A1) 7/8

a)

$$H := \begin{bmatrix} -\frac{2D}{M} & \frac{D}{M} & \frac{D}{M} \\ \frac{D}{m} & -\frac{D}{m} & 0 \\ \frac{D}{m} & 0 & -\frac{D}{m} \end{bmatrix} \qquad \ddot{\vec{x}} = H\vec{x}$$

besser → am Ende wird eher ω auch imaginär ↯ ↳ nicht sinnvoll

Ansatz: $\vec{x}(t) = \vec{a}\, e^{i\omega t}$ $\Rightarrow \ddot{\vec{x}}(t) = \omega^2 \vec{x}(t) = H\vec{x}(t)$

$\Rightarrow \vec{0} = (H - \omega^2 I)\,\vec{a}$

Gleichungssystem:

$$\begin{bmatrix} \left(-\frac{2D}{M} - \omega^2\right) a_1 + \frac{D}{M} a_2 + \frac{D}{M} a_3 = 0 \\ \frac{D}{m} a_1 + \left(-\frac{D}{m} - \omega^2\right) a_2 + 0 = 0 \\ \frac{D}{m} a_1 + 0 - \left(\frac{D}{m} - \omega^2\right) a_3 = 0 \end{bmatrix}$$

b) Eigenwerte bestimmen:

$$0 \overset{!}{=} \begin{vmatrix} -\frac{2D}{M} - \lambda & \frac{D}{M} & \frac{D}{M} \\ \frac{D}{m} & -\frac{D}{m} - \lambda & 0 \\ \frac{D}{m} & 0 & -\frac{D}{m} - \lambda \end{vmatrix} = \left(-\frac{D}{m} - \lambda\right)^2 \left(-\frac{2D}{M} - \lambda\right) - \frac{2D^2}{mM}\left(-\frac{D}{m} - \lambda\right)$$

$$\left(-\frac{D}{m} - \lambda\right)\left[\left(-\frac{D}{m} - \lambda\right)\left(-\frac{2D}{M} - \lambda\right) - \frac{2D}{mM}\right] = 0$$

Scharfes hinsehen: $\lambda_1 = -\frac{D}{m}$

$\Rightarrow \left(\frac{D}{m} + \lambda\right)\left(\frac{2D}{M} + \lambda\right) = \frac{2D^2}{mM}$

$\Rightarrow \lambda^2 + \cancel{\frac{2D^2}{mM}} + \lambda\left(\frac{D(2m+M)}{mM}\right) = \cancel{\frac{2D^2}{mM}}$

$\Rightarrow \lambda\left(\lambda + \frac{D(2m+M)}{mM}\right) = 0$

$\Rightarrow \lambda_3 = -\frac{D(2m+M)}{mM}$ und $\lambda_2 = 0$

$\Rightarrow \omega^2 \in \{\lambda_1, \lambda_2, \lambda_3\}$

$$\frac{\omega_3}{\omega_1} = \pm\sqrt{\frac{\lambda_3}{\lambda_1}} = \pm\sqrt{\frac{D(2m+M)}{mM}\frac{m}{D}} = \pm\sqrt{2\frac{m}{M} + 1}$$

?

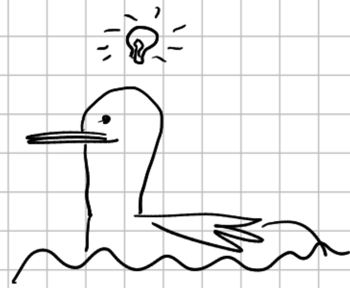
Negative Frequenzen ergeben keinen Sinn

c) Eigenvektoren:

zu $\lambda = -\frac{D}{m}$

$$\left[\begin{array}{ccc|c} -\frac{2D}{M}+\frac{D}{m} & \frac{D}{M} & \frac{D}{M} & 0 \\ \frac{D}{m} & -\frac{D}{m}+\frac{D}{m} & 0 & 0 \\ \frac{D}{m} & 0 & -\frac{D}{m}+\frac{D}{m} & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} -\frac{2D}{M}+\frac{D}{m} & \frac{D}{M} & \frac{D}{M} & 0 \\ \frac{D}{m} & 0 & 0 & 0 \\ \frac{D}{m} & 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{a}_1 = 0 \text{ und } \vec{a}_2 = -\vec{a}_3 \Rightarrow \vec{a} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$



