A1) a) $4\left[\cos(x)\cdot\cos(y) + \sin(y)\sin(x)\right]$

3/3

$= (e^{ix}+e^{-ix})(e^{iy}+e^{-iy}) - (e^{ix}-e^{-ix})(e^{iy}-e^{-iy})$

$= \cancel{e^{i(x+y)}} + e^{i(x-y)} + e^{-i(x-y)} + \cancel{e^{-(x+y)}} - \cancel{e^{i(x+y)}} + e^{i(x-y)} + e^{-i(x-y)} - \cancel{e^{-(x+y)}}$

$= 2\, e^{i(x-y)} + e^{-i(x-y)} = 4\cos(x-y)$

b) Hilfsbeweis:

$4i(\sin(x)\cos(y) + \sin(y)\cos(x))$

$= ((e^{ix}-e^{-ix})(e^{iy}+e^{-iy}) + (e^{iy}-e^{-iy})(e^{ix}+e^{-ix}))$

$= (e^{i(x+y)} + \cancel{e^{i(x-y)}} - \cancel{e^{-i(x-y)}} - e^{-i(x+y)} + e^{i(x+y)} + \cancel{e^{-i(x-y)}} - \cancel{e^{i(x-y)}} - e^{-i(x+y)})$

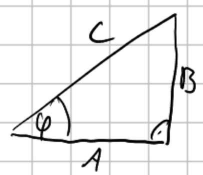
$2(e^{i(x+y)} - e^{-i(x+y)}) = 4i\sin(x+y)$

Vielleicht kurz was dazu sagen als Begründung

$C\cdot\sin(x+\varphi) = C\cdot(\sin(x)\cos(\varphi) + \cos(x)\sin(\varphi))$

$= C\cdot\left(\sin(x)\frac{B}{C} + \cos(x)\frac{A}{C}\right)$

$= B\sin(x) + A\cos(x)$



c) $\sin(x-y) + \sin(x+y)$ | Beweis siehe A1 b)

$= \sin(x)\cos(-y) + \sin(-y)\cos(x) + \sin(x)\cos(y) + \sin(y)\cos(x)$

$= \sin(x)\cos(y) - \cancel{\sin(y)\cos(x)} + \sin(x)\cos(y) + \cancel{\sin(y)\cos(x)}$

$= 2\sin(x)\cos(y)$

A2) $\frac{F_g}{m} = -g\frac{x}{R} = \ddot{x}$

4/4

$\dot{x}_0 = 0\,\frac{m}{s}$

$x_0 = R$

$\Rightarrow x(t) = A\cdot\cos\left(\sqrt{\frac{g}{R}}\,t\right)$

$\Rightarrow x_0 = A\cdot\cos(0) = R \Rightarrow A = R$

$x(t) = R\cos\left(\sqrt{\frac{g}{R}}\,t\right) \overset{!}{=} -R$

$\Rightarrow -1 = \cos\left(\sqrt{\frac{g}{R}}\,t_{end}\right)$

$\cos^{-1}(-1) = \pi = \sqrt{\frac{g}{R}}\,t_{end} \quad \Big| : \sqrt{\frac{g}{R}}$

$\Rightarrow \pi\sqrt{\frac{R}{g}} = t_{end} \approx 2537{,}5\,s$

$v(t) = -\sqrt{gR}\,\sin\left(\sqrt{\frac{g}{R}}\,t\right)$ $\quad v\left(\frac{t_{end}}{2}\right) = -\sqrt{gR}\,\sin\left(\frac{\pi}{2}\right) = -\sqrt{\frac{g}{R}} \approx -7923{,}6\,\frac{m}{s}$

5/5

A3) Es gibt zwei Anfangsbedingungen x_0 und v_0.
Ein Ansatz mit nur einem freien Parameter wäre überbestimmt.

$$\ddot{x} - 2a\dot{x} + \omega_0^2 x = 0 \quad \text{mit} \quad x = A e^{at}$$

$$\ddot{A}e^{at} + 2\dot{A}ae^{at} + Aa^2e^{at} - 2a\dot{A}e^{at} - 2a^2Ae^{at} + \omega_0^2 Ae^{at} = 0 \quad \Big| : e^{at}$$

$$\ddot{A} + 2\dot{A}a + Aa^2 - 2a\dot{A} - 2a^2A + \omega_0^2 A = 0$$

$$\ddot{A} + \underbrace{\dot{A}(2a - 2a)}_{=0} + \underbrace{A(a^2 - 2a^2 + \omega_0^2)}_{=0} = 0 \quad \Big| \text{ da } \omega_0^2 = a^2$$

