Vorlesung 2

Physik II

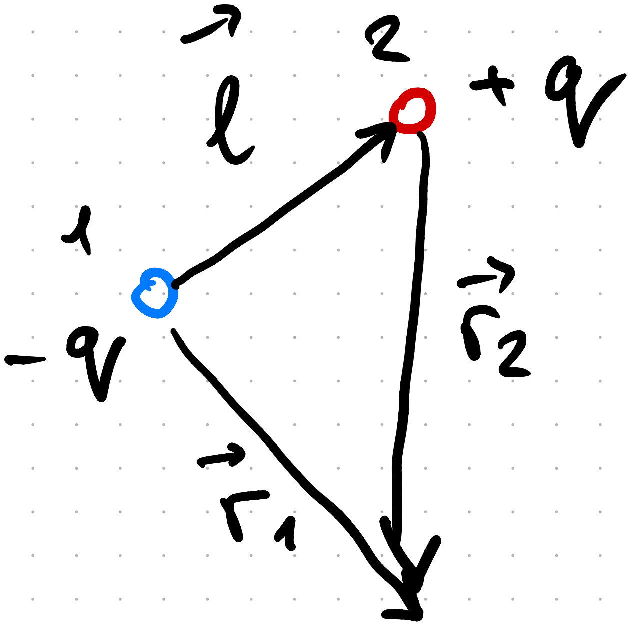
A. Ustinov

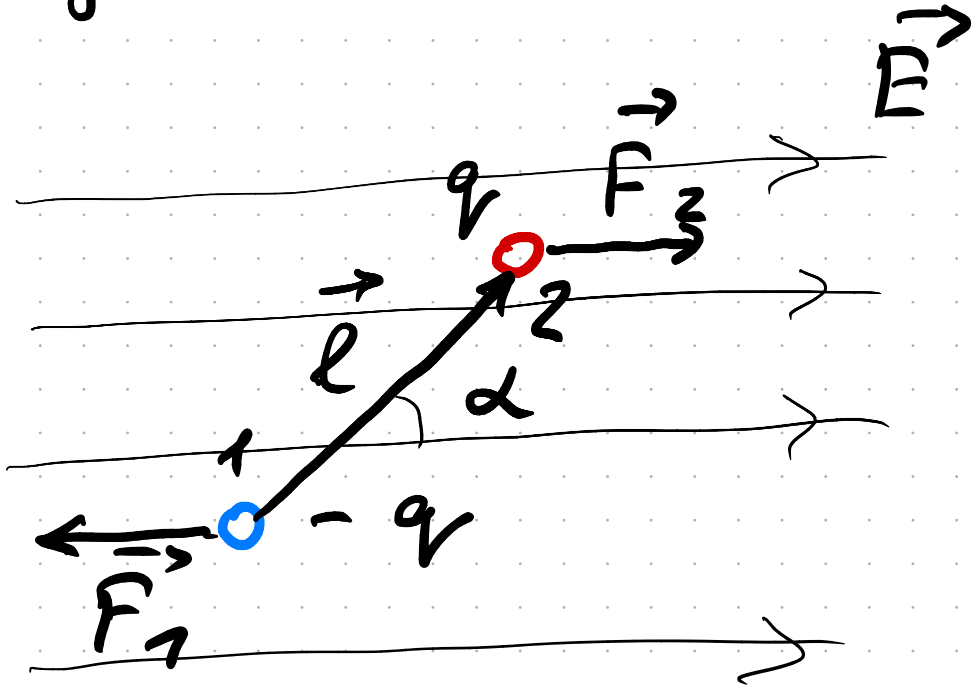
SS 2020

Das elektrische Feld des Dipols :

$$\vec{E}_c = \frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3} ;$$

a) Dipol im homogenen Feld





$$\vec{F}_1 = -q\vec{E} \quad ; \quad \vec{F}_2 = q\vec{E} \quad ; \quad \vec{r}_2 - \vec{r}_1 = \vec{\ell}$$

Drehmoment: $\vec{D} = q(\vec{r}_2 \times \vec{E}) - q(\vec{r}_1 \times \vec{E})$

$\vec{D} = (q \cdot \vec{\ell}) \times \vec{E} = \vec{p} \times \vec{E}$;

$|\vec{D}| = -pE\sin\alpha$;