Eigenvektor

$\lambda = -\frac{2Dm + DM}{mM}$

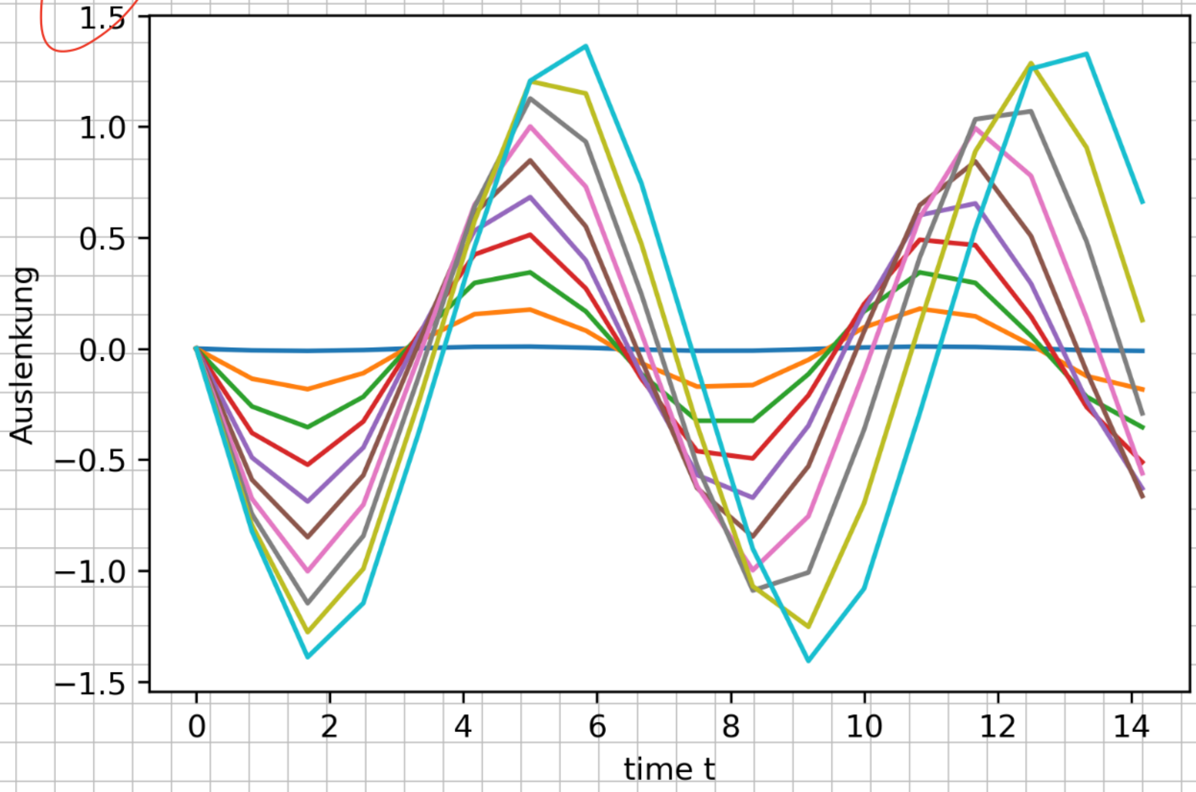
$$\left[\begin{array}{ccc|c} \frac{D}{m} & \frac{D}{M} & \frac{D}{M} & 0 \\ \frac{D}{m} & 2\frac{D}{M} & 0 & 0 \\ \frac{D}{m} & 0 & 2\frac{D}{M} & 0 \end{array}\right] \begin{array}{l} \ominus \\ \ominus \end{array}$$

