Aufgabe 1) (4/4)

$\frac{m}{\pi r^2} = \sigma \rightarrow \Theta = \sigma \int dV \, r_\perp^2$

a) $\frac{m}{\pi r^2} = \frac{m}{V} \int_0^r r \int_0^{2\pi} r^2 \, d\varphi \, dr = \frac{m}{\pi r^2} \int_0^r 2\pi r^3 \, dr = \frac{m}{\pi r^2} \frac{2}{4} \pi r^4 = \frac{1}{2} m r^2 = \Theta$

Drehimpulserhaltung:

$\frac{1}{2} \omega_1 m r^2 = \omega_g \cdot \left( \frac{1}{2} m r^2 + \frac{1}{2} m \left(\frac{r}{2}\right)^2 \right) = \frac{1}{2} \omega_g m \frac{5}{4} r^2$ ✓

$\Leftrightarrow \omega_g = \frac{4}{5} \omega_1 = 4 \, Hz$ ✓

b) $E_1 = \frac{1}{2} \Theta_1 \omega_1^2 \qquad E_2 = \frac{1}{2} \Theta_g \omega_g^2$

$\frac{E_2}{E_1} = \frac{\Theta_g \omega_g^2}{\Theta_1 \omega_1^2} = \frac{\frac{1}{2} m \frac{5}{4} r^2 \frac{16}{25} \omega_1^2}{\frac{1}{2} m r^2 \omega_1^2} = \frac{\frac{5}{4} \cdot \frac{16}{25}}{1} = \frac{4}{5}$

$\Rightarrow$ Es bleiben 80% der kin. Energie erhalten. ✓

Aufgabe 2 (5/5)

a) $\Theta = \frac{M}{V} \int_b^R r \int_0^{2\pi} r^2 \, d\varphi \, dr = \frac{M}{\pi (R^2 - b^2)} \int_b^R 2\pi r^3 \, dr = \frac{M}{\pi (R^2 - b^2)} \frac{1}{2} \pi (R^4 - b^4)$

$= \frac{M}{2} \left( \frac{R^4 - b^4}{R^2 - b^2} \right) = \frac{M}{2} \left( \frac{(R^2 + b^2)(R^2 - b^2)}{(R^2 - b^2)} \right) = \frac{M}{2} (R^2 + b^2)$ ✓

b) $m \cdot g = \frac{\Theta a}{R} + m \cdot a = \frac{\Theta a}{R^2} + ma = a \left[ \frac{M}{2R^2} (R^2 + b^2) + m \right]$

$\Rightarrow a = m \cdot g \cdot \left[ \frac{M}{2R^2} (R^2 + b^2) + m \right]^{-1} = \frac{1}{2} g = 4{,}905 \, \frac{m}{s^2}$

$F_S = \frac{1}{2} m g = 12{,}2625$ ✓

Aufgabe 3 (4,5/5)

a) $\omega = \frac{v_p - v_s}{r}$ ✓

b) $\Theta = \frac{7}{5} m r^2$

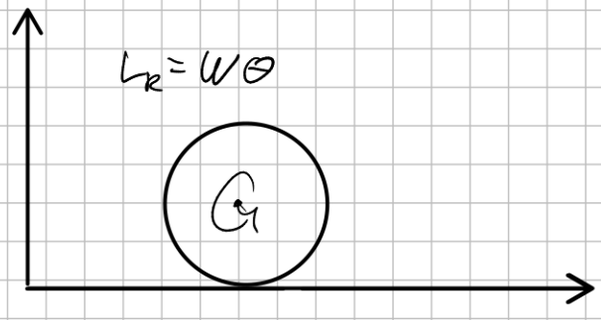
Der Drehimpuls um die Kontaktlinie bei $t = 0_s$ ist konstant, da die Kraft, die die Bewegung erzeugt radial (senkrecht) zu ihr liegt: ✓

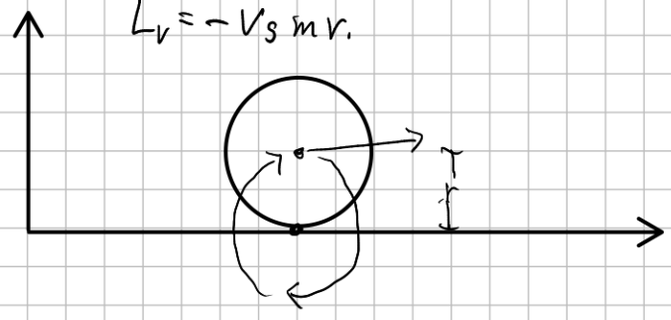
$L = \omega \Theta + (-v_s) m r \overset{!}{=} 0$ (da vor Beginn $\omega = 0$ und $v_s = 0$)

$= \frac{v_p - v_s}{r} \Theta - 2 v_s \frac{\Theta}{r}$

$\Leftrightarrow v_p - v_s = +2 v_s$

$\Leftrightarrow v_s = \frac{1}{3} v_p$ ✓



In[ ]:=
```
m = 2.5;
M = 3.2;
R = 0.2;
b = 0.15;
```

In[ ]:=
$$a := m\,g\left(\frac{M}{2\,R^2}\left(R^2+b^2\right)+m\right)^{-1}$$

In[ ]:=
```
a
```

Out[ ]=
$$0.5\,g$$

In[ ]:=
```
a /. g → 9.81
```

Out[ ]=
```
4.905
```