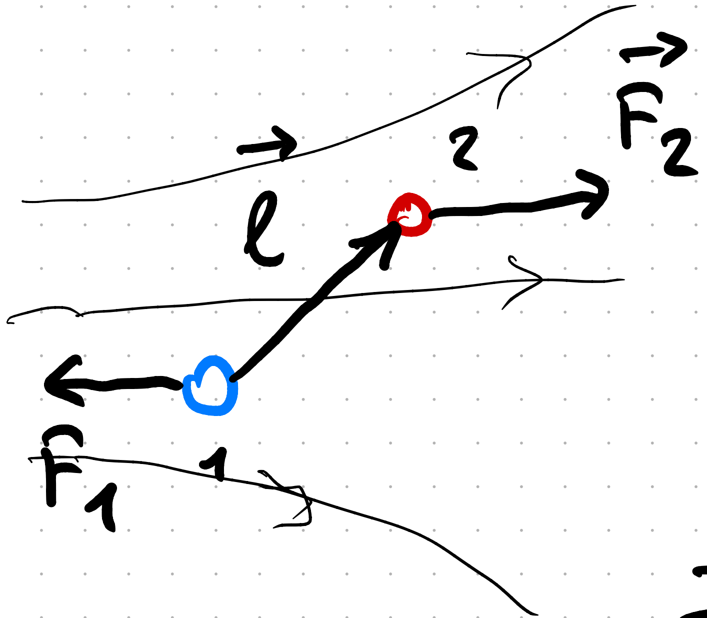
Die potentielle Energie des Dipols:

$\delta W_{pot} = pE\sin\alpha \cdot d\alpha$;

$W_{pot} = \int \delta W_{pot} = -pE\cos\alpha + \underset{0}{\underset{\parallel}{const}}$;

$W_{pot} = -(\vec{p} \cdot \vec{E})$;

b) Dipol im inhomogenen Feld



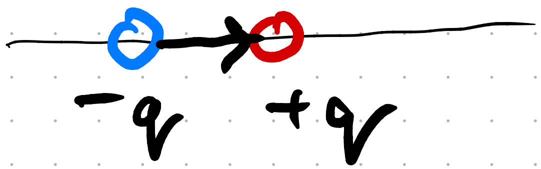
$$\vec{F}_2 \neq \vec{F}_1;$$

$$\vec{F} = q(\vec{E}_2 - \vec{E}_1) = q\, d\vec{E};$$

$$d\vec{E} = \ell_x \frac{\partial \vec{E}}{\partial x} + \ell_y \frac{\partial \vec{E}}{\partial y} + \ell_z \frac{\partial \vec{E}}{\partial z};$$

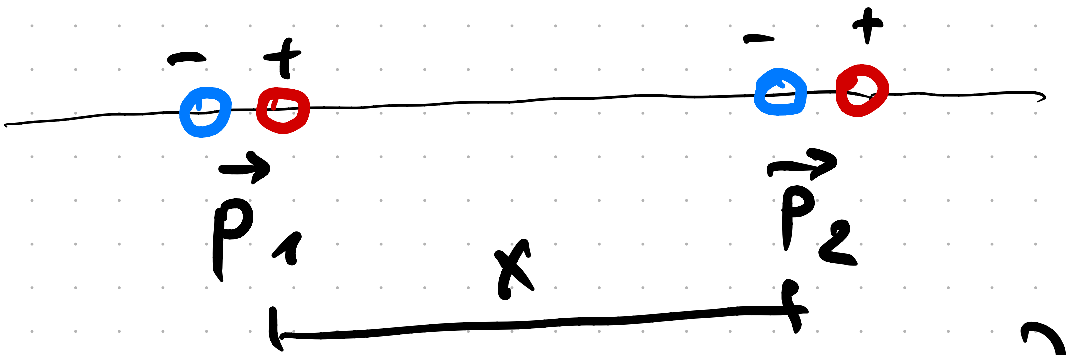
$$\vec{F} = p_x \frac{\partial \vec{E}}{\partial x} + p_y \frac{\partial \vec{E}}{\partial y} + p_z \frac{\partial \vec{E}}{\partial z};$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\right); \quad \vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E};$$



$$\vec{F} = p_x \frac{\partial \vec{E}}{\partial x};$$

c) Wechselwirkung zw. zwei Dipolen



$$F = p_2 \frac{\partial E_1}{\partial x} \, ; \quad E_1 = \frac{2p_1}{x^3} \, ;$$

$$\frac{\partial E_1}{\partial x} = -\frac{2 \cdot 3 \cdot p_1}{x^4} \, ;$$

$$\Rightarrow F = -\frac{6 p_1 p_2}{x^4} \, ;$$

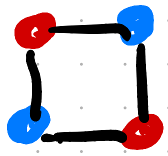
Ladung

$$E \sim \frac{1}{r^2}$$

Dipol

$$\sim \frac{1}{r^3}$$

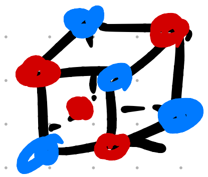
Quadrupol



$$\sim \frac{1}{r^4}$$

. . .

