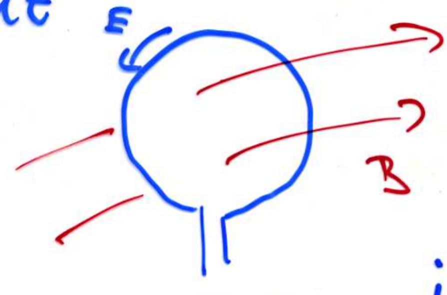
Faradaysche Induktionsgesetz

$$U_{ind} = -\frac{d}{dt}\int \vec{B}\,d\vec{A} = -\frac{d\phi_m}{dt}$$



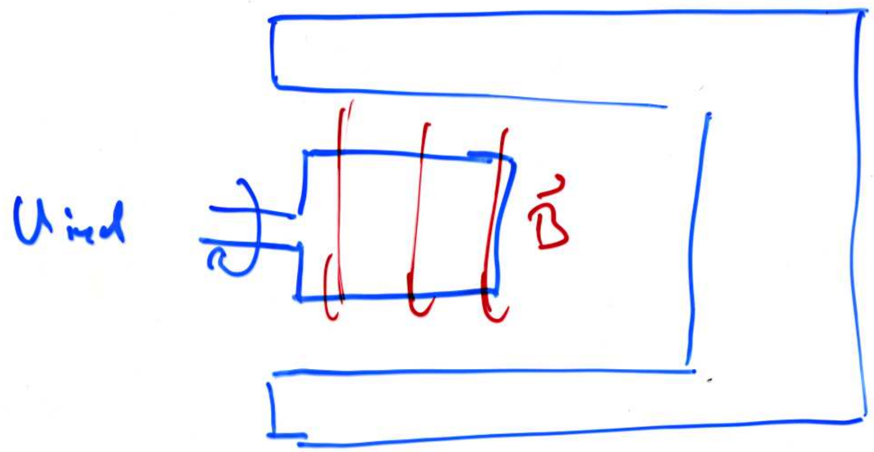
$$U_{ind} = -\dot{\phi}_m$$

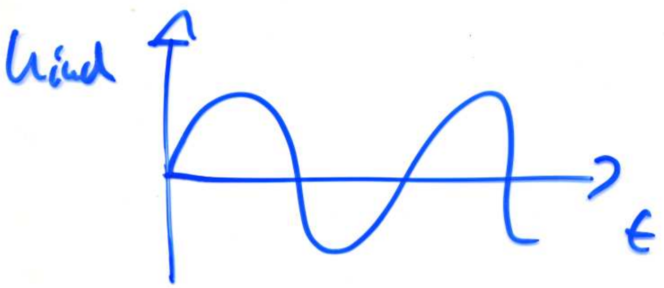
$$\mathrm{rot}\,\vec{E} = -\frac{d\vec{B}}{dt}$$

$$U_{ind} = \oint \vec{E}\,d\vec{s} \overset{\text{stokesche Satz}}{=} \int \mathrm{rot}\,\vec{E}\,d\vec{A}$$

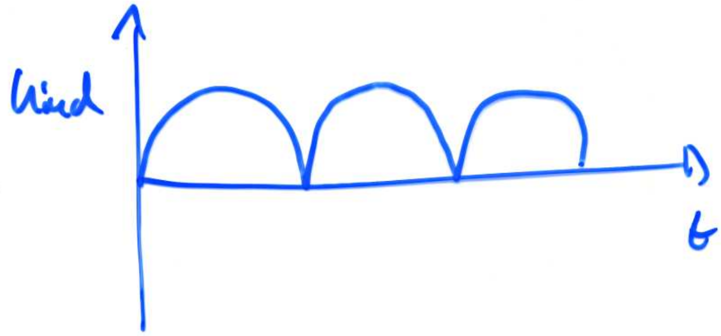
$$= \int -\frac{d\vec{B}}{dt}\,d\vec{A}$$

