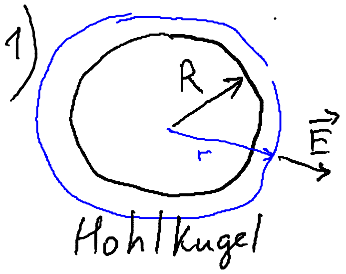
Beispiele von Feldern und Potential

1)

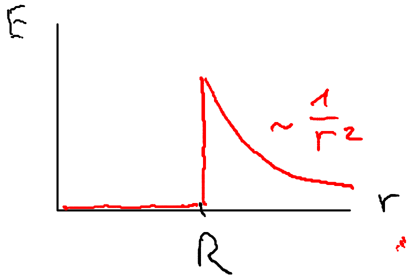



Flächenladungsdichte $\sigma$

$$Q = 4\pi R^2 \sigma$$

$$\Phi_{el} = \int \vec{E} \cdot d\vec{A} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{r}$$

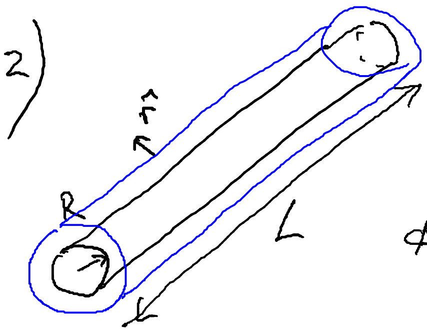



Potential

$$\phi(r) = \int_r^{\infty} E \cdot dr = \frac{Q}{4\pi\varepsilon_0 r}$$

$$|\vec{E}(\vec{r})| = \frac{\phi(r)}{r}$$

2)




Ladung pro Längeeinheit

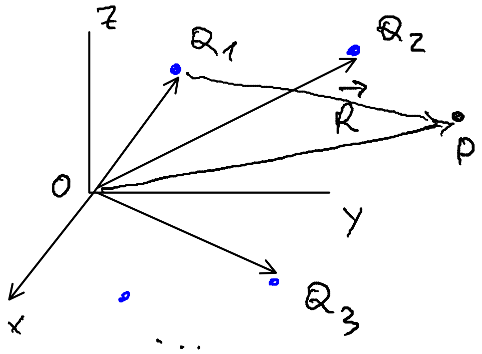
$$\lambda = \pi R^2 \rho$$

$$\Phi_{el} = \int \vec{E} \cdot d\vec{A} = \underline{E \cdot 2\pi r \cdot L} = \frac{Q}{\varepsilon_0} = \frac{\lambda \cdot L}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi \cdot \varepsilon_0 r}\,\hat{r}$$

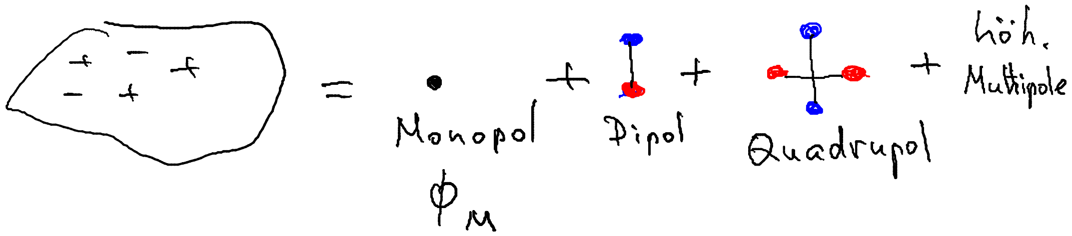
**! heir** $\phi(R) = 0$ ; $\phi(r) = -\frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\frac{r}{R} = -\int_R^r E(r)\,dr$

Multipole: Superpositionsprinzip

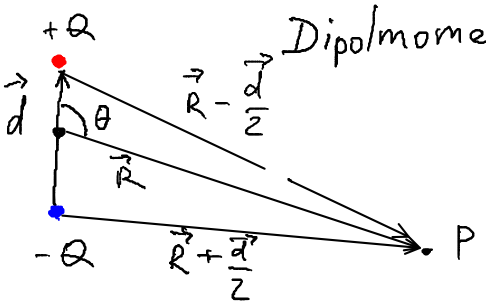



Gesamtpotential

$$\phi(\vec{R}) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_i}{(\vec{R} - \vec{r}_i)}$$



Elektrischer Dipol




Dipolmoment $\vec{p} = Q\cdot\vec{d}$

$$\phi_D(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\left|\vec{R}-\frac{\vec{d}}{2}\right|} - \frac{Q}{\left|\vec{R}+\frac{\vec{d}}{2}\right|}\right)$$

$R \gg d$

$$\frac{1}{\left|\vec{R}\pm\frac{\vec{d}}{2}\right|} = \frac{1}{R}\cdot\frac{1}{\sqrt{1\pm\frac{\vec{R}\cdot\vec{d}}{R^2}+\frac{d^2}{4R^2}}} = \frac{1}{R}\left(1\mp\frac{1}{2}\frac{\vec{R}\cdot\vec{d}}{R^2}+\ldots\right)$$

$$(1+\alpha)^{-\frac{1}{2}} \approx 1-\frac{1}{2}\alpha$$

$$\underbrace{\phi_D(\vec{R}) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{\vec{d}\cdot\vec{R}}{R^3}} = \frac{\vec{p}\cdot\vec{R}}{4\pi\varepsilon_0 R^3} = \frac{p\cdot\cos\theta}{4\pi\varepsilon_0 R^2}$$

$$\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$$

$$\phi_D(\vec{R}) = -\frac{Q}{4\pi\varepsilon_0}\vec{d}\cdot\nabla\left(\frac{1}{R}\right) = -\vec{d}\cdot\text{grad}\,\phi_M(\vec{R})$$

$$\vec{E} = -\text{grad}\,\phi_D$$

$$\text{grad}\,\phi_D = \frac{Q}{4\pi\varepsilon_0}\left[\underbrace{(\vec{d}\cdot\vec{R})}_{}\cdot\underbrace{\text{grad}\frac{1}{R^3}}_{} + \frac{1}{R^3}\cdot\text{grad}(\vec{d}\cdot\vec{R})\right]$$

$$Q\vec{d}\cdot\vec{R} = p\,R\cos\theta \qquad -\frac{3\vec{R}}{R^5}$$

$$Q\,\text{grad}(\vec{d}\cdot\vec{R}) = Q\left(d_1\frac{\partial}{\partial x}+d_2\frac{\partial}{\partial y}+d_3\frac{\partial}{\partial z}\right)\begin{pmatrix}x\\y\\z\end{pmatrix} = Q\cdot\vec{d} = \vec{p}$$

$$\boxed{E(\vec{R}) = \frac{1}{4\pi\varepsilon_0 R^3}\left(3p\,\hat{R}\cdot\cos\theta - \vec{p}\right)}$$

Polarkoordinaten $\vec{R}, \theta, \varphi$

Azimutwinkel

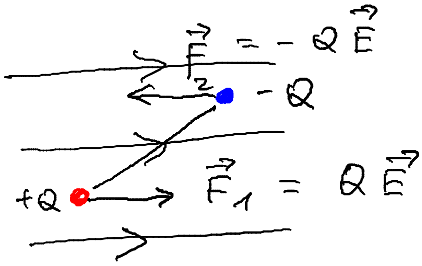
$$\vec{E} = -\text{grad}\,\phi = -\left\{\frac{\partial\phi}{\partial R}, \frac{1}{R}\frac{\partial\phi}{\partial\theta}, \frac{1}{R\sin\theta}\frac{\partial\phi}{\partial\varphi}\right\}$$

$$E_R = \frac{2p\cos\theta}{4\pi\varepsilon_0 R^3}; \quad E_\theta = \frac{p\cdot\sin\theta}{4\pi\varepsilon_0 R^3}; \quad E_\varphi = 0$$

Dipol im Feld

Dipol im homogenen Feld

Drehmoment

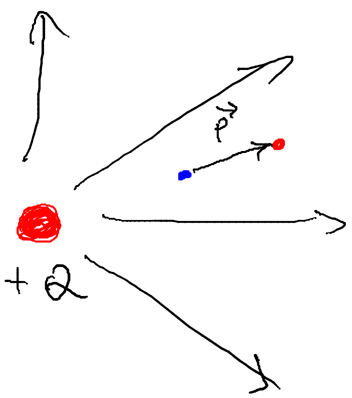



$$\vec{D} = Q(\vec{r}_1 \times \vec{E}) - Q(\vec{r}_2 \times \vec{E}) =$$

$$= Q \cdot \vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

Dipol im inhomogenen Feld

Kraft




$$\vec{F} = Q \cdot \left[\vec{E}(\vec{r}+\vec{d}) - \vec{E}(\vec{r})\right] =$$

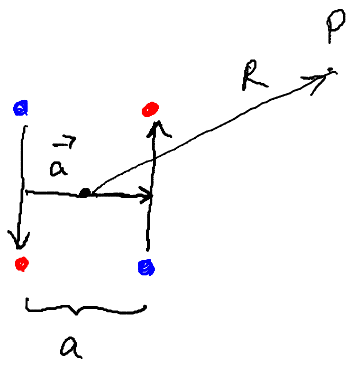
$$= Q \cdot \vec{d} \cdot \frac{d\vec{E}}{d\vec{r}} = \vec{p} \cdot \nabla\vec{E}$$

Vektorgradient von $\vec{E}$

$\downarrow$

Tensor

Elektrischer Quadrupol




$$\phi_Q(\vec{R}) = \phi_D\left(\vec{R} + \frac{\vec{a}}{2}\right) - \phi_D\left(\vec{R} - \frac{\vec{a}}{2}\right)$$

$$E \sim \frac{1}{R^5}$$

Leiter im elektrischen Feld

Influenz
für
(Leiter)