Octupol



Ladungsverteilungen:

Punktladung $\Rightarrow$ 0D $\quad q$

Draht $\Rightarrow$ 1D $\quad Q/L = \varkappa;$

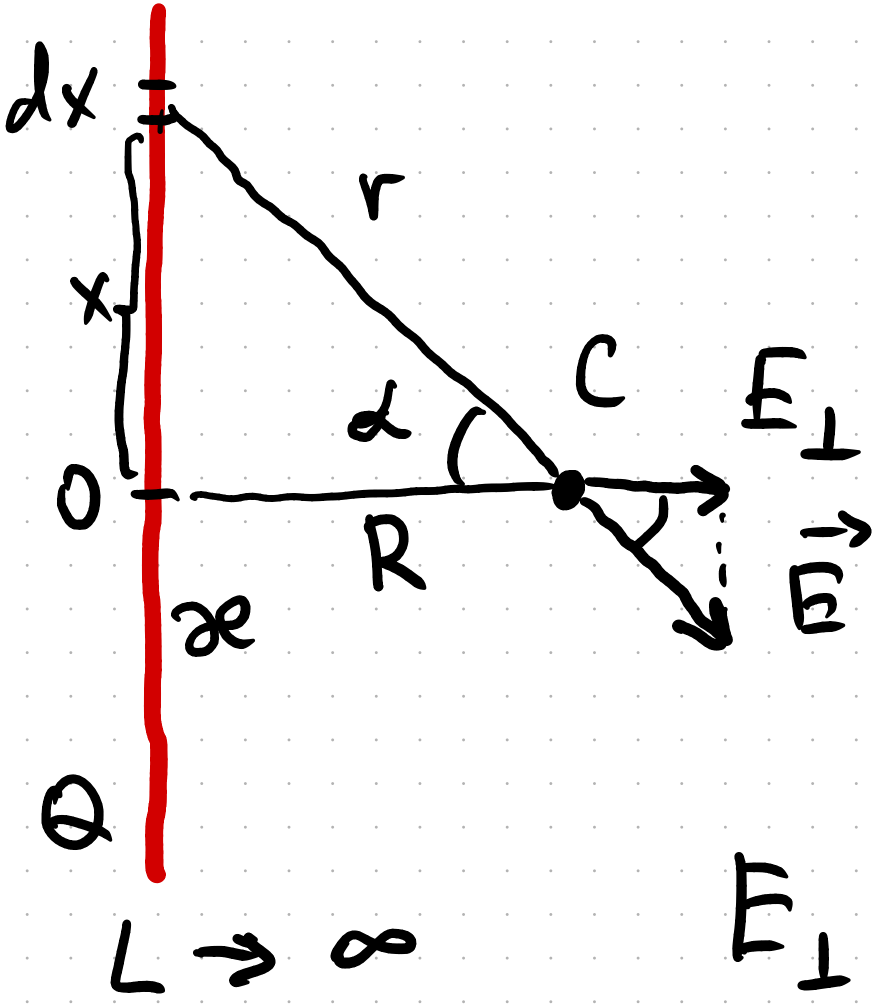
Ladungsdichte

Fläche $\Rightarrow$ 2D $\quad Q/S = \sigma;$

Volumen $\Rightarrow$ 3D $\quad Q/V = \rho;$

<u>Homogene 1D Draht</u>

$$dE_{\perp} = \varkappa\, dx \cdot \frac{1}{r^2} \cdot \cos\alpha \;;$$



$$E_\perp = \int_{-\infty}^{+\infty} \varkappa \, dx \cdot \frac{1}{r^2} \cos\alpha \, ;$$

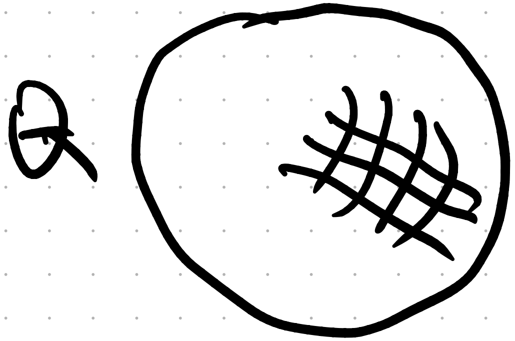
$$x = R \cdot \tan\alpha \, ; \quad dx = \frac{R \, d\alpha}{\cos^2\alpha} \, ;$$

$$r = \frac{R}{\cos\alpha} \, ;$$

$$E_\perp = \int_{-\pi/2}^{+\pi/2} \frac{\varkappa \, d\alpha \cdot \cancel{\frac{R}{\cos^2\alpha}} \, \cancel{\cos^2\alpha}}{R^{\cancel{2}}} \cos\alpha$$

$$E_\perp = \int_{-\pi/2}^{+\pi/2} \frac{\varkappa}{R} \cos\alpha \cdot d\alpha = \frac{2\varkappa}{R} \sim \frac{1}{R} \, ;$$

$[cgs] \Rightarrow E_\perp = \frac{2\varkappa}{R}$; $[SI] \Rightarrow E_\perp = \frac{2\varkappa}{4\pi\varepsilon_0 R}$

