5. 2. Wechselstrom und Schaltkreise

Jede periodische Wechselspannung:

$$u(t) = \sum_n \underbrace{u_n \sin(n \cdot \omega t + \phi_n)}$$

Größen:

Mittelwert: $$\langle u \rangle \equiv \frac{\int_0^T u(t)\,dt}{\int_0^T dt}$$

$$\frac{1}{T}\int_0^T u(t)\,dt$$

$\{= 0$ für $u = u_0 \sin(\,)\}$

Effektivwert:

$$u_{eff} = \sqrt{\frac{\int_0^T u^2(t)\,dt}{T}}$$

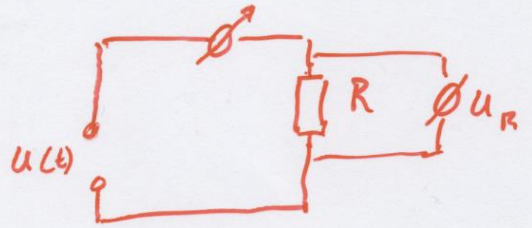
$$= \sqrt{\frac{u_0^2}{T}\int_0^T \sin^2(\omega t + \phi)} = \frac{u_0}{\sqrt{2}}$$

$$I_{eff} = \frac{I_0}{\sqrt{2}}$$

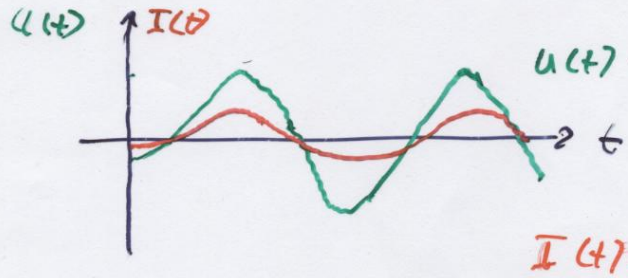
Bz. Hausspannung: $U_{eff} = 230\,V$

$U_0 = 325\,V$

2. Wechselspannung mit Widerstand



KH: $U_R = U(t)$



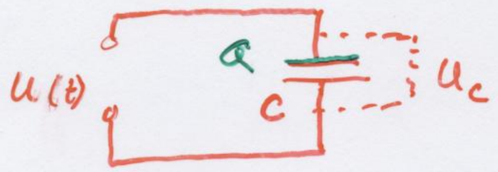
$$U(t) = U_0 \sin(\omega t + \phi)$$

$$I(t) = \frac{U(t)}{R} = \frac{U_0}{R} \sin(\omega t + \phi)$$

$$P(t) = U(t) \cdot I(t) = U_0 I_0 \sin^2(\omega t + \phi)$$

$$\langle P \rangle = \frac{1}{T} \cdot \int_0^T P(t)\,dt = \frac{1}{2} U_0 \cdot I_0 = U_{eff} \cdot I_{eff}$$

3. Wechselspannung mit Kondensator



$$U_C = U(t)$$
$$= U_0 \sin(\omega t + \phi)$$

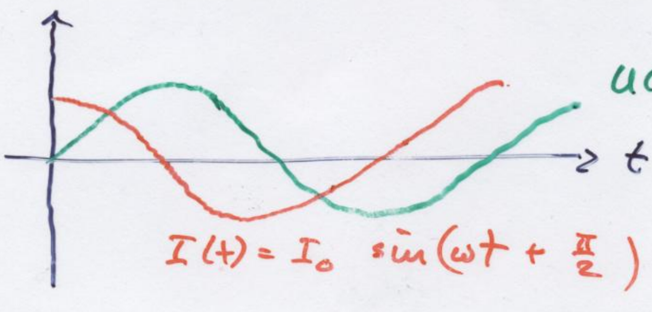
$$Q(t) = C \cdot U(t)$$
$$= C \cdot U_0 \sin(\omega t + \phi)$$