$$= -\frac{d}{dt}\int \vec{B}\,d\vec{A} = -\dot{\phi}_m$$



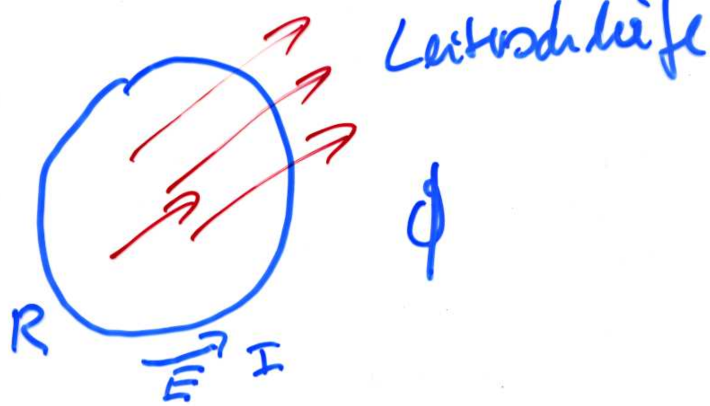


mit „Gleichrichter“ aus Schleifkontakten



Umkehrung des Prinzips ⇒ Elektromotor

Lenzsche Regel $\quad \mathrm{rot}\, E = - \frac{dB}{dt}$



Annahme: $\frac{d\phi}{dt} < 0 \quad$ B-Feld nimmt ab

$\mathrm{rot}\, E > 0 \qquad U_{ind} > 0$

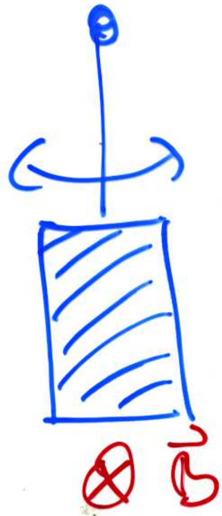
$\Rightarrow \quad I > 0$

$\Rightarrow$
induzierte Strom führt zu einem
induzierten B-Feld

$B_{ind} > 0$

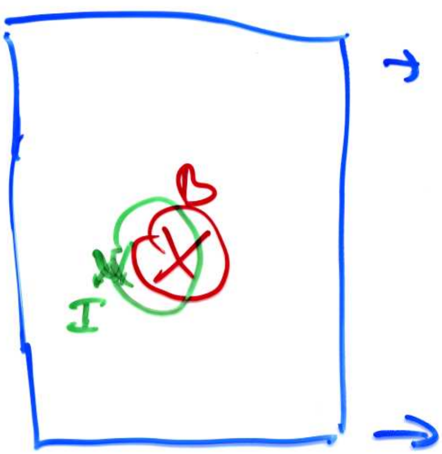
Lenzsche Regel: durch Induktion
erzeugte B-Feld wirkt
$\frac{dB}{dt}$ entgegen.

Versuch: Waltenhofsche Pendel



innerhalb des Pendels ist $\frac{dB}{dt} \neq 0$

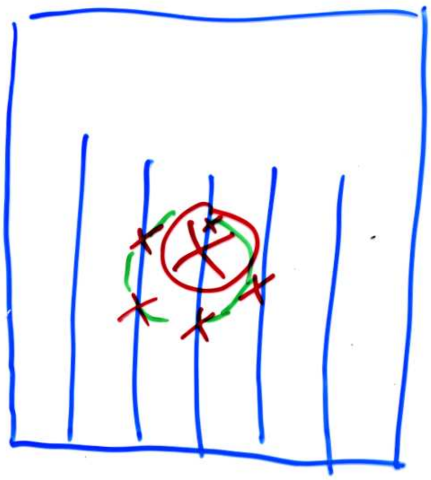
$$\mathrm{rot}\, E = -\frac{d}{dt}\vec{B}$$



Induzierte Wirbelströme erzeugen B_{ind}, was der Bewegung entgegen wirkt.

Aluminiumblech: $R \neq 0$

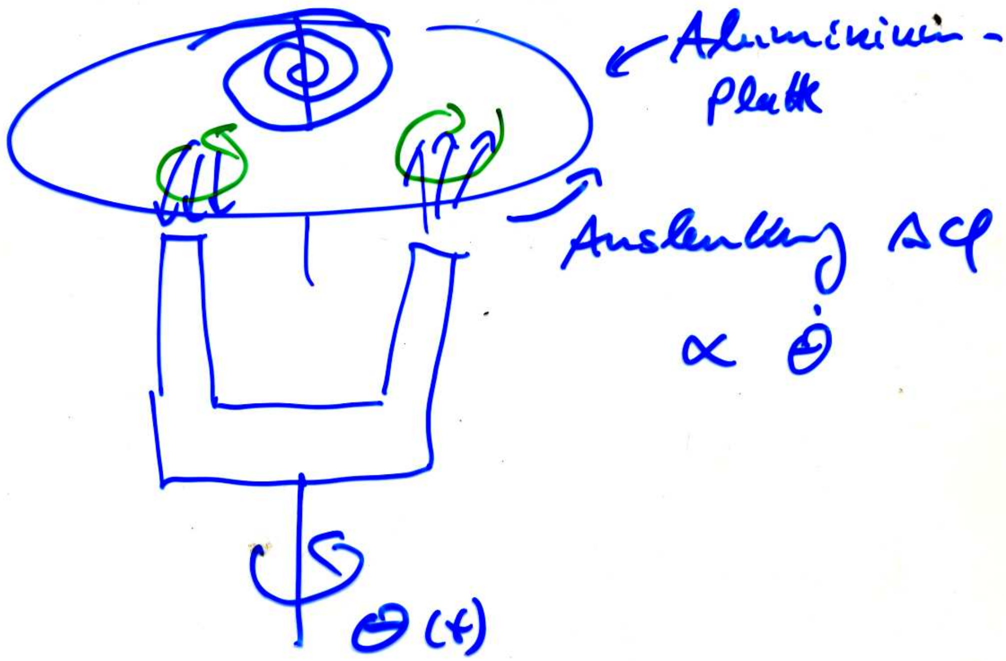
$\Rightarrow$ Kinetische Energie des Pendels wird in Wärme im Alublech umgewandelt.



