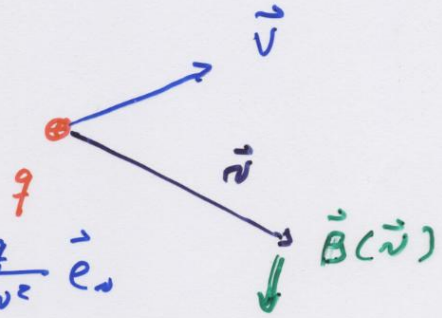
4.1.4 Magnetfeld von bewegten Ladungen

$$\vec{B} = \frac{1}{c^2}\,\vec{v} \times \vec{E}$$

Für Punktladung:



$$\vec{B} = \frac{1}{c^2}\,\vec{v} \times \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}\,\vec{e}_r$$

$$= \frac{1}{4\pi\varepsilon_0 c^2} \cdot \frac{q\,\vec{v} \times \vec{r}}{r^3}$$

$$\equiv \frac{\mu_0}{4\pi} \cdot \frac{q\,\vec{v} \times \vec{r}}{r^3}$$

$\mu_0 = 4\pi \cdot 10^{-7}\,\frac{\mathrm{Tm}}{\mathrm{A}}$ Magn.-Feldkonst.

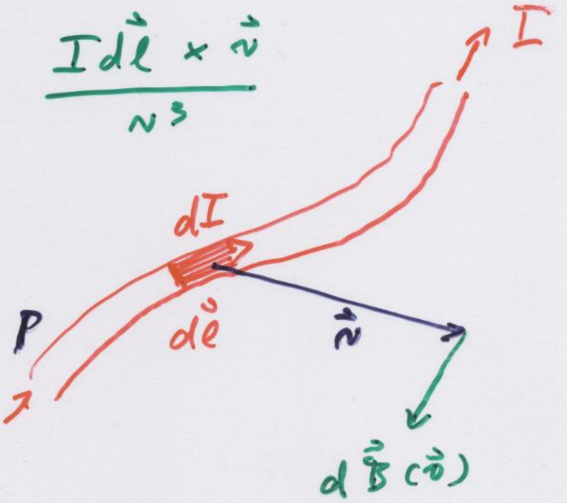
$\Rightarrow$ B prop. zu q, v, $\frac{1}{r^2}$

$\vec{B} \perp \vec{v}$, $\vec{r}$

4.1.5 Magnetfeld von Strömen: das Gesetz von Biot und Savart

mit $q \cdot \vec{v} = I \cdot d\vec{l}$

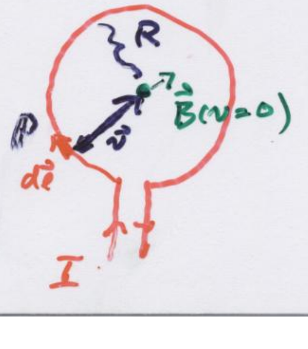
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^3}$$



$$\vec{B}(\vec{r}) = \int_P \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^3}$$

Biot - Savart

Beispiel: Magnetfeld einer Leiterschleife

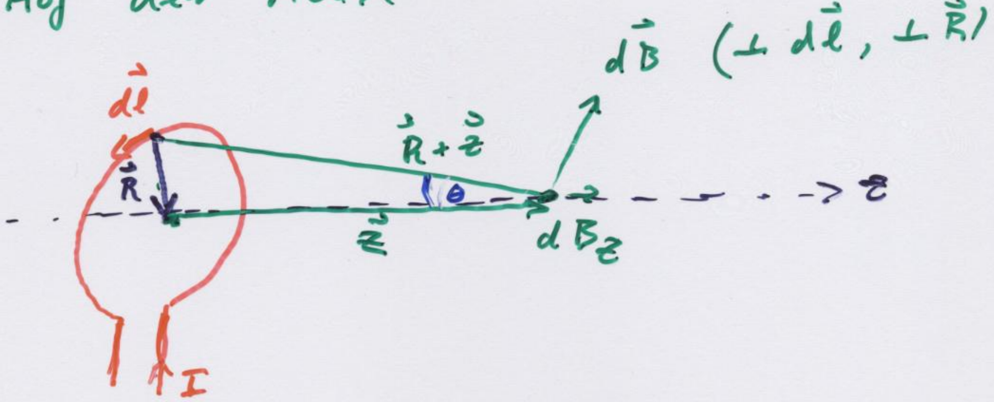


a) Im Zentrum

$$\underline{B} = \oint \frac{\mu_0}{4\pi} \frac{I}{R^2}\, dl$$

$$= \mu_0 \frac{I}{2R}$$

b) Auf der Achse



- $d\vec{B} = \frac{\mu_0}{4\pi} I \, d\vec{l} \times \frac{\vec{R}}{R^3}$

$\vec{B}(\vec{R}) = \mu_0 \frac{I}{R^2} \vec{e}_z$

- $d\vec{B}(\vec{R}+\vec{z}) = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times (\vec{R}+\vec{z})}{|\vec{R}+\vec{z}|^3}$