→ Strom $\quad I(t) = \frac{dQ}{dt}$

$$= \underbrace{\omega \cdot C \cdot U_0}_{I_0} \cos(\omega t + \phi)$$

Widerstand: $\quad R_C = \frac{U_0}{I_0} = \frac{1}{\omega \cdot C}$

Impedanz



$U(t) = U_0 \sin \omega t$

$\{\phi = 0\}$

$I(t) = I_0 \sin(\omega t + \frac{\pi}{2})$

4. W - Spannung mit Kondensator u. Widerstand

$$U(t) = I(t) \cdot R + \frac{Q(t)}{C}$$

$$\frac{dU}{dt} = \frac{dI}{dt} \cdot R + \frac{I}{C}$$

Mit $U = U_0 \sin \omega t$

$I = I_0 \sin(\omega t + \phi)$

$$\rightarrow I_0 = \frac{U_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\tan \phi = \frac{1}{\omega \cdot C \cdot R}$$

für $R \to 0$: $R = \frac{1}{\omega C}$

$\phi = \frac{\pi}{2}$

5. W - Spannung an der Spule

$$U_0 \sin \omega t = L \frac{dI}{dt}$$

$$I(t) = \frac{U_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

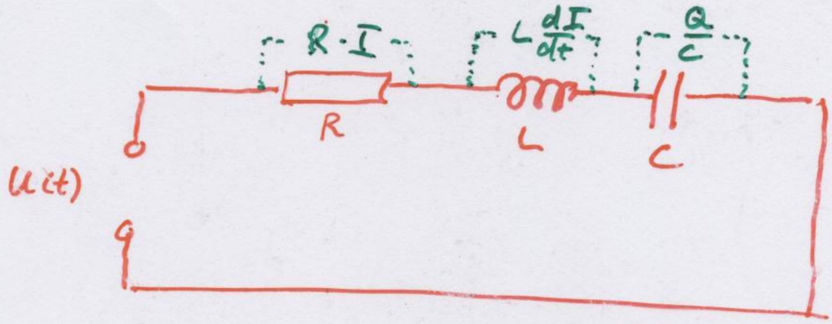
$R_L = \omega L$ (Induktiver Widerstand)

6. W- Spannung an Spule u. Widerstand

$$R_L = \sqrt{R^2 + (\omega L)^2}$$

$$\tan\phi = -\frac{\omega L}{R}$$

7. Serienschwingkreis

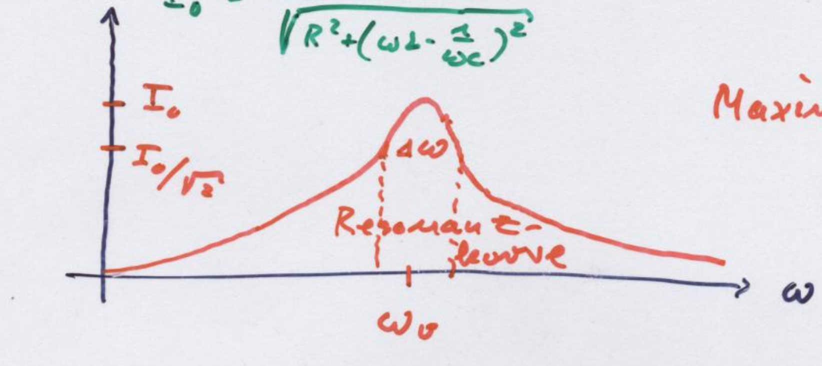


$$\text{DGL}: \frac{dU}{dt} = R\frac{dI}{dt} + L\frac{d^2I}{dt^2} + \frac{I}{C}$$

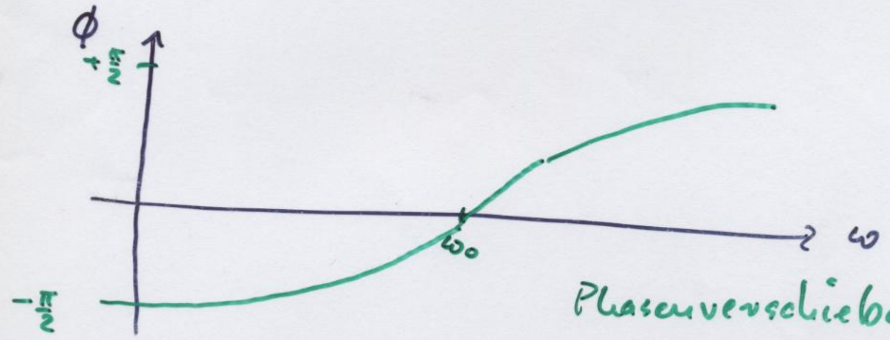
$$\rightarrow \frac{U_0}{I_0} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = |Z|$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$I_0 = \frac{U_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