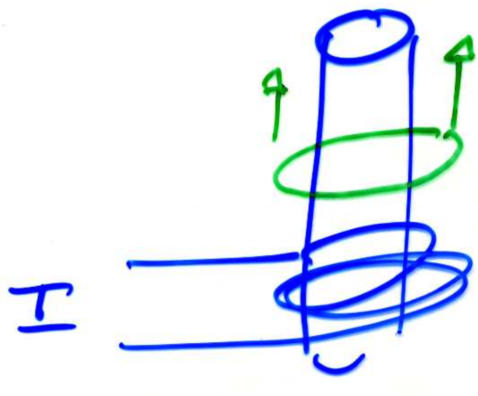
Durch Schlitze kann der Pfad der Wirbelströme unterbrochen werden und die Dissipation vermieden werden.

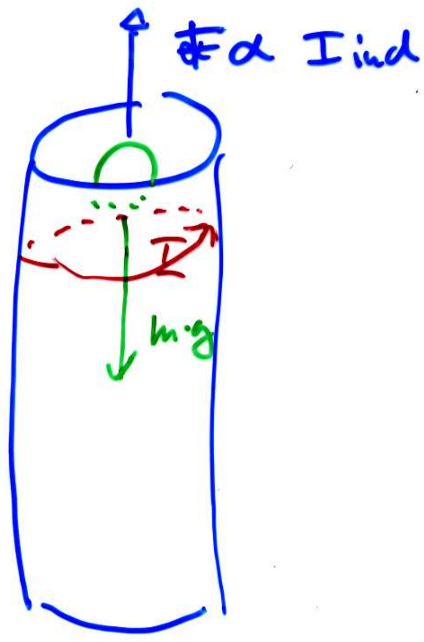
Ähnliches Prinzip wird bei Kernen von Transformatoren verwendet um Wirbelstromverluste zu vermeiden.

Tachometer



Elektromagnetische Kanone





U_{ind} $\qquad\qquad$ $U = R \cdot I$

$\Rightarrow I_{ind} = \dfrac{U_{ind}}{R}$, $\quad R \cdot I_{ind} = U_{ind}$

Potentielle Energie der Kugel wird in Wärme im Rohr umgesetzt.

$$U_{ind} \cdot I_{ind} = \cancel{\frac{U_{ind}}{R}} = R\, I_{ind}$$

Für $R = 0$ findet keine Umwandlung der potentiellen Energie in Wärme statt.

Was passiert?

Selbstinduktion

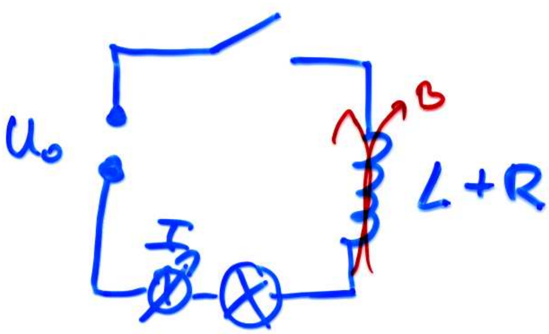
Stromdurchflossene Spule

$$\Phi_m = \int \vec{B}\,dF = \underline{\underline{L}}\, I$$

Selbstinduktivität L
(Induktivität)

$$[L] = 1\ \frac{Vs}{A} = 1\ \text{Henry} = 1\ H$$

$$U_{ind} = -L\dot{I}$$



$$U_0 = I \cdot R - U_{ind}$$
$$= I \cdot R + L\dot{I}$$

Anfangsbedingung $\quad I(0) = 0$

Ansatz: $I(t) = K e^{-t/\tau} + I_0$

$$\frac{U_0}{R} = K e^{-t/\tau} + I_0 + \frac{L}{R} K \left(-\frac{1}{\tau}\right) e^{-t/\tau}$$