$dB_z = dB \cdot \sin\theta$

$= \frac{\mu_0}{4\pi} I \frac{dl}{|\vec{R}+\vec{z}|^2} \cdot \frac{R}{\sqrt{R^2+z^2}}$

$= \frac{\mu_0}{4\pi} \frac{I \, dl \, R}{(R^2+z^2)^{3/2}}$

$$B_z = \oint dB_z = \frac{\mu_0}{4\pi} I \frac{R}{\left(\sqrt{R^2+z^2}\right)^{3}} \cdot 2\pi R$$

$$= \frac{\mu_0}{4\pi} \frac{2\pi R^2}{(z^2+R^2)^{3/2}} \cdot I$$

Große Abstände $(z \gg R)$:

$$B_z = \frac{\mu_0}{4\pi} \frac{2\pi R^2}{z^3} \cdot I$$

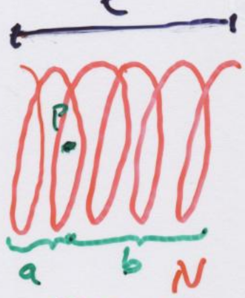
Magn. Moment $m = I \cdot \pi R^2$

$$B_z = \frac{\mu_0}{4\pi} 2 \cdot \frac{m}{z^3}$$ Magn. Dipolfeld

Analog :

$$E = \frac{1}{4\pi\varepsilon_0} 2 \cdot \frac{p}{z^3}$$ El. Dipolfeld

2. Zylinderspule



$$n = \frac{N}{l}$$

In der Spule:

$$B_z = \frac{1}{2}\mu_0 n \cdot I \left(\frac{b}{\sqrt{b^2+R^2}} + \frac{a}{\sqrt{a^2+R^2}} \right)$$

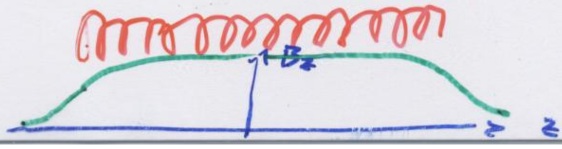
Sonderfall lange Spule ($l \gg R$)

a) $B_z \; (z=0) = \frac{1}{2}\mu_0 n \cdot I \left(\frac{b}{b} + \frac{a}{a} \right)$

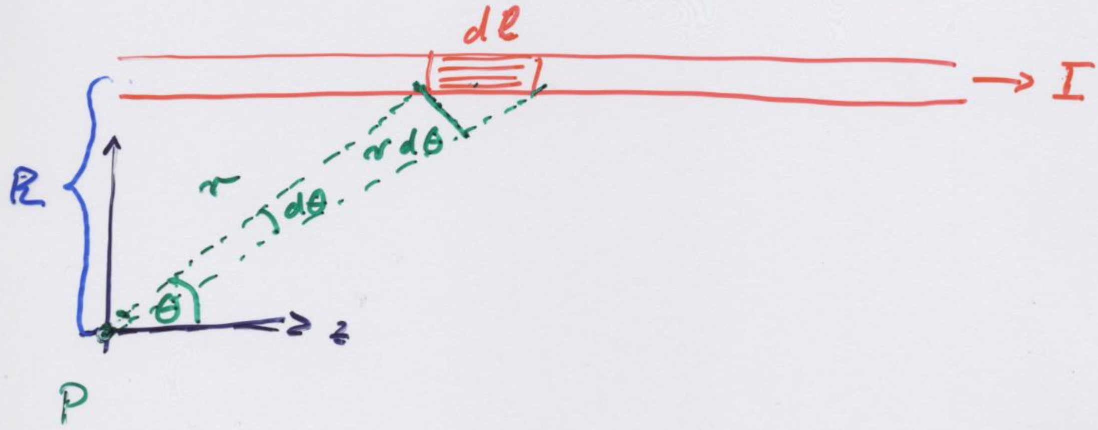
$$= \mu_0 \cdot I \cdot n$$

$$= \mu_0 \cdot \frac{I \cdot N}{l}$$

b) $B_z(z = -\frac{l}{2}) = \frac{1}{2}\mu_0 \cdot \frac{I \cdot N}{l} = \frac{1}{2} B_z(z=0)$



3. Unendlich langer gerader Leiter



$$dB = \frac{\mu_0}{4\pi} I \cdot \frac{dl \sin\theta}{r^2}$$

$$\sin\theta = \frac{R}{r}$$

$$= \frac{r \cdot \theta}{dl} \quad \Rightarrow \quad dl = \frac{r^2}{R} \, d\theta$$

$$dB = \frac{\mu_0}{4\pi} I \cdot \frac{\sin\theta \, d\theta}{R}$$

$$B = \int_0^{\pi} \frac{\mu_0 I}{4\pi R} \cdot \sin\theta \, d\theta = \frac{\mu_0 I}{2\pi R}$$