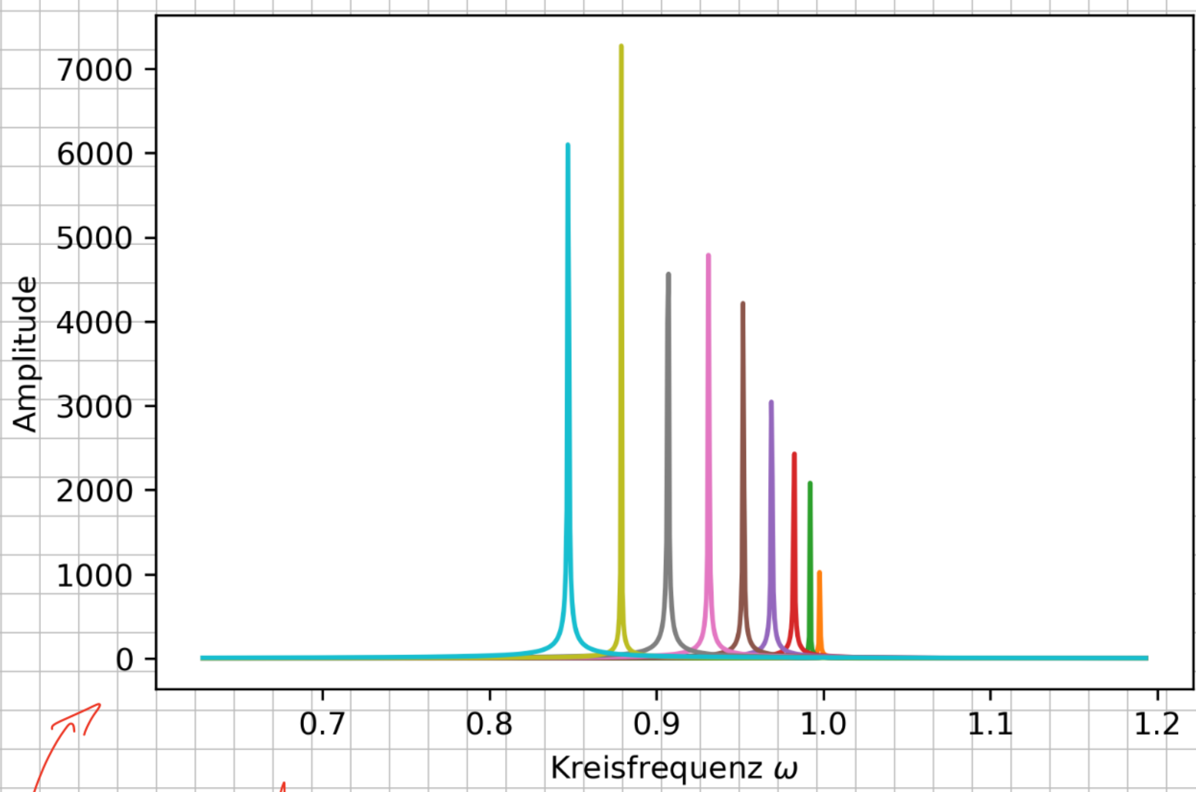
$$\left[\begin{array}{ccc|c} 0 & -\frac{D}{M} & \frac{D}{M} & 0 \\ \frac{D}{m} & 2\frac{D}{M} & 0 & 0 \\ 0 & -2\frac{D}{M} & 2\frac{D}{M} & 0 \end{array}\right] \begin{array}{l} 2\times \\ \ominus\, 2\times \quad \oplus \end{array}$$

$$\left[\begin{array}{ccc|c} \frac{D}{m} & 0 & 2\frac{D}{M} & 0 \\ \frac{D}{m} & 2\frac{D}{M} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} :\frac{D}{m} \\ :\frac{D}{m} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2\frac{m}{M} & 0 \\ 1 & 2\frac{m}{M} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{a} = \begin{bmatrix} 1 \\ -\frac{M}{2m} \\ -\frac{M}{2m} \end{bmatrix}$$