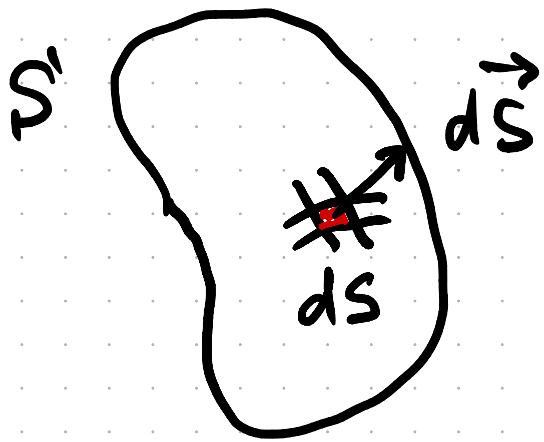


geladene Kugel

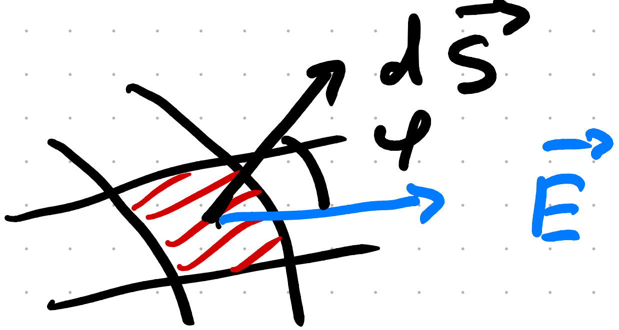
• x, y, z $\Rightarrow$ complex integral

$\rightsquigarrow$ vereinfachung erwünscht!

1.3. Gauß Theorem (Satz)



Flächenelement dS ; Flächennormalenvektor $d\vec{S}$; elektrischen Fluss : $d\Phi_{el} = \vec{E}\cdot d\vec{S}$;



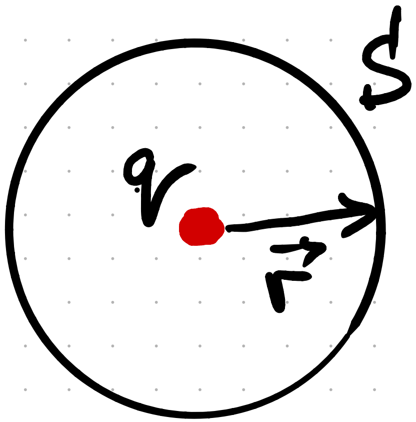
$$d\Phi_{el} = |d\vec{S}| \cdot \underbrace{|\vec{E}| \cdot \cos\varphi}_{E_n} ;$$

[cgs] $$\boxed{\Phi = \oint E_n \, dS = 4\pi q} ;$$

geschlossene Oberfläche

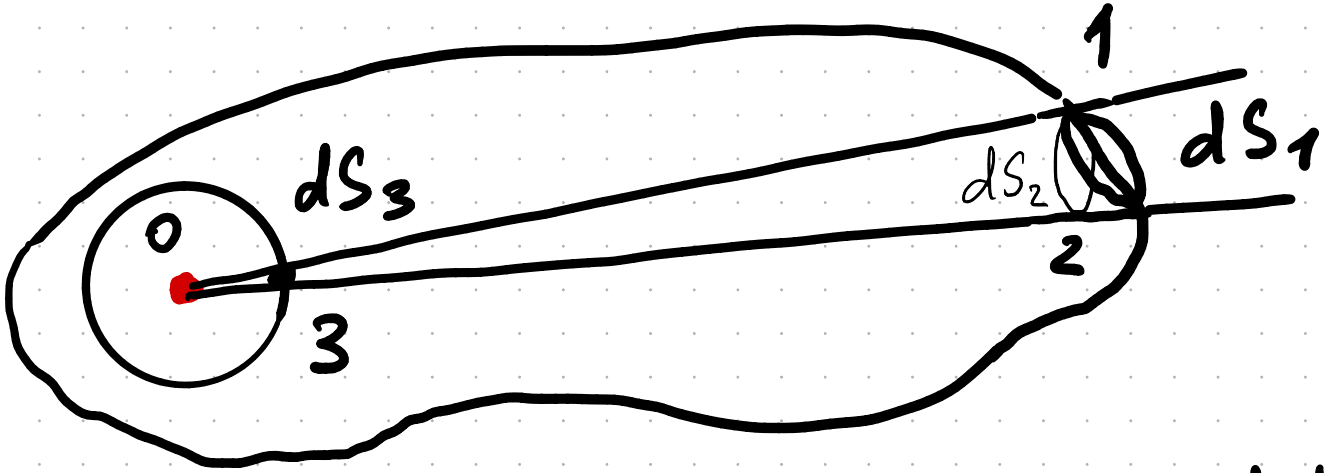
[SI] $$\boxed{\Phi = \frac{q}{\varepsilon_0}} ;$$