Maximal bei $\omega_0 L = \frac{1}{\omega_0 C}$



Phasenverschiebung

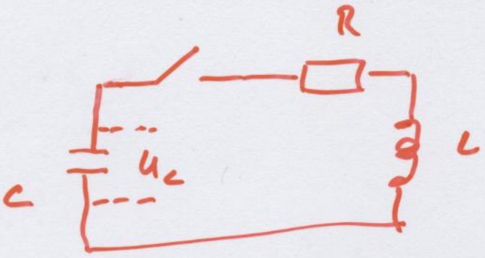
$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Kreisgüte $Q = \frac{\omega_0}{\Delta\omega}$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C \cdot R}$$

$\rightarrow \infty$ für $R \rightarrow 0$

8 Schwingkreis

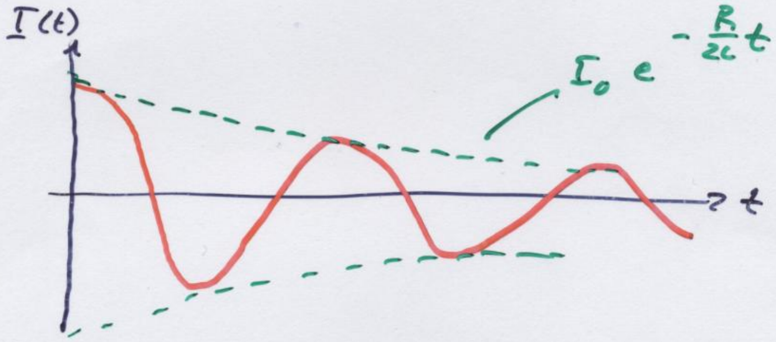


$$U_R + U_C + U_L = 0$$

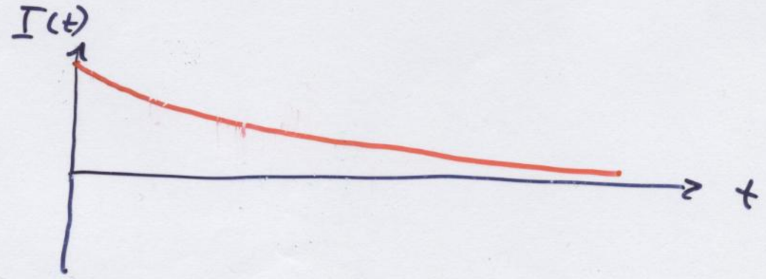
$$R \cdot \dot{I} + \frac{I}{C} + L\ddot{I} = 0$$

$$\left(\dot{I} = \frac{dI}{dt}\right)$$

Lösung : $I(t) = I_0 \cdot e^{-\frac{R}{2L}t} \cdot \cos\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \cdot t$

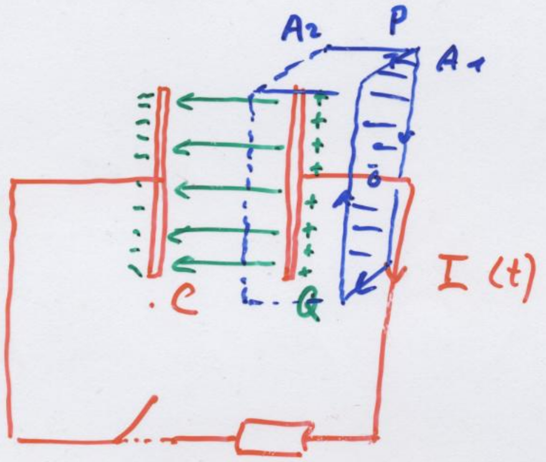


Sonderfall $\frac{1}{LC} = \frac{R^2}{4L^2}$



5.3 Maxwellscher Verschiebungsstrom

Berücksichtigung zeitl. abhängiger el. Felder



$$\oint_P \vec{B}\, d\vec{s} = \mu_0 \cdot I$$

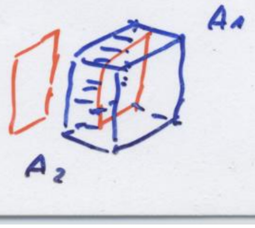
$$= \mu_0 \int_A \vec{j}\, d\vec{A}$$

A_1: $\mu_0 \int_{A_1} \vec{j}\, d\vec{A} = \mu_0 I$

↯

A_2: $\mu_0 \int_{A_2} \vec{j}\, d\vec{A} = 0$ (kein Strom fließt durch A_2)

$$\neq \oint_P \vec{B}\, d\vec{s}$$



Maxwell:

„Verschiebungsstrom" zwischen den Platten des Kondensators.

$$I_v = \frac{dQ}{dt}$$

$$= \frac{d}{dt} \left(\varepsilon_0 \vec{A}_c \cdot \vec{E} \right)$$

$$= \varepsilon_0 \vec{A} \frac{\partial \vec{E}}{\partial t}$$

$$\vec{j}_v = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \oint \vec{B} \, d\vec{s} = \mu_0 \cdot I$$

$$= \mu_0 \int \left(\vec{j} + \vec{j}_v \right) d\vec{A}$$

(Stokes)

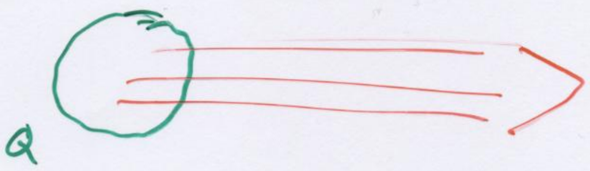
$$\operatorname{rot} \vec{B} = \mu_0 \cdot \vec{j} + \underbrace{\mu_0 \cdot \varepsilon_0}_{\frac{1}{c^2}} \frac{\partial \vec{E}}{\partial t}$$

4. MW-Gl.

→ Kontinuitätsgleichung

$$\underbrace{\vec{\nabla} \operatorname{rot} \vec{B}}_{=0} = \mu_0 \vec{\nabla} \vec{j} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{\nabla} \vec{E}$$

$$\Rightarrow \boxed{\vec{\nabla} \vec{j} + \frac{d}{dt} \rho = 0}$$



Kontinuitätsgleichung $\Longleftrightarrow$ Ladungserhaltung

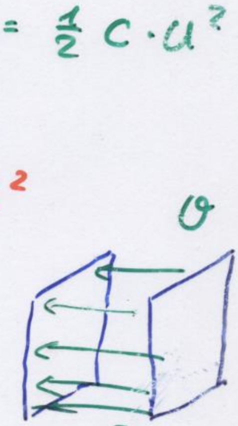
5.3 Energie der el. und magn. Felder

Erinnerung: Aufladung eines Kondensators

$$dW = dQ \cdot U$$
$$= dQ \cdot \frac{Q}{C}$$
$$W = \frac{1}{C} \int Q \, dQ = \frac{1}{2} \frac{Q^2}{C}$$
$$= \frac{1}{2} C \cdot U^2$$

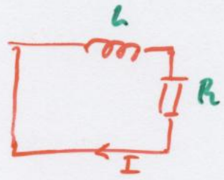
Bsp Plattenkondensator:

$$W = \frac{1}{2} \frac{\varepsilon_0 A}{d} \cdot E^2 d^2$$
$$= \frac{1}{2} \varepsilon_0 \cdot V \cdot E^2$$



Energiedichte $w = \frac{1}{2} \varepsilon_0 E^2$

Analog: Energie des B-Feldes:

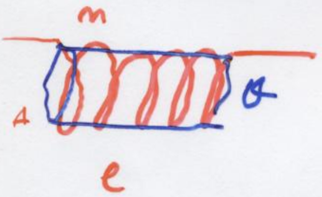


$$W = \int_0^\infty I \cdot U \, dt = \int_0^\infty I^2 R \, dt$$

allgemein

$$= \int_0^\infty R \cdot I_0^2 \, e^{-\frac{2R}{L} t} \, dt$$

$$= \underline{\frac{1}{2} L \cdot I_0^2}$$



$$= \frac{1}{2} \mu_0 \cdot n^2 \cdot A \cdot l \cdot \left(\frac{B}{\mu_0 n}\right)^2$$

$$= \frac{1}{2\mu_0} B^2 \cdot V$$

Energiedichte: $\underline{w = \frac{1}{2\mu_0} B^2}$