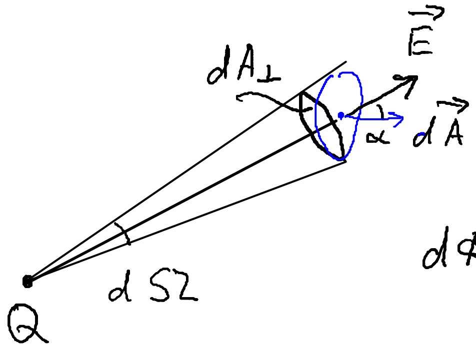
elektrischer Fluss: $d\Phi_{el} = \vec{E} \cdot d\vec{A}$

$$= E \cdot dA \cdot \cos\alpha$$
$$= E \cdot dA_\perp$$
$$dA_\perp = \hat{r} \cdot d\vec{A}$$



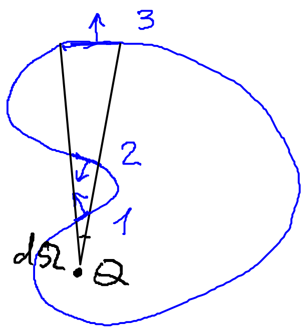
$$d\Phi_{el} = Q \frac{dA_\perp}{4\pi\varepsilon_0 r^2}$$

Raumwinkel $d\Omega$

für jede geschlossene Oberfläche A: $\boxed{\Phi_{el} = \frac{Q}{4\pi\varepsilon_0} \oint_A d\Omega = \frac{Q}{\varepsilon_0}}$

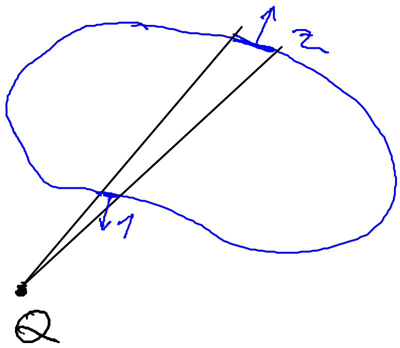
2 Beispiele

a)



$$d\Phi_1 = -d\Phi_2 = d\Phi_3$$

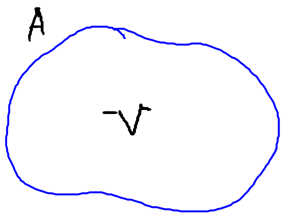
b)

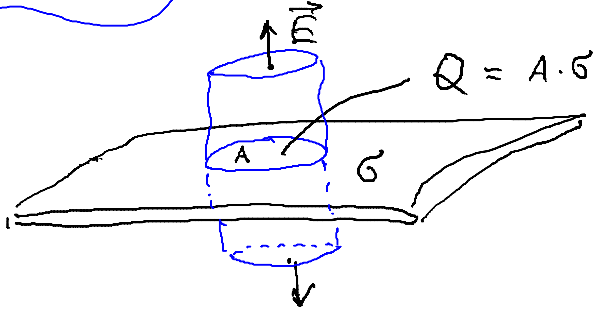


Der G. Satz → differentiale Form

$$\Phi_{el} = \oint_A \vec{E} \cdot d\vec{A} = \int_V \operatorname{div} \vec{E} \cdot dV$$

$$\frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_V \rho \cdot dV \longrightarrow \boxed{\operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0}}$$





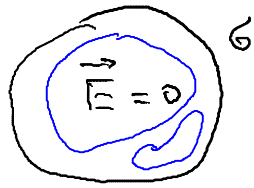
$Q = A \cdot \sigma$

$$\Phi_{el} = 2\,E \cdot A = \frac{A\sigma}{\varepsilon_0}$$

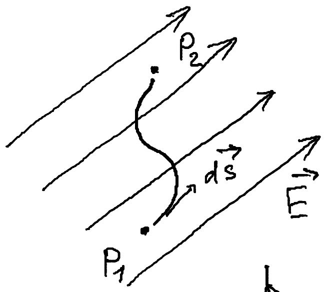
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Geladene Hohlkugel

Geladene Hohlkugel



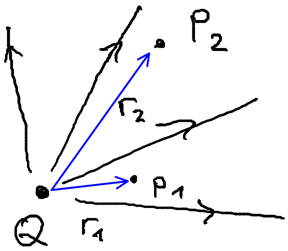
Elektrostatisches Potential



q von P_1 nach P_2

$$W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{s} = q\int_{P_1}^{P} \vec{E}\cdot d\vec{s}$$

Punktladung



$$W = \frac{qQ}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Für Konservative Kraftfelder $\longrightarrow$ $\text{rot}(\vec{E}) = 0$

Potential: $\boxed{\phi(P) = \int_{P}^{\infty} E\cdot d\vec{s}}$

$$\phi(\infty) = 0$$

$$W = q\cdot\phi(P)$$

Potentialdifferenz zwischen zwei Punkten P_1 und P_2

$$U = \phi(P_1) - \phi(P_2) = \int_{P_1}^{P_2}\vec{E}\cdot d\vec{s}$$

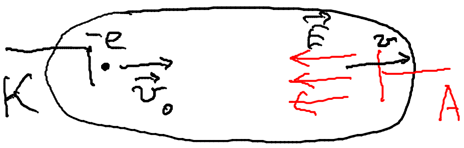
eletrische Spannung

Elektrische Spannung

$$U = \phi(P_1) - \phi(P_2) = \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s}$$

potentielle Energie $\Delta E_{pot} = -q \cdot U$

Gesamtenergie $E_{ges} = E_{kin} + E_{pot}$



$$\frac{m v^2}{2} = \frac{m v_0^2}{2} + eU$$

Die Einheit der Spannung = V (Volt)

$$[U] = \left[\frac{E_{pot}}{q}\right] = 1\,\frac{N \cdot m}{A \cdot s} = 1\,V\,\frac{\cancel{A \cdot s}}{\cancel{A \cdot s}} = 1\,V$$

Elektronenvolt $\underline{1\,eV} \simeq 1{,}6 \cdot 10^{-19}\,C \cdot 1\,V \simeq 1{,}6 \cdot 10^{-19}\,J$

Potentialgleichung

$$\phi(P) = \int_P^{\infty} E \cdot d\vec{s} \qquad \boxed{\vec{E} = -\,\mathrm{grad}(\phi) = -\nabla\phi}$$

$$\mathrm{div}\,\vec{E} = \frac{\rho}{\varepsilon_0} = -\,\mathrm{div}\,\mathrm{grad}\,\phi = -\Delta\phi$$

$$\Delta\phi = \nabla^2\phi = \nabla\cdot\nabla\phi$$

Δ der Laplace-Operator (Laplacian)

Cartesian coordinates (engl.) x_i

$$\Delta\phi = \sum_{i=1}^{i=3} \frac{\partial^2\phi}{\partial x_i^2}$$

$$\boxed{\Delta\phi = -\frac{\rho}{\varepsilon}}$$ Poisson-Gleichung

$\rho(x,y,z)$

$\hookrightarrow$ Rechnung von $\phi(x,y,z)$ oder $\vec{E}(x,y,z)$

keine Ladungen $\longrightarrow$ $\boxed{\Delta\phi = 0}$

Laplace-Gleichung

Äquipotentialflächen