \\ V_0 \sin(\alpha) t - \frac{1}{2} g t^2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow t = \frac{x}{V_0 \cos(\alpha)}$$

$$\Rightarrow y = \tan(\alpha) \cdot x - \frac{g}{2} \frac{x^2}{V_0^2 \cos(\alpha)^2}$$

$$y' = \tan(\alpha) - \frac{g \cdot x}{V_0^2 \cos(\alpha)^2} \overset{!}{=} 0 \Leftrightarrow \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{g x}{\cos(\alpha)^2 V_0^2}$$

$$\Rightarrow x_{max} = \frac{\sin(\alpha) \cdot V_0^2 \cdot \cos(\alpha)}{g}$$

$$y(x_{max}) = \frac{\sin(\alpha)^2 \cdot V_0^2}{g} - \frac{1}{2} \frac{\sin(\alpha)^2 \cdot V_0^2}{g} = \frac{\sin(\alpha)^2 \cdot V_0^2}{2g} = 0{,}01m \Rightarrow V_0^2 = \frac{2g \cdot 0{,}01m}{\sin(\alpha)^2}$$

$$\Rightarrow y = \tan(\alpha) \cdot x - \frac{25}{m} x^2 \cdot \tan(\alpha)^2$$

$$r(t_{end}) = \begin{bmatrix} 0{,}1m \\ -0{,}05m \end{bmatrix} \Rightarrow -0{,}05m = \tan(\alpha) \cdot 0{,}1m - \frac{1m^2}{4m} \cdot \tan(\alpha)^2$$

$$\Leftrightarrow \quad = \tan(\alpha)^2 - 0{,}4 \tan(\alpha) - 0{,}2$$

$$\Rightarrow \tan(\alpha) = 0{,}2 \pm \sqrt{0{,}04 + 0{,}2} = 0{,}2 \pm \sqrt{0{,}24} \geq 0 \Rightarrow \tan(\alpha) = 0{,}2 + \sqrt{0{,}24}$$

$$\Rightarrow \alpha = \arctan(0{,}2 + \sqrt{0{,}24}) \approx 0{,}6039 \approx 34{,}6017°$$

$$\Rightarrow V_0 = \sqrt{\frac{2g \cdot 0{,}01m}{\sin(\alpha)^2}} \approx 0{,}7800 \frac{m}{s}$$

