Das elektrische Feld

Probeladung q $\longrightarrow$ Kraft $\vec{F}(\vec{r}) = \frac{Q \cdot q}{4\pi\varepsilon_0 r^2}\,\hat{r}$

Q

$\vec{r} = 0$

$$\frac{\vec{F}(\vec{r})}{q} = \underline{\underline{\vec{E}(\vec{r})}}$$ elektrische Feld

$$[E] = \frac{[F]}{[q]} = \frac{1\,N}{1\,A\cdot s} = 1\frac{V}{m}$$

$$\vec{F} = q \cdot \vec{E}$$

Q_1 Q_2

Q_3

$\vdots$

Q_i

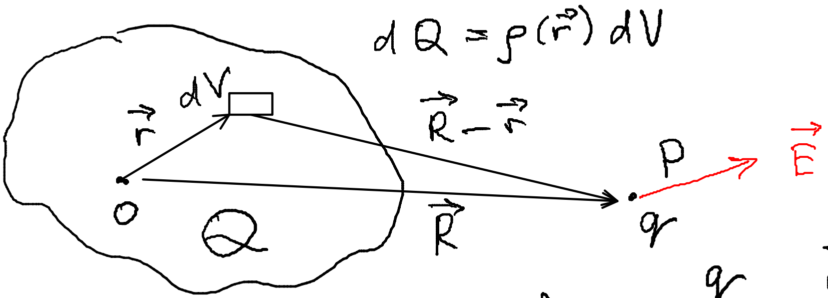
$$\vec{F} = \frac{q}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i^2}\,\hat{r}_i$$

Ladungsdichte $\rho(\vec{r})$

Q Gesamtladung

$$Q = \int_V \rho(\vec{r})\, dV$$

$$dQ = \rho(\vec{r})\, dV$$

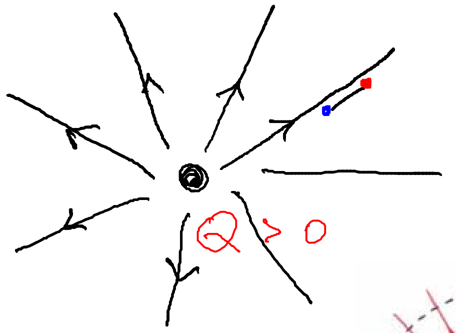


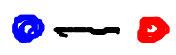
$$F(dV) = \frac{q}{4\pi\varepsilon_0} \frac{\vec{R}-\vec{r}}{|\vec{R}-\vec{r}|^3} \rho(\vec{r})\, dV$$

$$\vec{F}(\vec{R}) = \frac{q}{4\pi\varepsilon_0} \int_V \frac{\vec{R}-\vec{r}}{|\vec{R}-\vec{r}|^3} \rho(\vec{r})\, dV$$

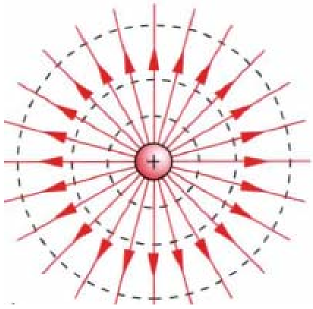
$$\vec{E}(\vec{R}) = \frac{\vec{F}(\vec{R})}{q}$$