1)



$$\Phi = \oint E_n \, dS = \frac{q}{r^2} 4\pi r^2$$
$$= 4\pi q ;$$

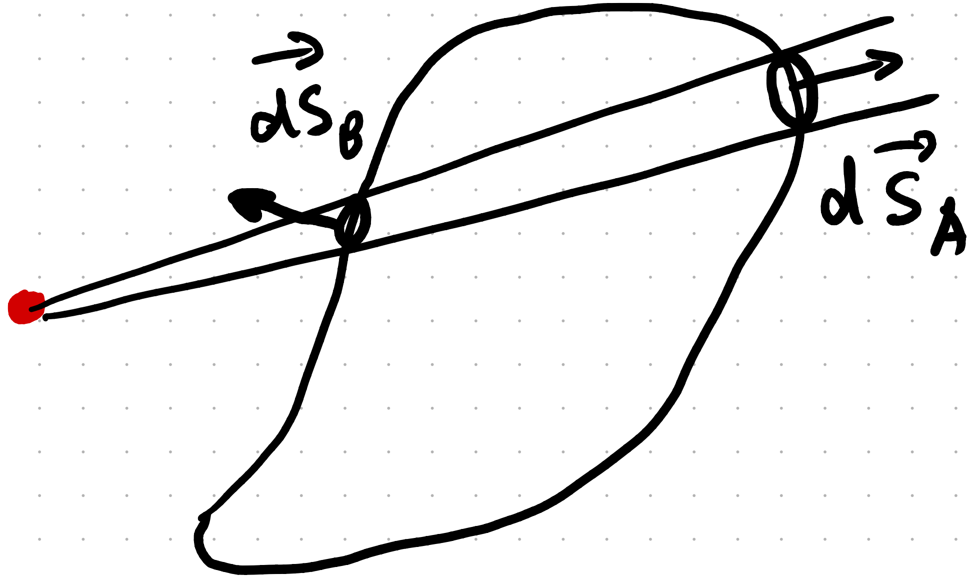
2)



$E_1 \cdot dS_1 \cos\varphi = E_1 \cdot dS_2$; $dS_i \sim r_i^2$; $i = 1, 2, 3$;

$E_i \sim \frac{1}{r^2}$; $\Rightarrow$ $dS_i \cdot E_i = \text{const}$

3)



$\delta\Phi = 0$;

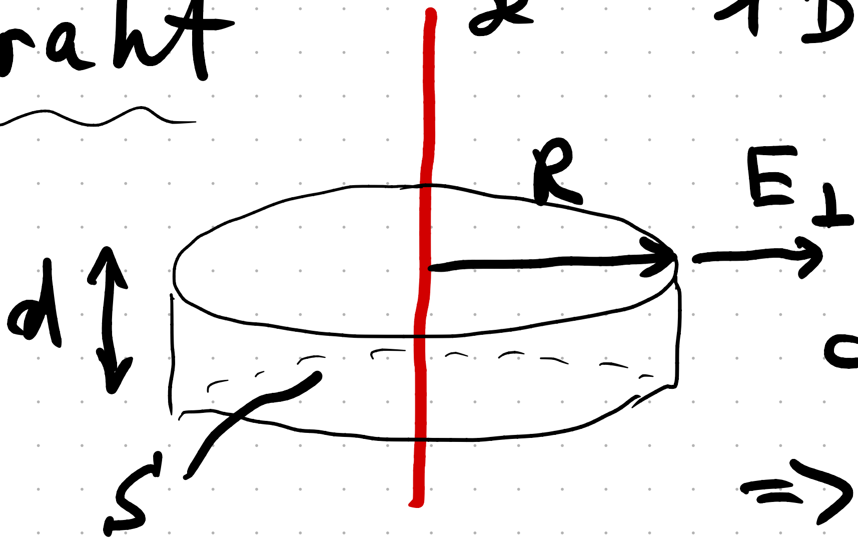
Superp. Prinzip

$\vec{E} = \sum_i \vec{E_i}$;

Gauß Theorem: Fluß aus beliebiger geschlossener Fläche ist proportional zur umgeschlossenen Ladung Q, unabhängig von der Ladungsverteilung.

$[cgs]$: $\Phi = 4\pi Q$; $[SI]$: $\Phi = \frac{Q}{\varepsilon_0}$;