A4) $v_0 = 20\,\frac{m}{s}$

3,5 / 4

$$\dot v = -g - \frac{v\gamma_s}{m}$$

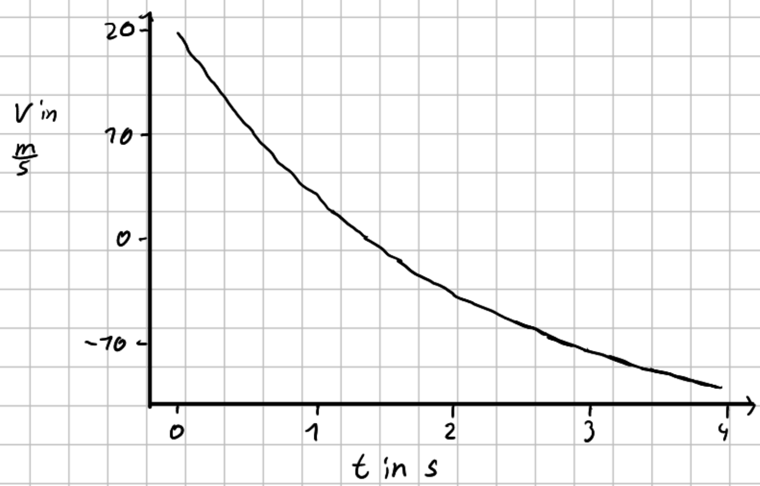
$$\dot v + \frac{\gamma_s}{m} v = -g$$

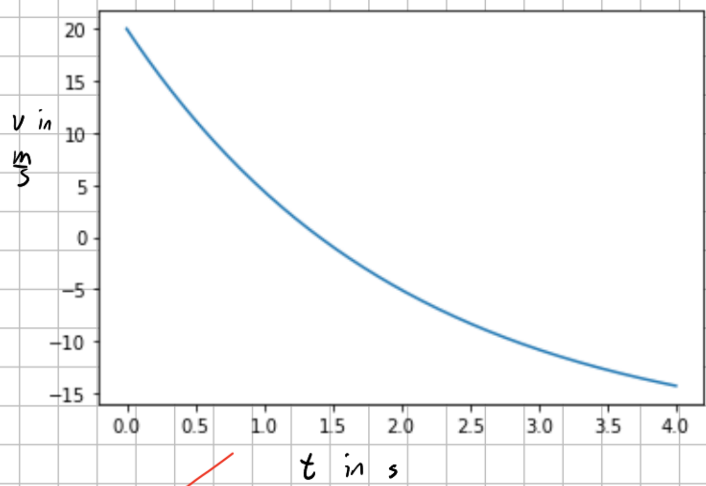
Warum setzt ihr einmal Zahlen ein und einmal nicht?

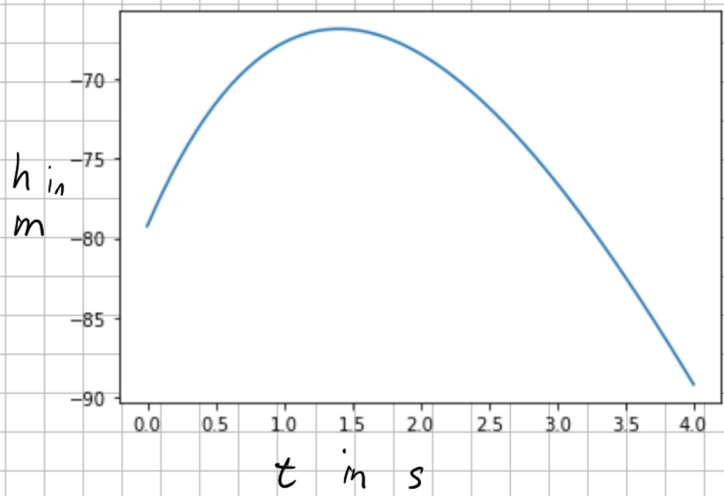
$$\Rightarrow v = e^{-\frac{\gamma_s}{m}t} \cdot \left[\int -g\, e^{\frac{\gamma_s}{m}t}\, dt + C\right] = C e^{-\frac{\gamma_s}{m}t} - g\,\frac{m}{\gamma_s}$$

$$v(0) = -\frac{m}{\gamma_s}\, g + C \overset{!}{=} 20\,\frac{m}{s}$$

$$\Leftrightarrow C = 20\,\frac{m}{s} + \frac{1\,kg}{0{,}5\,\frac{kg}{s}} \cdot 9{,}81\,\frac{m}{s^2} = 39{,}62\,\frac{m}{s} \qquad \Rightarrow 39{,}62\,\frac{m}{s} \cdot e^{-\frac{\gamma_s}{m}t} - g\,\frac{m}{\gamma_s}$$