$$\ddot{A} = 0$$

$$x = (B + Ct)e^{at}$$

$$\dot{x} = a(B + Ct)e^{at} + Ce^{at}$$

$$x(0) = B = x_0$$

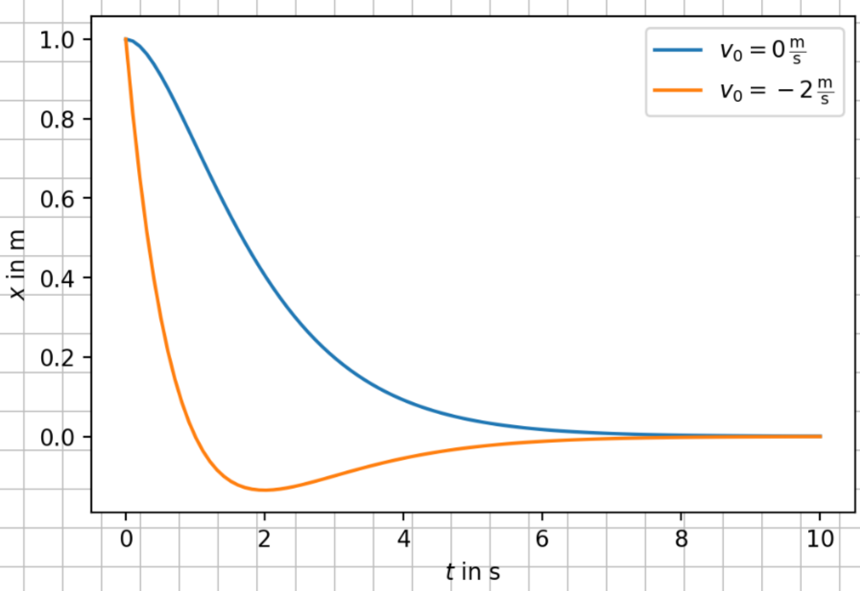
$$\dot{x}(0) = ax_0 + C = v_0$$

$$\Rightarrow C = v_0 - ax_0$$

$$\Rightarrow x(t) = (x_0 + (v_0 - ax_0)t)\, e^{at}$$

mit $a = -\omega_0$ ergibt sich

$$\Rightarrow x(t) = (x_0 + (v_0 + \omega_0 x_0)t)\, e^{-\omega_0 t}$$



A4) 3/3

$D = 5 \cdot 10^4 \frac{N}{m}$ $\quad M = 800\,kg$

=> Vereinfachung mit 1. Feder und $m = 200\,kg$

$\frac{D}{m} = \omega_0^2 \overset{!}{=} \gamma^2 = \frac{\gamma_s^2}{4m^2}$

$2\sqrt{Dm} = \gamma_s = 2\sqrt{5 \cdot 10^4 \frac{N}{m} \cdot 200\,kg} \approx 6324.55 \frac{kg}{s}$

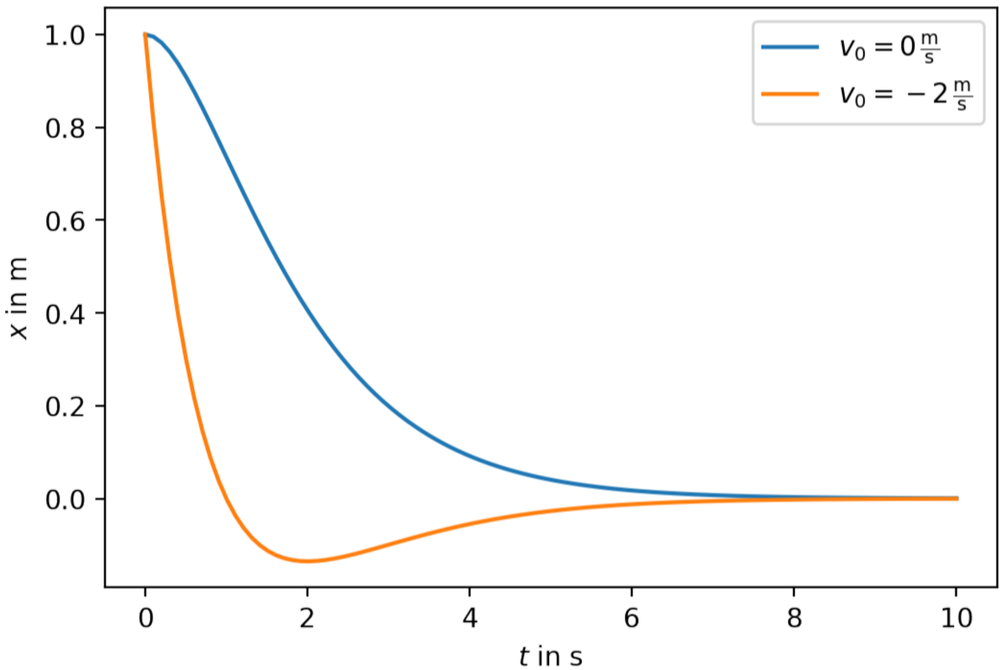
Ex_7

December 10, 2022

[11]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy import e
import matplotlib as mpl
mpl.rcParams['figure.dpi'] = 200

w = 1
v0s = [0, -2]
x0 = 1
t = np.linspace(0,10,100)

for v0 in v0s:
    x = (x0 + (v0+w*x0)*t )*e**(-w*t)
    plt.plot(t,x, label=f"$v_0 = {v0}␣
 ↪\\,\\frac{{\\mathrm{{m}}}}{{\\mathrm{{s}}}}$")
    plt.legend()
    plt.ylabel("$x$ in m")
    plt.xlabel("$t$ in s")
```


1.0
0.8
0.6
0.4
0.2
0.0
x in m
0
2
4
6
8
10
t in s
$v_0 = 0\,\frac{m}{s}$
$v_0 = -2\,\frac{m}{s}$