Aufgabe 1 $\frac{3{,}5}{5}$

$$\vec{F}_c = -2m\,\vec{\omega}\times\vec{v}\,' = -2m\,\vec{\omega}\times\vec{e}_\phi v = 2m\omega v\,\hat{e}_r' = 0{,}002\,N\cdot\hat{e}_r'$$

$$\vec{F}_z = -m\,\vec{\omega}\times(\vec{\omega}\times\vec{r}\,') = m\omega^2\vec{r}\,' = m\omega^2 r\,\hat{e}_r' = 0{,}04\cdot\hat{e}_r'$$

$$\vec{F}_s = \vec{F}_c + \vec{F}_z = 0{,}042N\;\hat{e}_r'$$

Die Zentrifugalkraft des Käfers wegen seiner Geschwindigkeit fehlt

$$m_s = \frac{|\vec{F}_s|}{g} = 0{,}00428\,kg = 4{,}28\cdot 10^{-3}\,kg$$

Aufgabe 2 $\frac{5}{5}$

Kathesisch: Zylinder Reicht

$$q := \tfrac{1}{2}L \qquad g(z) := \cos(\sin^{-1}(\tfrac{z}{R}))R = R\sqrt{1-(\tfrac{z}{R})^2}$$

$$\Theta = \int_{-R}^{R}\int_{-q}^{q}\int_{-g(\frac{z}{R})}^{g(\frac{z}{R})}(x^2+y^2)\frac{m}{V}\,dx\,dy\,dz$$

$$= \frac{m}{V}\int_{-R}^{R}\int_{-g(\frac{z}{R})}^{g(\frac{z}{R})}\int_{-q}^{q}x^2+y^2\,dy\,dx\,dz$$

$$= \frac{m}{V}\int_{-R}^{R}\int_{-g(\frac{z}{R})}^{g(\frac{z}{R})}\left[yx^2+\tfrac{1}{3}y^3\right]_{-q}^{q}dx\,dz$$

$$= \frac{m}{V}\int_{-R}^{R}\int_{-g(\frac{z}{R})}^{g(\frac{z}{R})}2q\,x^2+\tfrac{2}{3}q^3\,dx\,dz$$

$$= \frac{m}{V}\int_{-R}^{R}\left[\tfrac{2}{3}q\,x^3+\tfrac{2}{3}q^3x\right]_{-g(\frac{z}{R})}^{g(\frac{z}{R})}dz$$

$$= \frac{m}{V}\int_{-R}^{R}\tfrac{4}{3}q\,g(\tfrac{z}{R})^3+\tfrac{4}{3}q^3g(\tfrac{z}{R})\,dz$$

Mathematica

$$\overset{\downarrow}{=} \frac{1}{12}\frac{m}{V}L\pi R^2(L^2+3R^2)$$

$$= \frac{m}{12}(L^2+3R^2)$$

Zylinder:

$$\Theta = \frac{m}{V}\int_0^R r\int_0^{2\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\lambda^2+\cos(\phi)^2r^2\,d\lambda\,d\phi\,dr$$

$$= \frac{m}{V}\int_0^R r\int_0^{2\pi}\left[\tfrac{1}{3}\lambda^3+\cos^2(\phi)r^2\right]_{-\frac{L}{2}}^{\frac{L}{2}}d\phi\,dr$$

$$= \frac{m}{V}\int_0^R r\int_0^{2\pi}\tfrac{2}{3}\tfrac{L^3}{8}+\cos^2(\phi)r^2L\,d\phi\,dr$$

$$= \frac{m}{V}\int_0^R r\left[\tfrac{2}{3}\tfrac{L^3}{8}\phi+r^2L\left(\tfrac{\phi}{2}+\tfrac{1}{4}\sin(2\phi)\right)\right]_0^{2\pi}dr$$

$$= \frac{m}{V}\int_0^R r\,\tfrac{4}{3}\tfrac{L^3}{8}\pi+r^3L\pi\,dr$$

$$= \frac{m}{V}\left[r^2\tfrac{2}{3}\tfrac{L^3}{8}\pi+\tfrac{1}{4}r^4\pi L\right]_0^R$$

$$= \frac{m}{V}\left[R^2\tfrac{1}{12}L^3\pi+\tfrac{1}{4}R^4\pi L\right]$$

$$= \frac{1}{12}\frac{m}{V}L\pi R^2(L^2+3R^2)$$

$$= \frac{m}{12}(L^2+3R^2)$$

Aufgabe 3 $\frac{5}{5}$

$$\Theta = r\int_0^{2\pi} r^2 \frac{m}{2\pi r}\, d\phi = 2\pi r^2 \frac{m}{2\pi} = r^2 m$$

$$E = F_H \cdot s = \frac{1}{2}\Theta \cdot \omega^2 + \frac{1}{2}v^2 m = \frac{1}{2}\Theta \cdot \frac{v^2}{r^2} + \frac{1}{2}v^2 m$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = m\dot{s}^2 \overset{!}{=} s \cdot F_H$$

Geht auch einfacher, aber warum nicht :)

Ansatz: $at^n = s(t)$

$$\Rightarrow F_H\, at^n = m(an\, t^{(n-1)})^2$$

$$= m(an)^2\, t^{2n-2}$$

$$\Rightarrow n = 2n-2 \quad | +2 - n$$

$$2 = n$$

$$\Rightarrow F_H\, a = 4ma^2$$

$$\Rightarrow a = 0 \text{ oder } a = \frac{F_H}{4m}$$

$$\Rightarrow s(t) = \frac{F_H}{4m} t^2 \quad (\text{Vollständig, da } s(0) \overset{!}{=} 0 \text{ und } \dot{s}(0) \overset{!}{=} 0)$$

$$\Rightarrow a = \frac{F}{2m}$$

$F_H = m \cdot g \cdot \sin\varphi$

$$\Rightarrow \alpha = \dot{\omega} = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{v}{r}\right) = \frac{a}{r} = \frac{F}{2mr}$$

$$\Rightarrow M_C = \dot{\omega}\Theta = \frac{F}{2mr} r^2 m = \frac{1}{2}Fr \quad \text{mit } M_C \hat{=} \text{ Drehmoment um Zylindermitte}$$

$$M = \dot{\omega}(\Theta + m \cdot r^2) = \frac{F}{2mr} 2mr^2 = F \cdot r$$

In[ ]:= `A = Integrate[x^2 + y^2, {y, -L/2, L/2}]`

Out[ ]= $\frac{L^3}{12} + L\,x^2$

In[ ]:= `B = Integrate[A, {x, -Cos[ArcSin[z/R]] R, Cos[ArcSin[z/R]] R}]`

Out[ ]= $\frac{1}{6}\,L^3\,R\,\sqrt{1-\frac{z^2}{R^2}} + \frac{2}{3}\,L\,R^3\left(1-\frac{z^2}{R^2}\right)^{3/2}$

In[ ]:= `Integrate[B, {z, -R, R}]`

Out[ ]= $\frac{1}{12}\,L\,\pi\,R^2\left(L^2+3\,R^2\right)$