

### Aufgabe 1) (3/3)

$$-F_g = F_z = \frac{m v_L^2}{r} \Leftrightarrow v_L^2 = \frac{F_g \cdot r}{m} = g \cdot r \Rightarrow E_{kin} = \frac{1}{2} m v_L^2 = \frac{1}{2} m g r$$

Die kinetische Energie am höchsten Punkt des Loopings ist

$$E_{kin} = m \cdot g \cdot (h - 2r) = \frac{1}{2} m g r$$

$$\Leftrightarrow h - 2r = \frac{1}{2} r$$

$$\Leftrightarrow h = \frac{5}{2} r$$

### Aufgabe 2) (5,5/6)

$$a) \frac{(-a m_1 + (L+b) m_2)}{m_1 + m_2} = s$$

$$b) 0 \leq s \leq L$$

$$c) F_a = \left(1 - \frac{s}{L}\right) \cdot (m_1 + m_2) g \quad F_b = \frac{s}{L} \cdot (m_1 + m_2) g$$

Redung  
dieses schreiben!

$$s = 0,7 \text{ m}$$

$$F_a = 8,829 \text{ N}$$

$$F_b = 20,601 \text{ N}$$

### Aufgabe 3) (7/7)

$$a) \vec{V}_0 = \hat{e}_2 \omega \times \vec{r}_0 = \begin{bmatrix} \omega R \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta t = \frac{\pi}{\omega}$$

$$b) \vec{V}_{ball} = \frac{-2 \vec{r}_0}{\Delta t} = \begin{bmatrix} 0 \\ \frac{2}{\pi} R \omega \\ 0 \end{bmatrix}$$

$$\vec{r}(t) = \vec{V}_{ball} \cdot t + \vec{r}_0 \quad \text{mit } t \in [0, \Delta t]$$

$$c) \vec{V}' = \vec{V}_{ball} - \vec{V}_0 = \begin{bmatrix} -\omega R \\ \frac{2}{\pi} R \omega \\ 0 \end{bmatrix}$$

$$\alpha = \arccos \frac{\vec{V}' \cdot \vec{e}_y}{|\vec{V}'|} = \arccos \left[ \frac{\frac{2}{\pi} R \omega}{\sqrt{(\omega R)^2 + \left(\frac{2}{\pi} R \omega\right)^2}} \right]$$

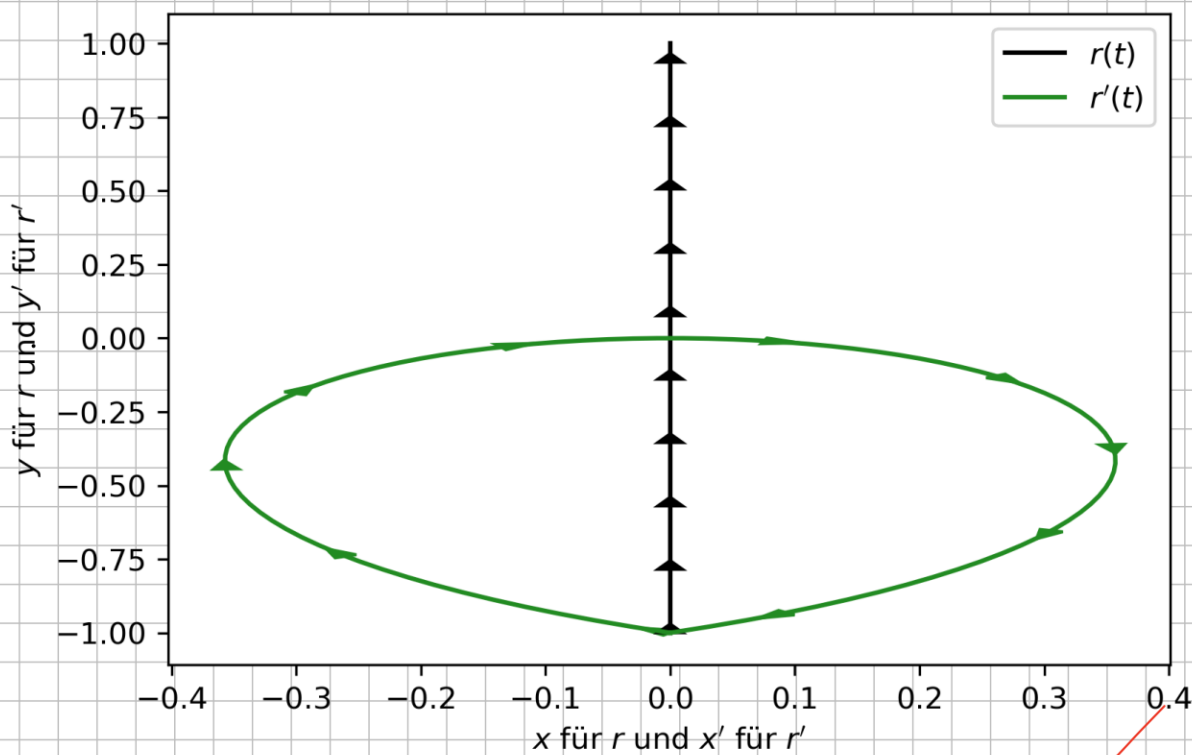
$$= \arccos \frac{2}{\pi \sqrt{1 + \frac{4}{\pi^2}}} \approx 57,518^\circ$$

d) Mit  $\vec{r}(t) = \left[ 0 \quad \frac{2}{\pi} R \omega t \right]^T$

$\vec{r}'(t) = A \cdot \vec{r}$  wobei  $A$  die Rotationsmatrix  $A: \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix}$   
mit  $\varphi = -\omega t$  da sich das Laborsystem  
relativ zum Karusellsystem mit  $-\omega$  dreht.