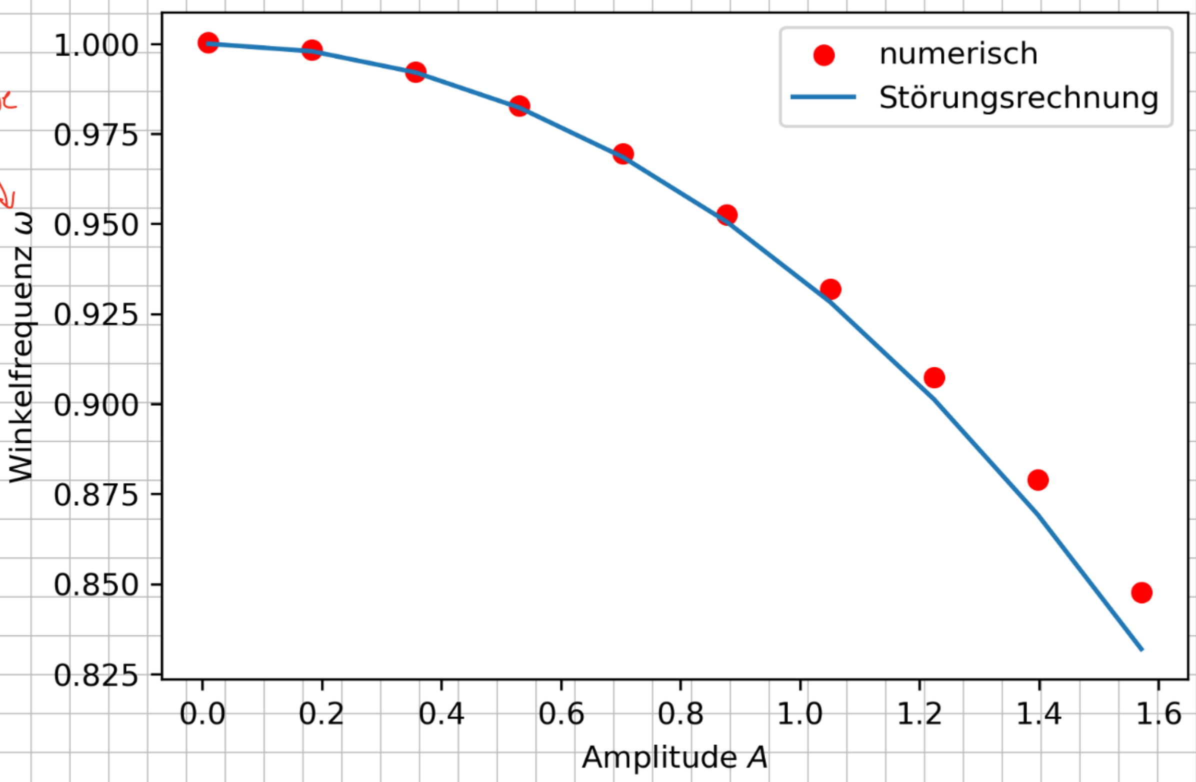
Aufgabe 2 (6/6)