$\Rightarrow$ $t \to \infty$

$\Rightarrow$ $I_0 = \frac{U_0}{R}$

$t=0$, $I(0)=0$

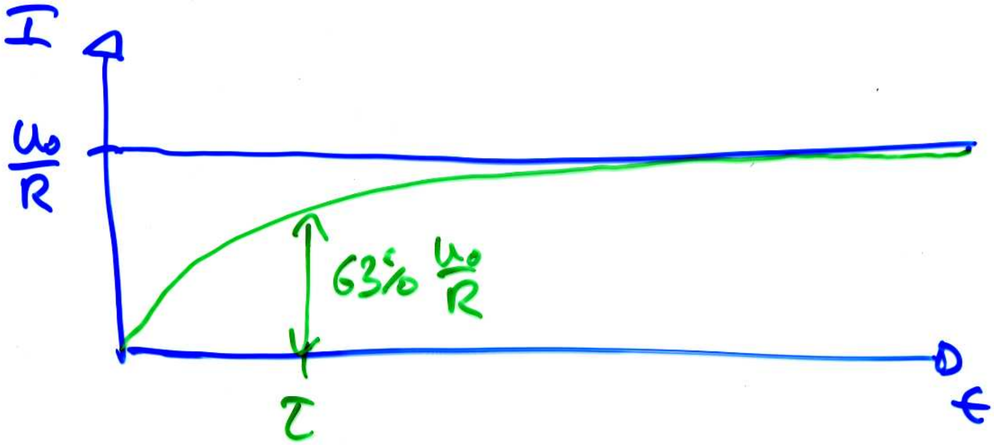
$$K + \frac{L}{R}\frac{K}{\tau} = 0$$

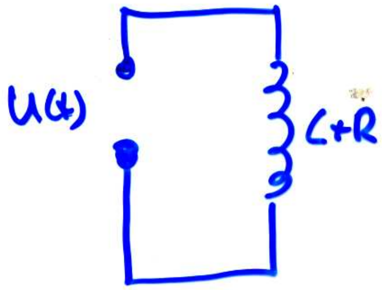
$\Rightarrow$ $\tau = \frac{L}{R}$ Zeitkonstante

$t=0$: $I(t) = K + I_0 = 0$

$\Rightarrow$ $K = -\frac{U_0}{R}$

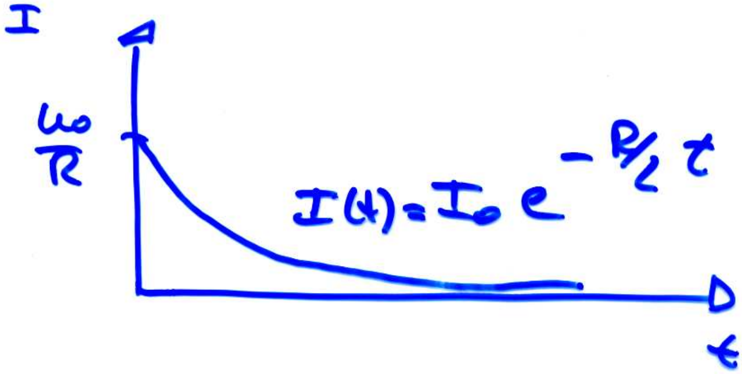
also: $I(t) = \frac{U_0}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right)$





$t<0 \quad U = U_0$

$t=0 \quad U=0$

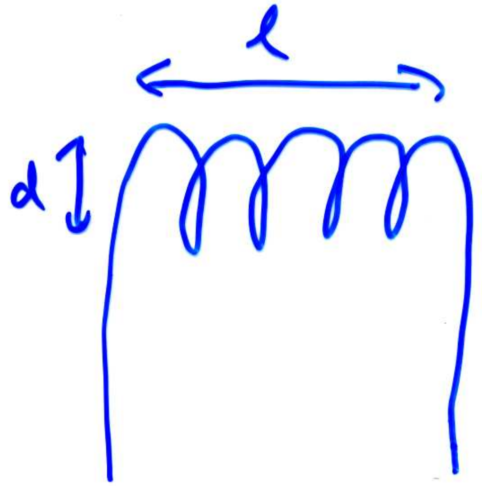


$$0 = I(t)R - U_{ind}$$

$$= IR + L\dot{I}$$

$$\Rightarrow I = I_0 e^{-R/L\, t}$$

Wie groß ist die Induktivität einer Spule?



$N = n \cdot l$ Windungen

$A = \pi (d/2)^2$

$$B = \mu_0 n I$$

$$\phi = B \cdot A = \mu_0 n A I$$

$\frac{dI}{dt}$ bewirkt $\quad \frac{d\phi}{dt} = \mu_0 n A \frac{dI}{dt}$

$$\Rightarrow U_{ind} = -N \frac{d\phi}{dt} = -\mu_0 N n A \frac{dI}{dt} = -L \frac{dI}{dt}$$

$\Rightarrow L = \mu_0 N n A \qquad N = n \cdot l$

$= \mu_0 n^2 l A \qquad l \cdot A = V$ (Volumen der Spule)

$= \mu_0 n^2 V$

$= \mu_0 \frac{N^2}{l} A$