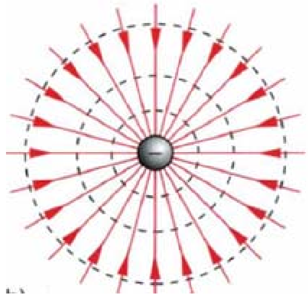
Feldlinien

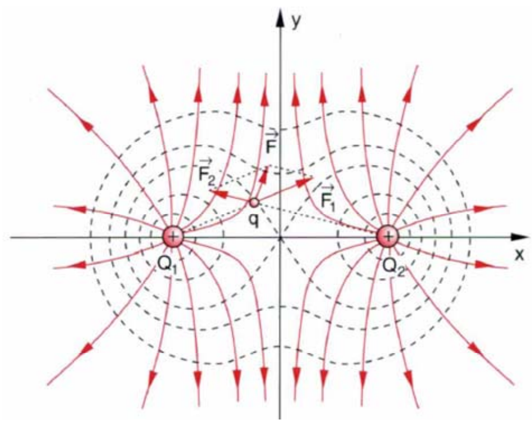


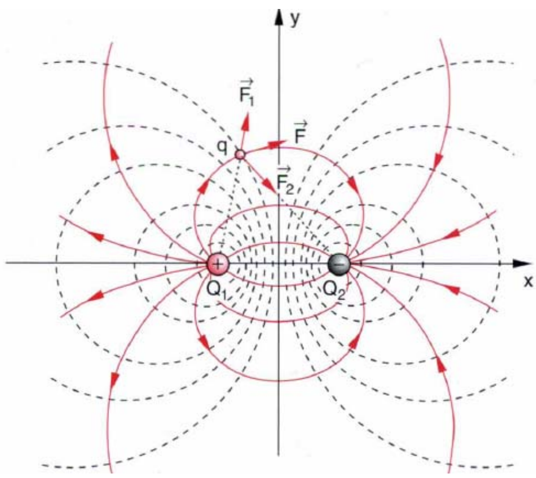


Dipol

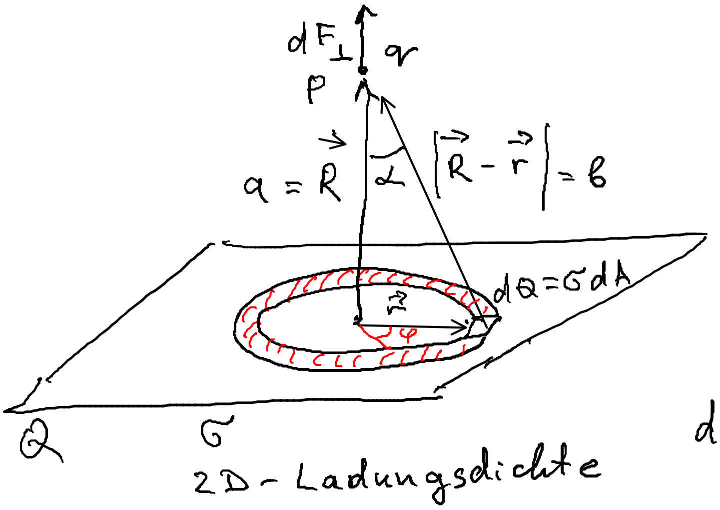








Platte mit der homogener
Ladungsverteilung



$$d\vec F = \frac{q}{4\pi\varepsilon_0}\cdot\frac{\sigma\, dA}{b^2}\cdot\frac{\vec R-\vec r}{b}$$

$$b = \frac{a}{\cos\alpha}$$

Vertikalkomponente

$$|d\vec F_\perp| = |d\vec F|\cdot\cos\alpha$$

$$dA = r\cdot d\varphi\cdot dr$$

Fläche $\quad 2\pi r\cdot dr$

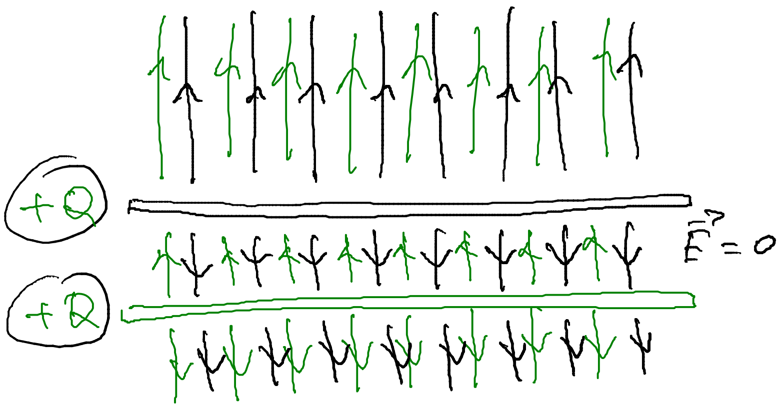
$$dF_\perp = \frac{2\pi \vec r\cdot dr}{4\pi\varepsilon_0 b^2}\, q\cdot\sigma\cdot\cos\alpha$$

$$= \frac{q\cdot\sigma}{2\varepsilon_0 a^2}\,(\cos^3\alpha)\cdot r\cdot dr$$

$$r = a\cdot\tan\alpha \qquad dr = \frac{a}{\cos^2\alpha}\cdot d\alpha$$

$$dF_\perp = \frac{q\sigma\cdot d\alpha}{2\varepsilon_0 \cancel{a^2}}\,\frac{\cancel{\cos^3\alpha}}{\cancel{\cos^2\alpha}}\,\cancel{a}\cdot\cancel{a}\cdot\frac{\sin\alpha}{\cancel{\cos\alpha}} = \frac{q\sigma}{2\varepsilon_0}\sin\alpha\, d\alpha$$

$$F_\perp = \int_0^{\pi/2} dF_\perp = \int_0^{\pi/2}\frac{q\sigma}{2\varepsilon_0}\sin\alpha\, d\alpha = -\frac{q\sigma}{2\varepsilon_0}\cos\alpha\Big|_0^{\pi/2} = \frac{q\sigma}{2\varepsilon_0}$$

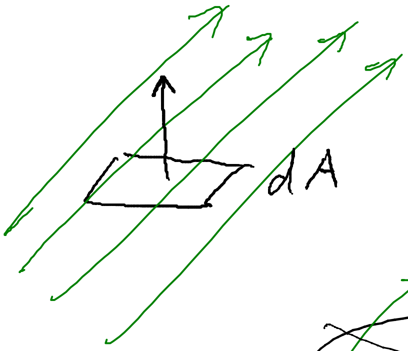


$$E = \frac{\sigma}{2\varepsilon_0}\cdot\frac{\Omega}{2\pi}$$

Elektrischer Fluss

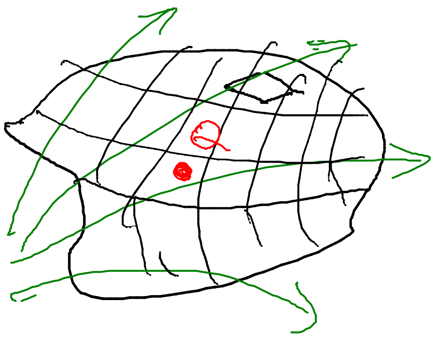
$$d\Phi_{el} = \vec{E}\cdot d\vec{A}$$

$d\vec{A}$ Flächennormalenvektor



$$\Phi_{el} = \int \vec{E}\cdot d\vec{A} = \frac{Q}{4\pi\varepsilon_0}\int \frac{\hat{r}}{r^2}\, d\vec{A}$$

$$= \frac{Q}{4\pi\varepsilon_0}\int d\Omega = \frac{Q}{\varepsilon_0}$$



$$\frac{\hat{r}\, d\vec{A}}{r^2} = d\Omega$$

Raumwinkel