# Ex_5

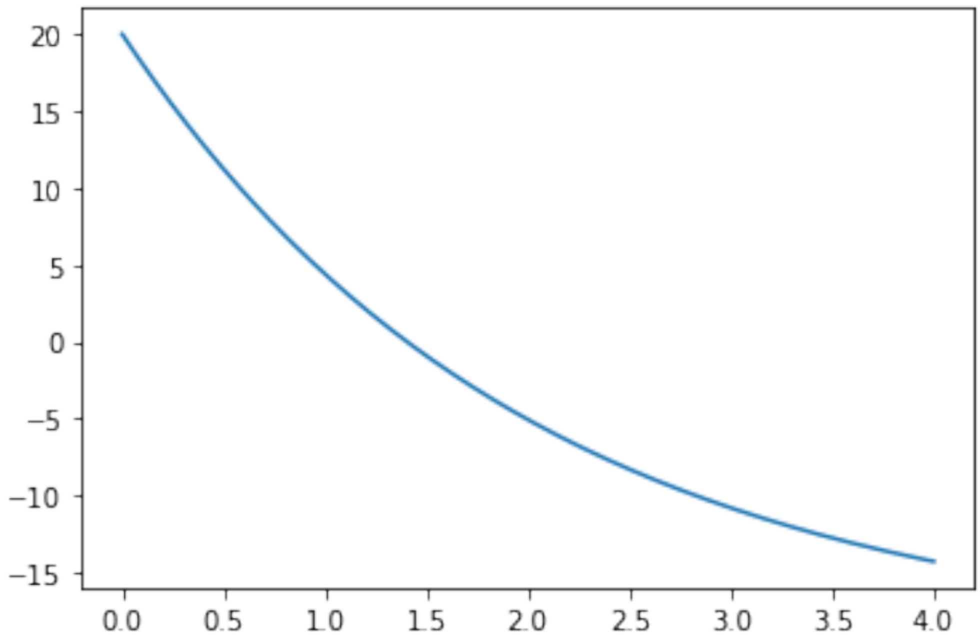
November 28, 2022

[8]:
```
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0,4,200)
v = -1/0.5*9.81 + 39.62*np.e**(-0.5*x)


plt.plot(x,v)
```

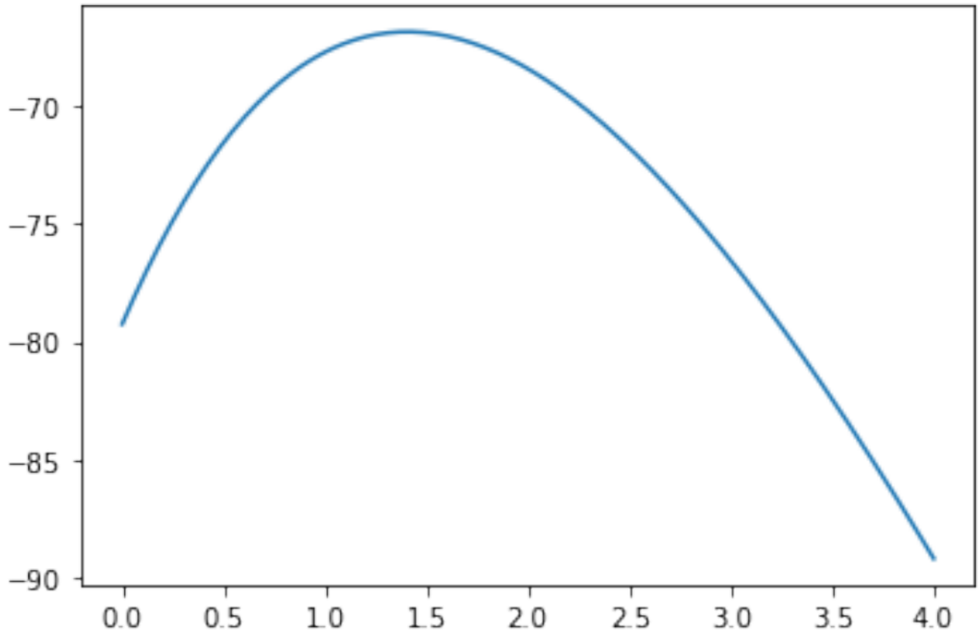
[8]: `[<matplotlib.lines.Line2D at 0x7f0b0a92a8c0>]`




[9]:
```
y = -1/0.5*9.81*x + -2*39.62*np.e**(-0.5*x)

plt.plot(x,y)
```

[9]: `[<matplotlib.lines.Line2D at 0x7f0b02808bb0>]`




[29]:
```
x = np.linspace(0,0.1,100)
y = np.tan(0.6039)*x-9.81/2*x**2/0.78**2/cos(0.6039)**2
plt.plot(x,y)
```

[29]: 'Aufgabe 3'