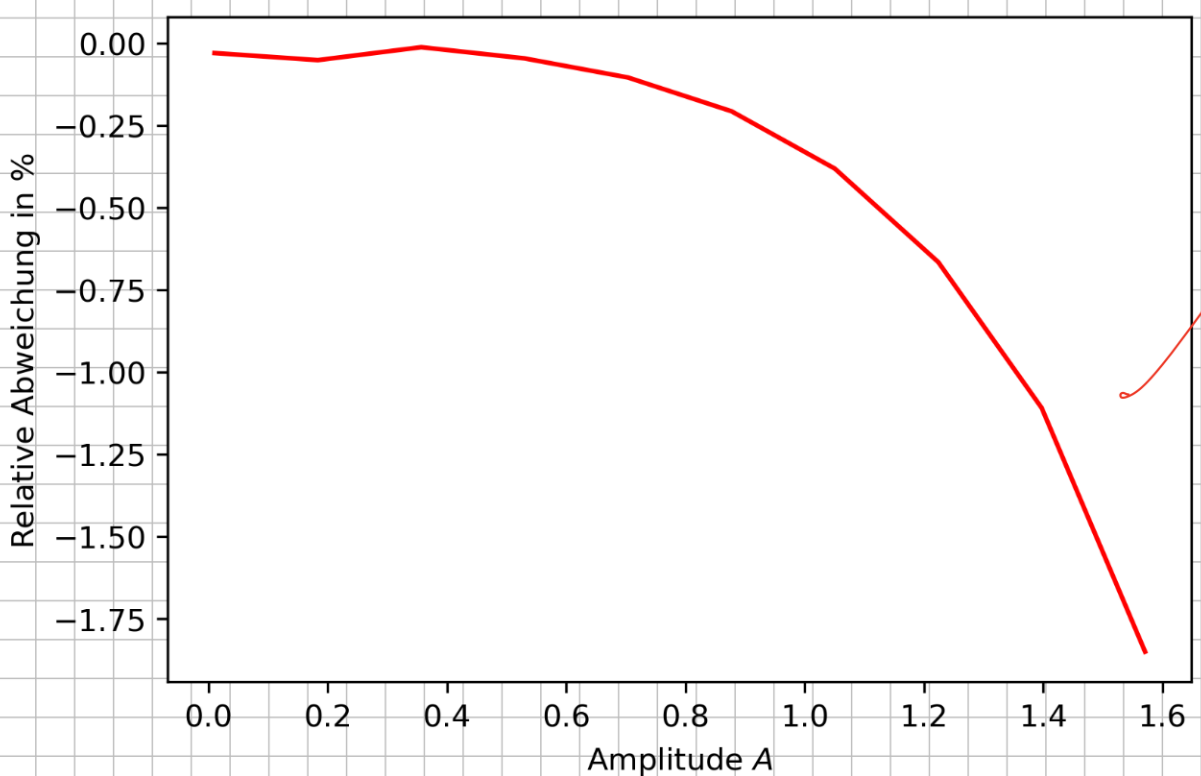




Schreibt dazu was die Farben bedeuten

ich glaube das ist ϵ, nicht ω





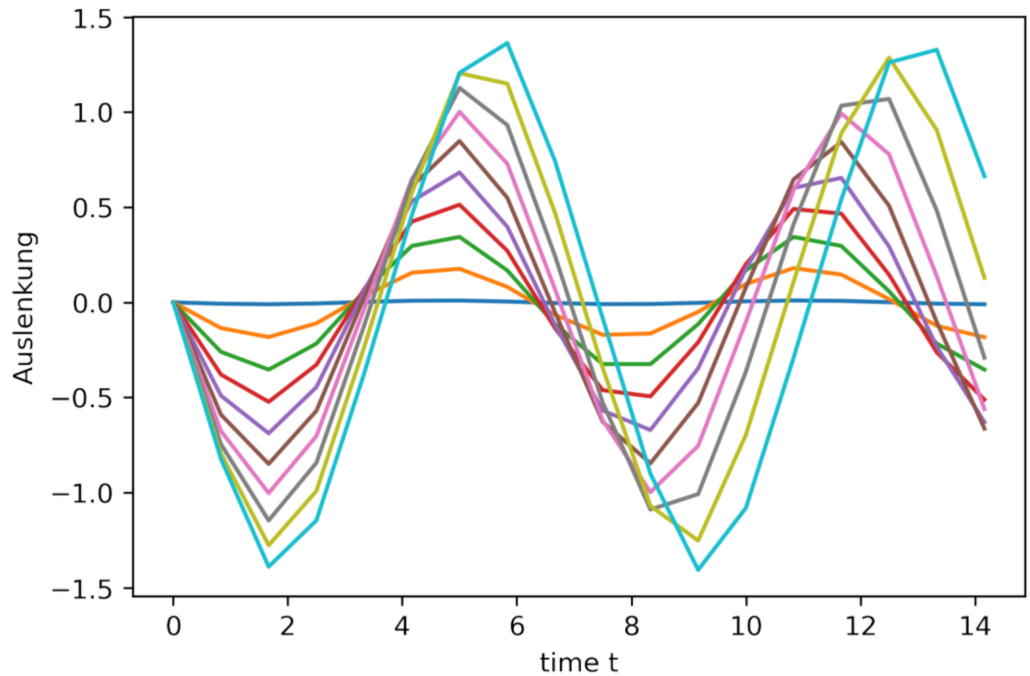
Ex_9

January 8, 2023

[2]:

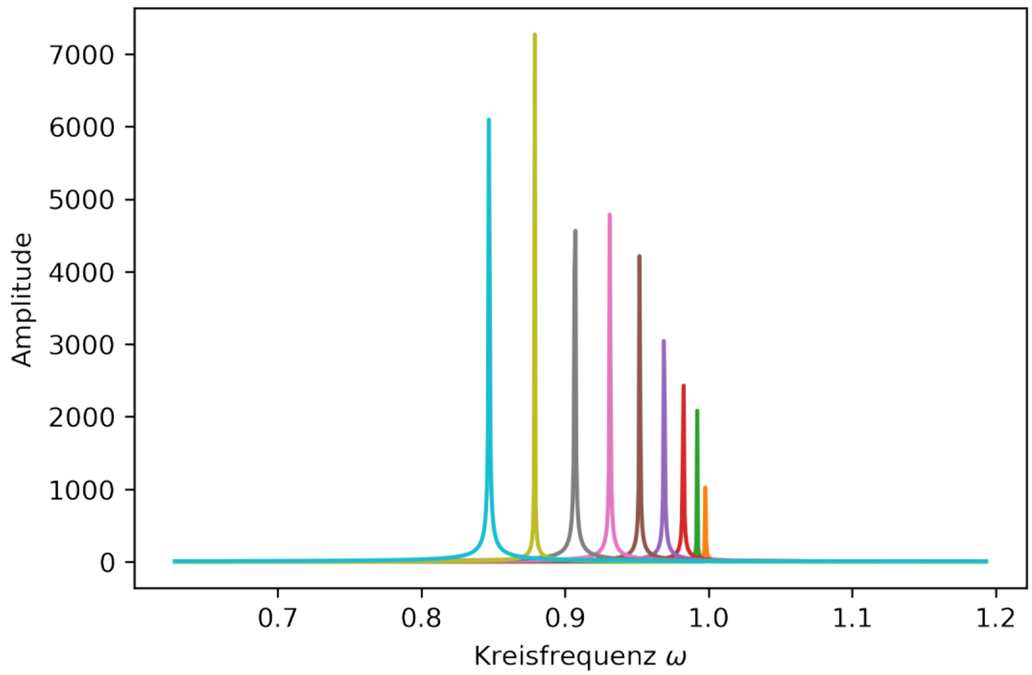
```
import numpy as np
import sympy
import matplotlib.pyplot as plt
from scipy.fft import fft, ifft, fftfreq
from scipy.integrate import odeint
import matplotlib as mpl
mpl.rcParams['figure.dpi'] = 300


frequency_resolution = 0.0001
maximum_freq = 0.6

T = 1/ (2*maximum_freq)
N = round(1/frequency_resolution/T)

print(f"{T=}, {N=}")

A0s = np.linspace(0.01, np.pi/2, 10)
t = np.linspace(0, N*T, N, endpoint=False)

curves = []

for A0 in A0s:
    def step(y, t):
        A, omega = y
        return [omega,-np.sin(A)]
    curve = odeint(step, [A0, 0.0], t)[:,1]

    curves.append(curve)

    plt.plot(t[:round(15/T)], curve[:round(15/T)])
plt.xlabel("time t")
plt.ylabel("Auslenkung")
plt.show()
```