$$\Rightarrow \vec{r}'(t) = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \cdot \vec{r}(t).$$

$$= \begin{bmatrix} \sin(\omega t) \left( \frac{2}{\pi} R \omega t - R \right) \\ \cos(\omega t) \left( \frac{2}{\pi} R \omega t - R \right) \end{bmatrix}$$



## Ex\_10

January 11, 2023

```
[48]: import numpy as np
import matplotlib.pyplot as plt
from numpy import pi, sin, cos
import matplotlib as mpl
mpl.rcParams['figure.dpi'] = 300

omega = 1
R = 1
t_max = pi / omega
t = np.linspace(0, t_max, 100)

x_ = sin(omega*t)*(2/pi*R*omega*t - R )
y_ = cos(omega*t)*(2/pi*R*omega*t - R )

(omega*cos(omega*t)*(2/pi*R*omega*t - R ) + sin(omega*t)*2/pi*R*omega) * 0.0001
(-omega*sin(omega*t)*(2/pi*R*omega*t - R ) + cos(omega*t)*2/pi*R*omega) * 0.0001

y = 2/pi*R*omega*t - R
x = t*0

plt.plot(x,y, color="black", label="$r\\left(t\\right)$")
plt.plot(x_,y_, color="forestgreen", label="$r'\\left(t\\right)$")
plt.legend()
plt.xlabel("$x$ für $r$ und $x'$ für $r'$")
plt.ylabel("$y$ für $r$ und $y'$ für $r'$")

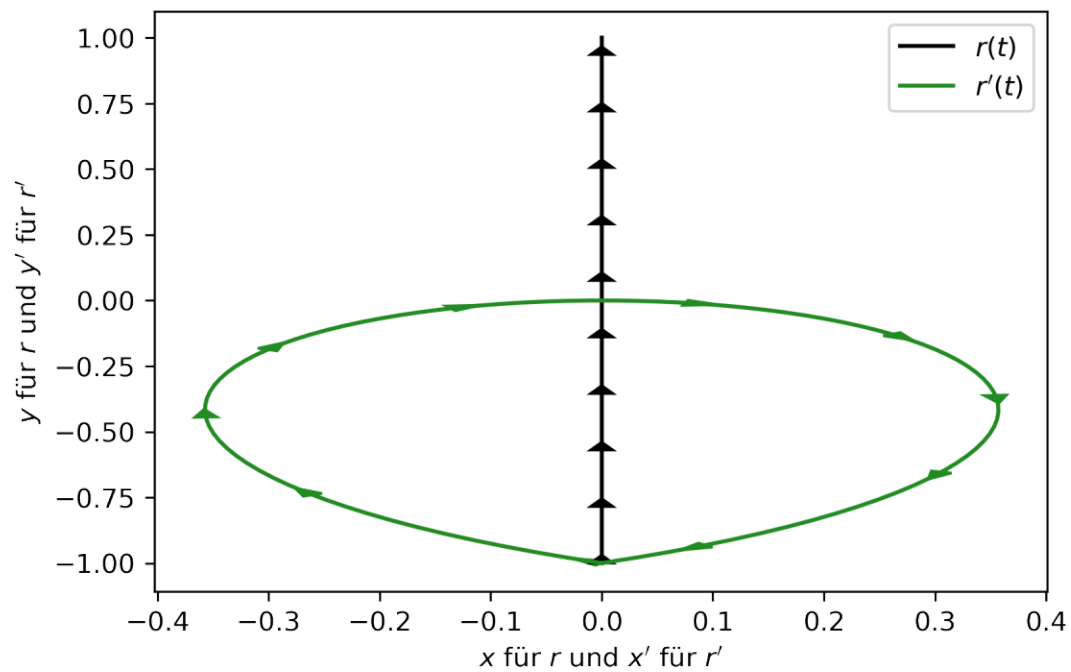
# Ab hier nur Code für Pfeile
numarrows = 10
[plt.arrow(
    sin(omega*i)*(2/pi*R*omega*i - R ),
    cos(omega*i)*(2/pi*R*omega*i - R ),
    (omega*cos(omega*i)*(2/pi*R*omega*i - R ) + sin(omega*i)*2/pi*R*omega) * 0.
↪0001,
    (-omega*sin(omega*i)*(2/pi*R*omega*i - R ) + cos(omega*i)*2/pi*R*omega) * 0.
↪0001,
    overhang=0, head_width=0.02, color="forestgreen")
```

```

for i in np.linspace(0, t_max-0.1, numarrows)]

[plt.arrow(0, 2/pi*R*omega*i - R, 0, 0.0001, overhang=0, head_width=0.02,
↪color="black") for i in np.linspace(0, t_max-0.1, numarrows)]
None

```



[ ]: