Rechnenregeln für den Nabla-Operator

$$\text{div}\,\vec{A} \equiv \vec{\nabla}\cdot\vec{A} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergenz

$$\text{rot}\,\vec{A} \equiv \vec{\nabla}\times\vec{A} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}.$$

Rotation

$$= \begin{pmatrix} \frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \\ \frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \\ \frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \end{pmatrix}$$

$$\text{grad}\,V \equiv \vec{\nabla} V = \begin{pmatrix} \frac{\partial}{\partial x} V \\ \frac{\partial}{\partial y} V \\ \frac{\partial V}{\partial z} \end{pmatrix}$$

Gradient

$$\vec{\nabla}\cdot\left(\vec{A}\varphi\right) = \varphi\,\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}\varphi = \varphi\,\text{div}\,\vec{A} + \vec{A}\,\text{grad}\,\varphi$$

$$\vec{\nabla}\times\left(\vec{A}\varphi\right) = \varphi\vec{\nabla}\times\vec{A} - \vec{A}\times\vec{\nabla}\varphi = \ldots$$

$$\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right) = \vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right)$$

$$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right) = \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} - \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} + \vec{A}\left(\vec{\nabla}\cdot\vec{B}\right) - \vec{B}\left(\vec{\nabla}\cdot\vec{A}\right)$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\varphi\right) = \text{div}\left(\text{grad}\,\varphi\right) = \Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$$

Laplace-Operator

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) \equiv \mathrm{div}\,(\mathrm{rot}\,\vec{A}) = (\vec{\nabla}\times\vec{\nabla})\cdot\vec{A} = 0$$

$$\vec{\nabla}\times(\vec{\nabla}\varphi) = \mathrm{rot}\,\mathrm{grad}\,\varphi = (\vec{\nabla}\times\vec{\nabla})\,\varphi = 0$$

Konservative Felder haben keine Wirbel

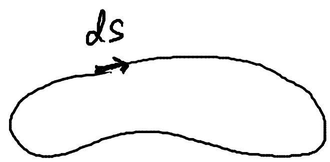
$$\vec{\nabla}\times\vec{E} = \vec{0} \; ; \; \vec{\nabla}\times(\vec{\nabla}\phi) = 0$$

Wirbelfelder haben keine Divergenz

$$\vec{\nabla}\cdot\vec{B} = 0 \; ; \; \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$$

Gaussscher Satz des elektrischen Feldes $\int_V \mathrm{div}\,\vec{E}\,dV = \oint \vec{E}\cdot d\vec{A}$

Stokesscher Satz des Magnetfeldes $\int_A \mathrm{rot}\,\vec{B}\cdot d\vec{A} = \oint_P \vec{B}\,d\vec{s}$



Biot-Savart-Gesetz

$$\vec{B} = \text{rot}\,\vec{A}$$

$$B(r_1) = \frac{\mu_0}{4\pi} \int \vec{\nabla} \times \frac{\vec{j}(\vec{r}_2) \cdot dV_2}{r_{12}}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{r}_2) \times \hat{e}_{12}}{r_{12}^2}$$

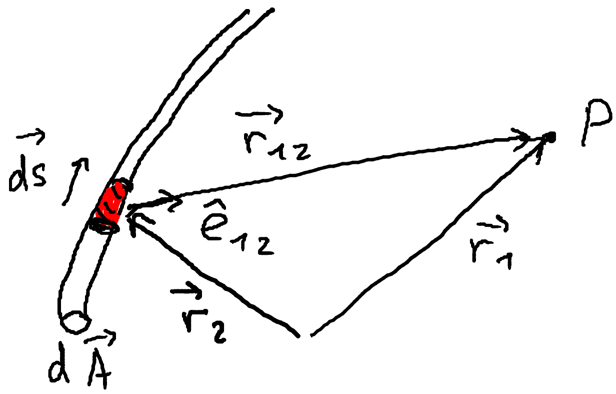
$$r_{12}{}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

Einheitsvektor $\hat{e}_{12} = \frac{\vec{r}_{12}}{r_{12}}$

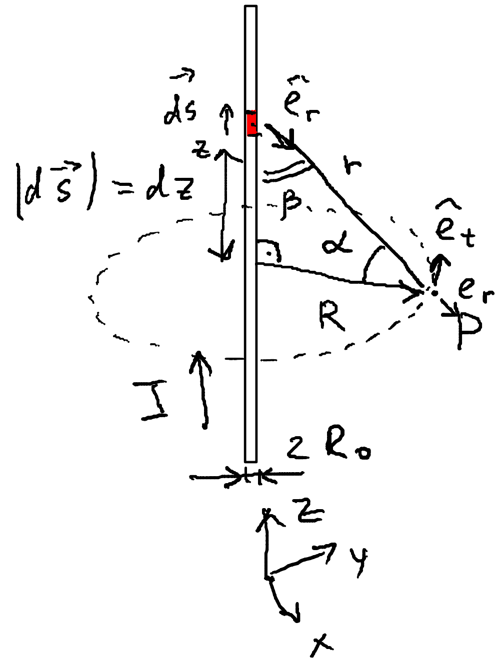
$$\vec{j}\,dV = j\,\vec{dA} \cdot \vec{ds} = I\,d\vec{s}$$

$$\boxed{\vec{B}(r_1) = -\frac{\mu_0}{4\pi} I \int \frac{\hat{e}_{12} \times d\vec{s}}{r_{12}^2}}$$

Biot-Savart-Gesetz


$\vec{ds}$
$\vec{r}_{12}$
P
$\hat{e}_{12}$
$\vec{r}_1$
$\vec{r}_2$
$d\vec{A}$

Das Magnetfeld eines geraden Leiters



$$|\hat{e}_{12} \times d\vec{s}| = dz \cdot \sin\beta$$

$$\hat{e}_r \qquad = \cos\alpha \cdot dz$$

$$\vec{B}(\vec{R}) = \frac{\mu_0 I}{4\pi} \hat{e}_t \int \frac{\cos\alpha}{r^2} dz$$

$$r = \frac{R}{\cos\alpha} \; ; \; z = R \cdot \tan\alpha$$

$$\downarrow$$

$$dz = \frac{R \, d\alpha}{\cos^2\alpha}$$

$$B = \frac{\mu_0 I}{4\pi R} \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha = \frac{\mu_0 I}{2\pi R}$$

$$\vec{A} = \{0, 0, A_z\}$$

$$\vec{B} = \operatorname{rot} \vec{A} \; ; \longrightarrow \quad B_x = \frac{\partial A_z}{\partial y} \; ; \; B_y = -\frac{\partial A_z}{\partial x}$$

$$B_z = 0$$

$x, y, z \longrightarrow$ Zylinderkoordinaten

$$r, \varphi, z$$

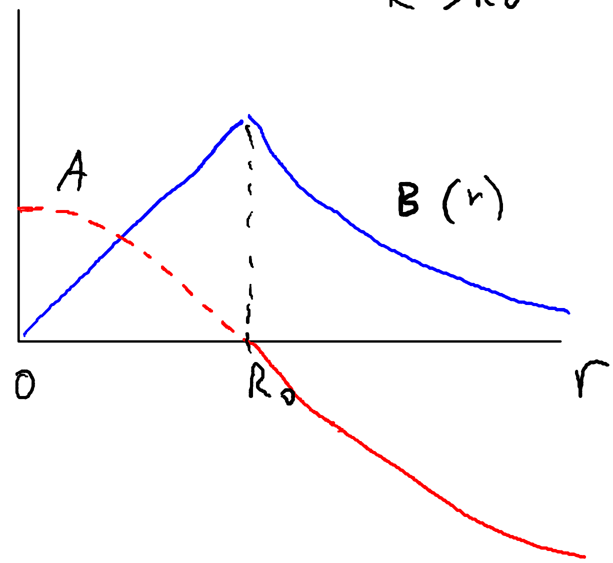
$$B_r = \frac{1}{r} \frac{\partial A_z}{\partial \varphi} \; ; \; B_\varphi = -\frac{\partial A_z}{\partial r}$$

Symmetrie $\frac{\partial A_z}{\partial \varphi} = 0 \; , \rightarrow \quad B_r = 0$

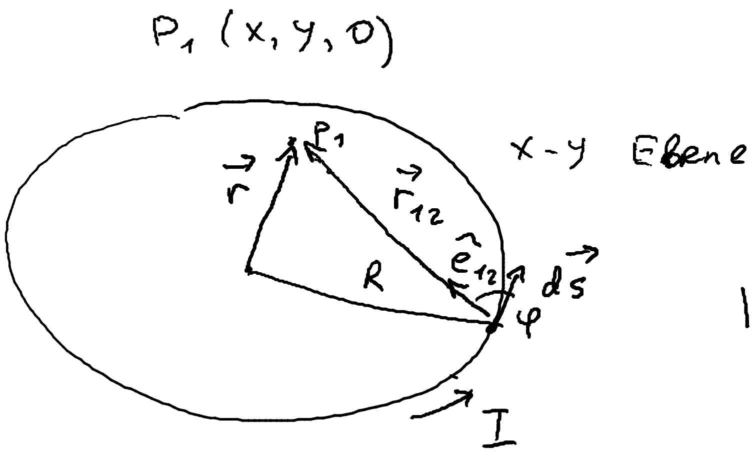
$$B = B_\varphi = -\left.\frac{\partial A_z}{\partial r}\right|_{r=R} = \frac{\mu_0 I}{2\pi R}$$

$$A_z = -\int_{R_0} B\,dR = -\frac{\mu_0 I}{2\pi}\ln\frac{R}{R_0}$$

$R > R_0$



Das Magnetfeld einer kreisförmigen Stromschleife

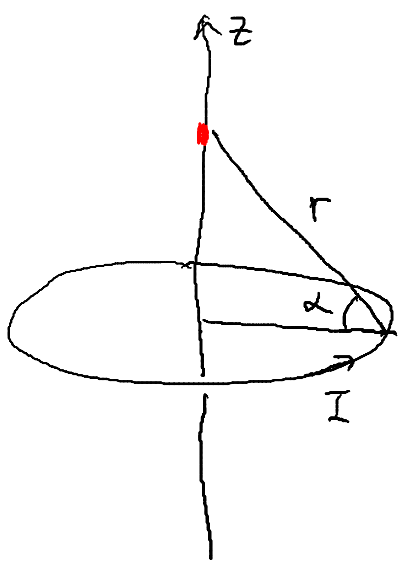


$$|\vec{e}_{12} \times d\vec{s}| = \sin\varphi$$

$$B_z = \frac{\mu_0 I}{4\pi} \oint \frac{\sin\varphi \, ds}{r_{12}^2}$$

Für $r_{12} = R \quad ; \quad \varphi = \pi/2$

$$B_z = \frac{\mu_0 I}{2R}$$



Auf der Symmetrieachse

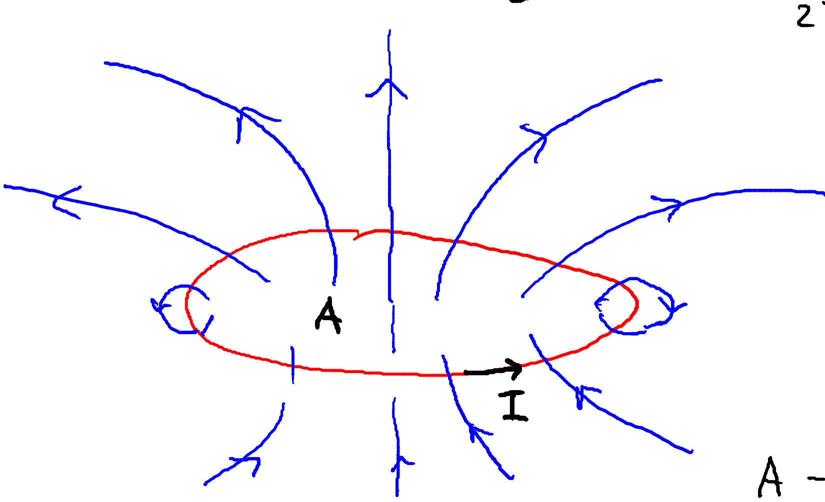
$$d\vec{B} = -\frac{\mu_0 I}{4\pi} \cdot \frac{\vec{r}\, d\vec{s}}{r^3}$$