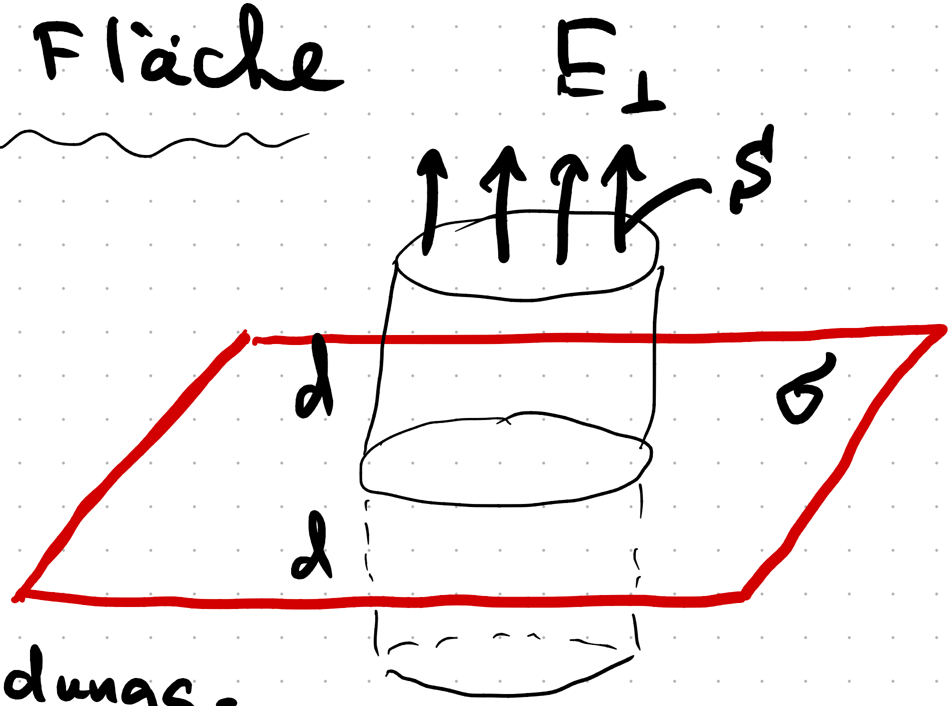
1D Draht $\varkappa$ 1D Ladungsdichte



$$\Phi = 2\pi R d E_\perp = 4\pi \varkappa d;$$

$$\Rightarrow E_\perp = \frac{2\varkappa}{R};$$

2D Fläche



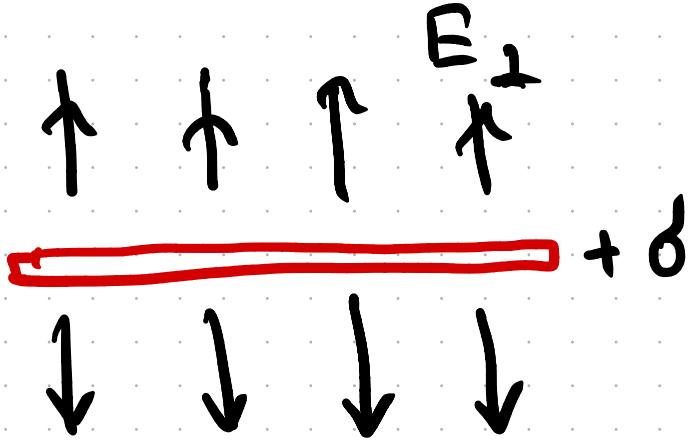
σ

2D Ladungs-
dichte

$$\Phi = 2 \cancel{S} E_\perp = 4\pi\sigma \cancel{S};$$

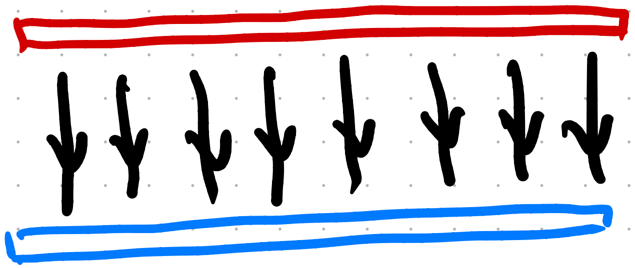
$$\boxed{E_\perp = 2\pi\sigma} = \text{const};$$

2D



$$|E_{+\sigma}| = |E_{-\sigma}| = 2\pi\sigma$$

$E_\perp = 0$



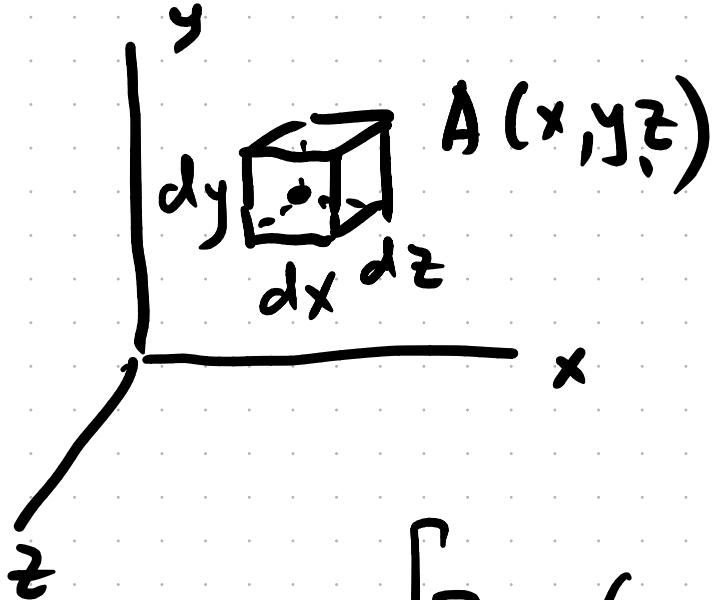
$\boxed{E = 4\pi\sigma}$

$E_\perp = 0$

Kondensator

Plattenkondensator

Der Gaußsche Satz in Differentialform



3D Ladungsverteilung

$\rho(x,y,z)$

Fluß durch Flächen $\perp x$

$$\left[E_x(x+dx) - E_x(x)\right] dy\,dz$$

$$= \frac{\partial E_x}{\partial x} \cdot dx \cdot dy \cdot dz = \frac{\partial E_x}{\partial x} \cdot dV;$$