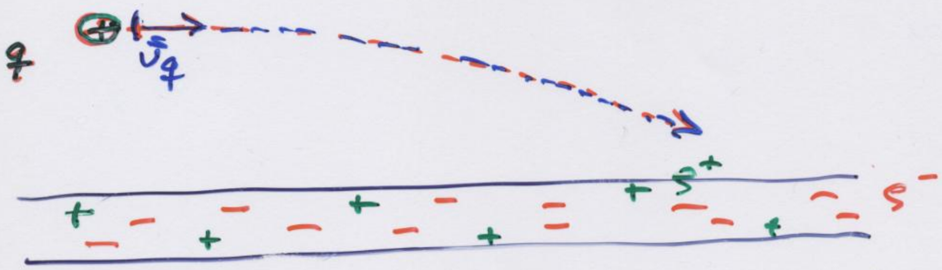
4. Relativiste Betrachtung

a. Bewegung von q längs einem neutralen Draht

$E = 0$, da $\rho^+ + \rho^- = 0$

$F = 0$

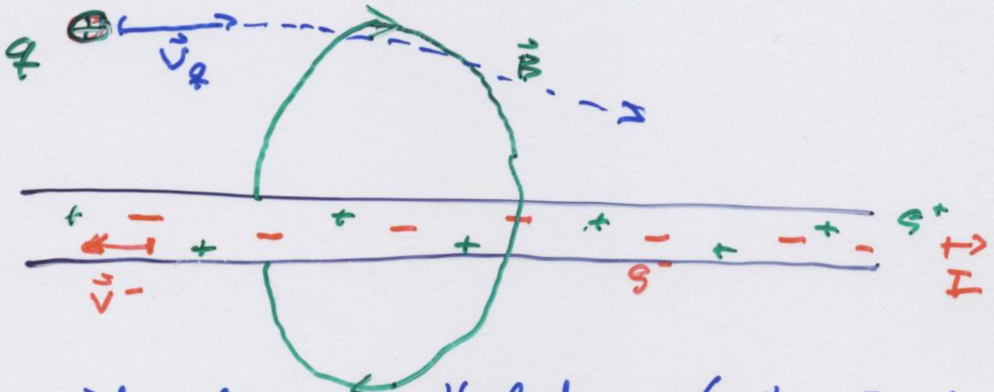
b. Bewegung längs geladenem Draht



Anziehung durch $E = \dfrac{\rho^- + \rho^+}{2\pi\varepsilon_0 r}$

$F = q \cdot E$

c. Draht neutral und stromdurchflossen

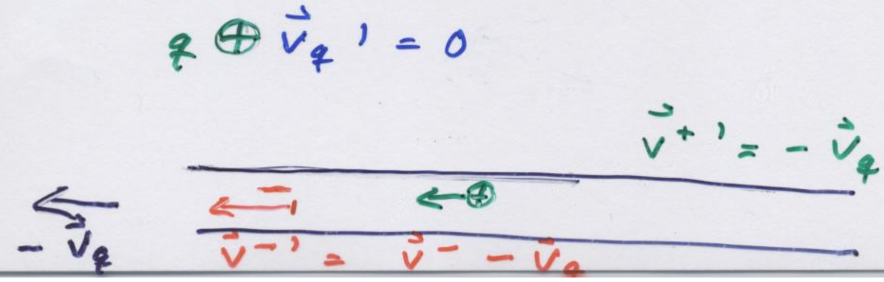


q sieht keine Nettoladung ($\rho^+ + \rho^- = 0$)

→ erwarte keine Elektrostat. Kraft,
wohl aber Lorentzkraft.

$$\vec F = q \cdot \vec v_q \times \vec B$$

d. Draht neutral, stromdurchflossen, gesehen von Ladung q



Elektronen im Draht $v^{-\,\prime} = -(v_q + v^-)$

Pos. Ladungen: $v^{+\,\prime} = -v_q$

$\Rightarrow$ Ladungsdichte erhöht

$$\rho^{+\,\prime} = \frac{\rho^+}{\sqrt{1 - \frac{v_q^2}{c^2}}}$$

$$\rho^{-\,\prime} = \frac{\rho^-}{\sqrt{1 - \frac{(v_q + v^-)^2}{c^2}}}$$

$$\rho_0' = \frac{\rho^+}{\sqrt{}} + \frac{\rho^-}{\sqrt{}}$$

[Auch: $\rho^+ = -\rho^-$]

$$\rho_0' = \frac{\rho^+}{\sqrt{}} - \frac{\rho^+}{\sqrt{}}$$

Mit: $\frac{1}{\sqrt{1-\varepsilon}} \approx 1 + \frac{1}{2}\varepsilon$

$$\varrho_0' \approx \varrho^+ \left(\left(1 + \frac{1}{2}\frac{v_q^2}{c^2}\right) - \left(1 + \frac{1}{2}\frac{(v_q + v^-)^2}{c^2}\right) \right)$$

$$\approx - \varrho^+ \cdot \frac{v_q \cdot v^-}{c^2} \neq 0 \ !$$

Ladung q „sieht" Nettoladung im Draht $\Rightarrow$

$$\vec{E} = - \frac{\varrho^+}{2\pi\varepsilon_0 r} \cdot \frac{v_q \, v^-}{c^2} \, \vec{e}_r$$

$$\Rightarrow \quad \vec{F} = \vec{E} \cdot q$$