$$B_{\parallel} = B_z = \int |dB| \cos\alpha$$

$$B = \frac{\mu_0 I}{4\pi r^3} \oint \vec{R} \cdot d\vec{s} = \frac{\mu_0 I R}{4\pi r^3} \cdot 2\pi R$$

$$r^2 = R^2 + z^2$$

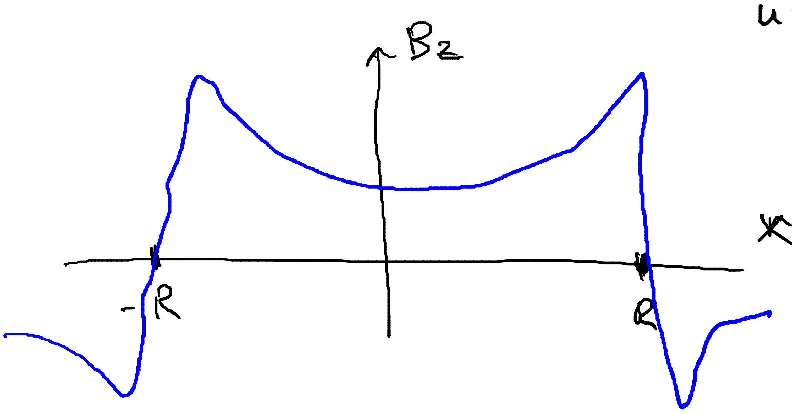
$$B_z(z) = \frac{\mu_0 I \pi R^2}{2\pi (R^2 + z^2)^{3/2}}$$



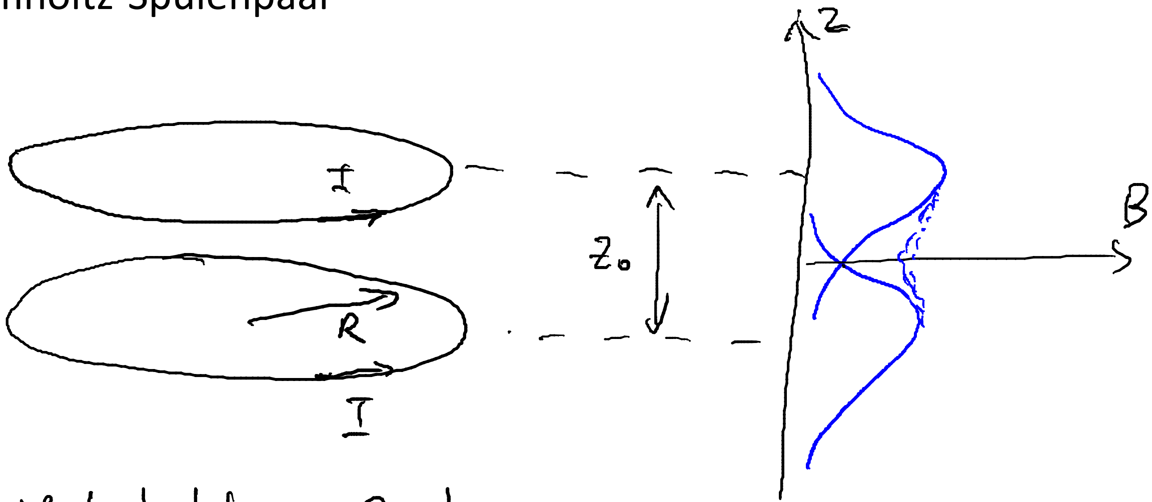
magnetische Dipolmoment

$$\vec{p}_m = I \cdot \vec{A}$$

A – vom Strom umschossene Fläche



Ein Helmholtz-Spulenpaar



Helmholtz - Spulenpaar

$$z_0 = R$$

$$B(z) \simeq \frac{\mu_0 I}{\left(5/4\right)^{3/2} R} \left[1 - \frac{144}{125} \cdot \frac{z^4}{R^4} \right]$$

! bei $z = 0$