$$d\Phi = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) \cancel{dV} = 4\pi\rho\, \cancel{dV};$$

$$\left(\vec{\nabla}, \vec{E}\right) \hat{=} \operatorname{div} \vec{E} = 4\pi\rho;$$ (Skalarprodukt "Nabla" und $\vec{E}$)

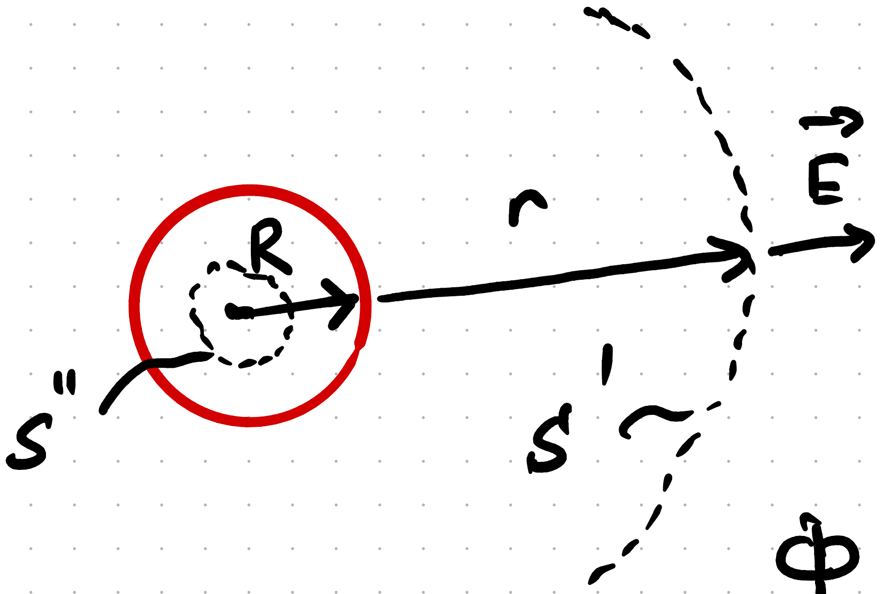
$$\boxed{\operatorname{div} \vec{E} = 4\pi \rho}$$

phys. Bed.: $\operatorname{div} \vec{E} = \lim\limits_{V \to 0} \dfrac{\oint_S d\vec{E} \cdot \vec{ds}}{V}$;

$\Rightarrow$ die im Raum verteilten Ladungen sind die Quellen (für $\rho > 0$) bzw. Senken (für $\rho < 0$) des elektrostatischen Feldes.

Spezielle Ladungsverteilungen:

a) Geladene Hohlkugel mit radius R und Flächenladungsdichte σ:



$$Q = 4\pi R^2 \sigma \,;$$

für $r \geq R$;