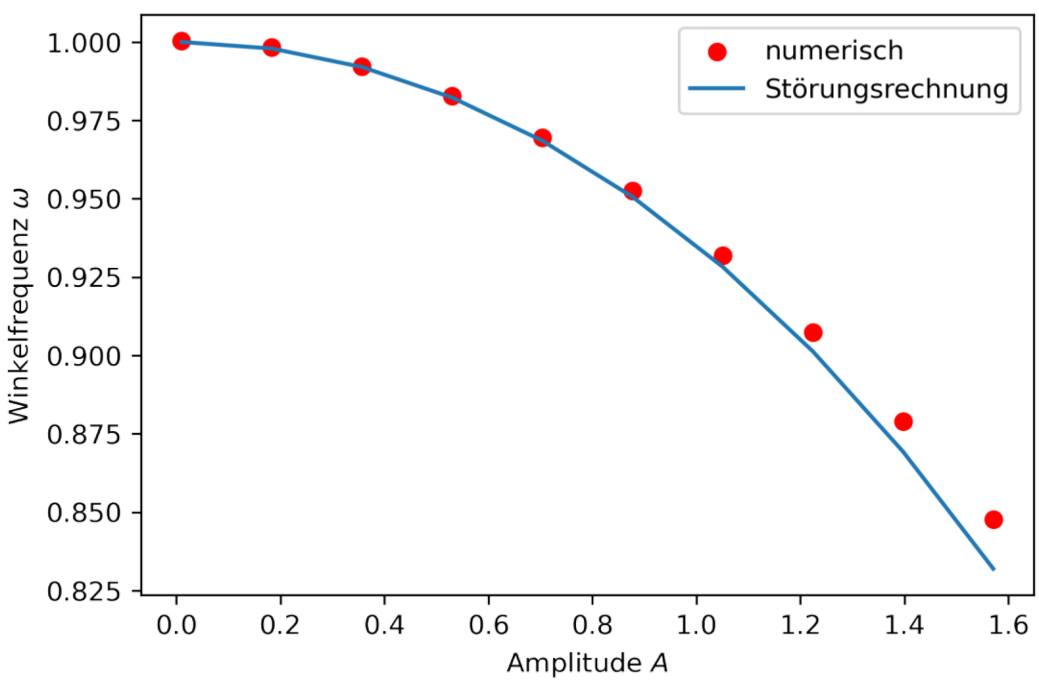
```
T=0.8333333333333334, N=12000
```

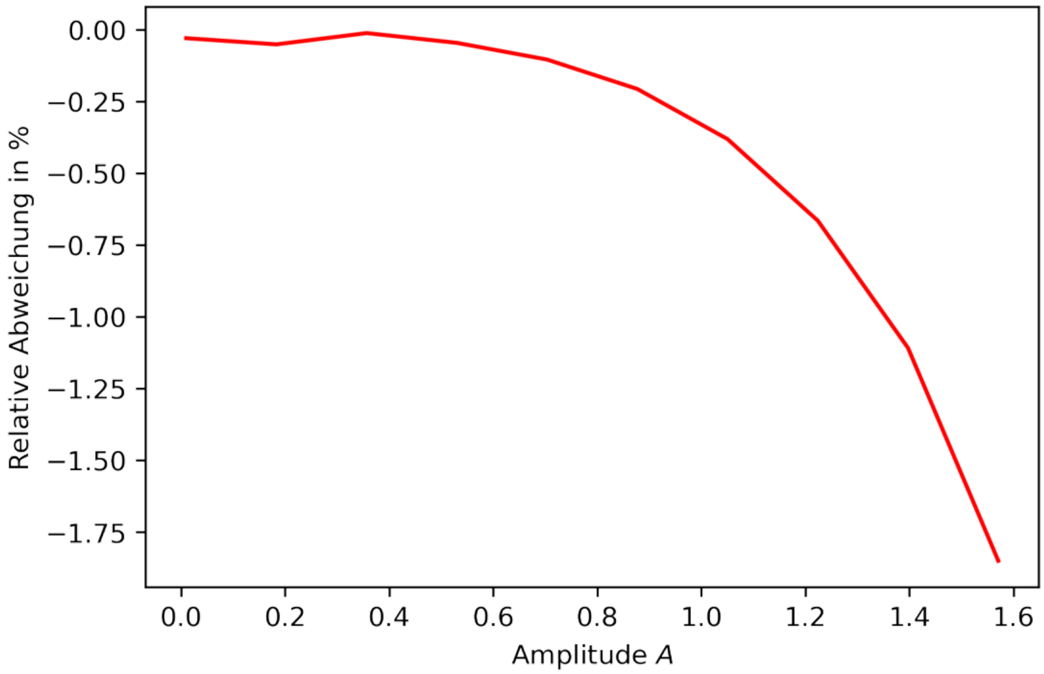


[7]:
```
eig_freqs = []

for curve in curves:
    ws = fftfreq(N, T)[:N//2]*2*np.pi
    As = fft(curve)[:N//2]

    plt.plot(ws[1000:1900],np.abs(As)[1000:1900])
    eig_freq = max(zip(ws, As), key=lambda tup: tup[1])[0]
    eig_freqs.append(eig_freq)
plt.ylabel("Amplitude")
plt.xlabel("Kreisfrequenz $\omega$")
plt.show()

w_stoer = 1-A0s**2/16-7*A0s**4/3072

fig, ax1 = plt.subplots()
ax1.set_ylabel("Winkelfrequenz $\omega$")
ax1.set_xlabel("Amplitude $A$")
ax1.scatter(A0s, np.array(eig_freqs), color="red", label="numerisch")
ax1.plot(A0s, w_stoer, label="Störungsrechnung")
ax1.legend()
plt.show()

fig, ax1 = plt.subplots()
```

```
ax1.set_xlabel("Amplitude $A$")
ax1.set_ylabel("Relative Abweichung in %")
ax1.plot(A0s, (-1+w_stoer/(np.array(eig_freqs)))*100, "-r")
plt.show()
```







0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
Relative Abweichung in %
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Amplitude A


```
[ ]:
```