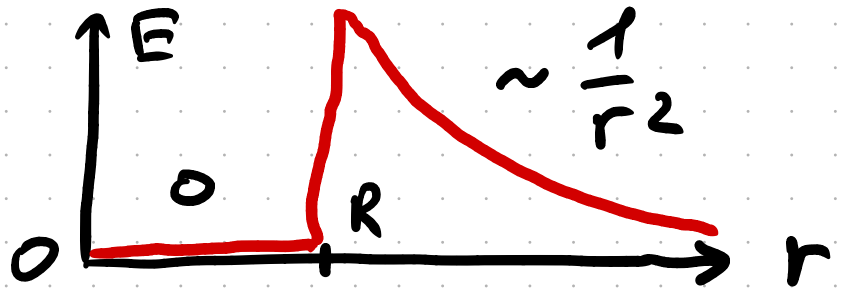
$$\vec{E} \parallel \vec{r}$$

$$\Phi_{S'} = \oint_{S'} \vec{E} \cdot \vec{ds} = E\, 4\pi r^2 = 4\pi Q \,;$$

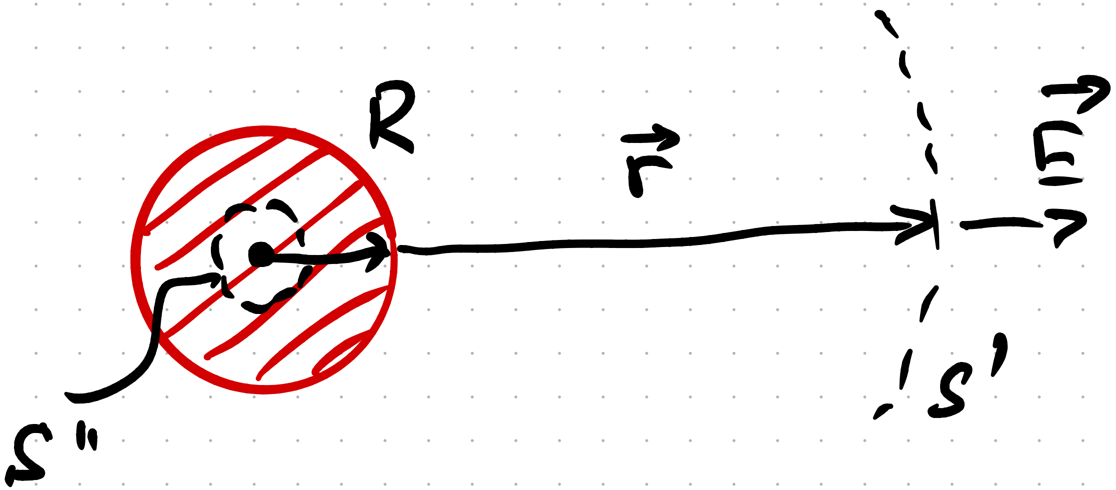
$$\Downarrow$$

$$\vec{E} = 4\pi\sigma \frac{R^2}{r^2} \frac{\vec{r}}{r}$$

$$\Phi_{S''} = \oint_{S''} \vec{E} \cdot \vec{dS} = 0$$



b) eine homogen geladene nicht-leitende Vollkugel mit Volumenladungsdichte ρ:



$$Q = \frac{4}{3}\pi R^3 \rho;$$

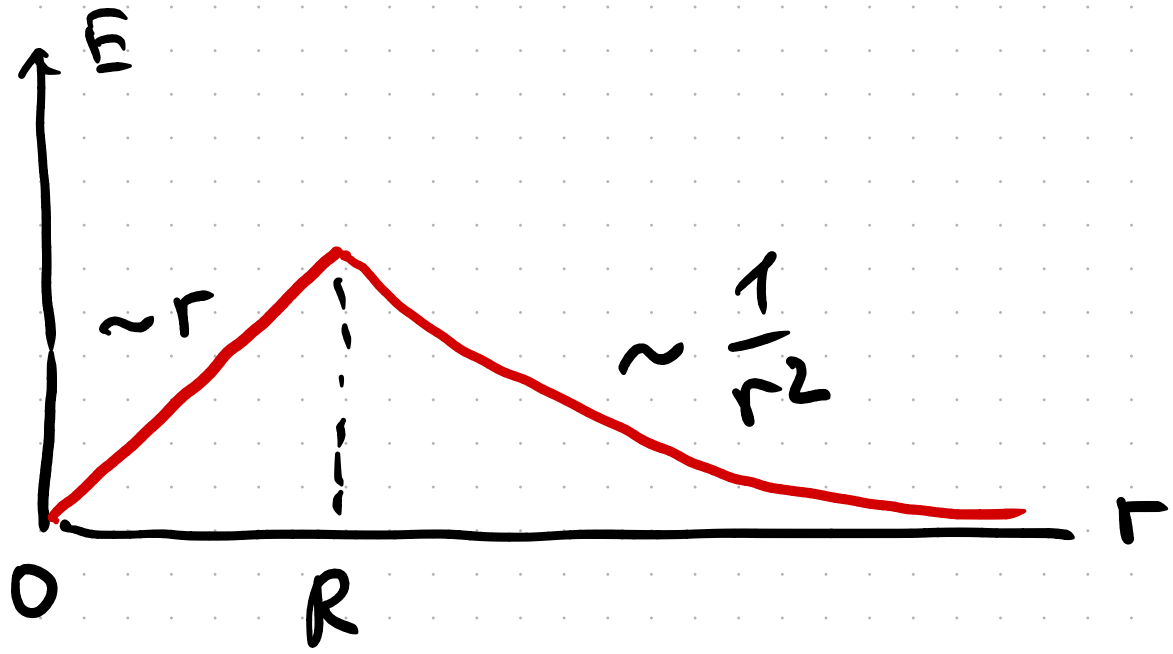
für $r \geq R$: analog zu (a)

$$\vec{E} = \frac{Q}{r^3}\vec{r};$$

$$Q'' = \frac{4}{3}\pi r^3 \rho; \quad r < R$$

$$\oint_{S''} \vec{E}\cdot d\vec{s} = E \cdot 4\pi r^2 = 4\pi \frac{4}{3}\cancel{\pi} r^3 \rho;$$

$$\Rightarrow \quad E''\Big|_{r<R} = \frac{4}{3}\pi r \rho;$$



E
$\sim r$
$\sim \frac{1}{r^2}$
r
0
R