Vorlesung 11

Physik II

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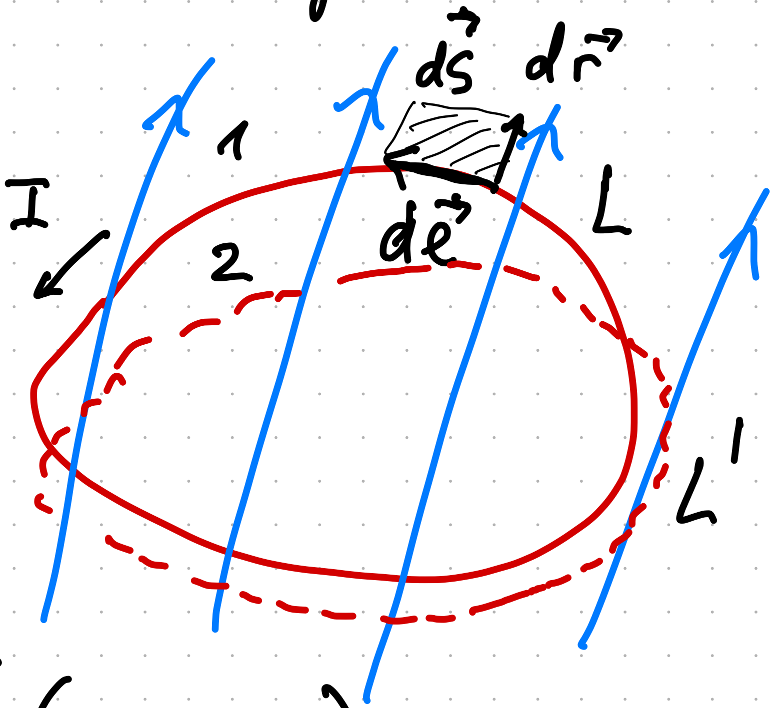
SS 2020

4. Zeitabhängige elektromagnetische Felder

4.1. Induktionsgesetze

2 Beispiele:

a)



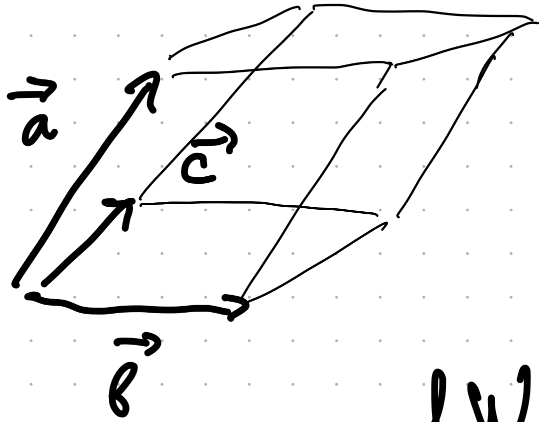
$\vec{B}(x,y,z)$

elem. Verschiebung;

elem. Arbeit:

$$dW = (d\vec{F} \cdot \vec{dr})$$

$$= \frac{I}{c}\left([\vec{dl}\,\vec{B}]\,\vec{dr}\right);$$



das Spatprodukt:

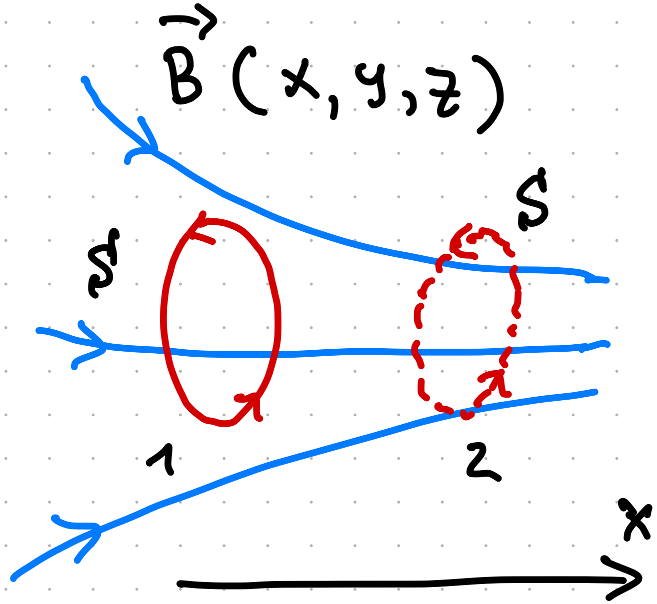
$$\vec{a}\cdot[\vec{b}\times\vec{c}] = \vec{b}\cdot[\vec{c}\times\vec{a}] = \vec{c}\cdot[\vec{a}\times\vec{b}];$$

$$dW = \frac{I}{c}\left(\vec{B}\cdot[\vec{dr}\times\vec{dl}]\right) = \frac{I}{c}\left(\vec{B}\cdot\vec{ds}\right);$$

Verschiebung der Stromkreis: $L \to L'$

$$\Delta W_{2,1} = \frac{I}{c}(\Phi_2 - \Phi_1) = \frac{I}{c}\Delta\Phi;$$

b)



allgemein: $\vec{F} = (\vec{p}_m \nabla)\vec{B};$

$$\Delta W = \underbrace{\underbrace{\frac{IS}{c}}_{p_m}\cdot\frac{dB_x}{\partial x}}_{F_x}\cdot\Delta x = F_x\,\Delta x;$$

$$\Rightarrow \quad \Delta W_{2,1} = \frac{I}{c}\left(S B_2 - S B_1\right) = \frac{I}{c}\,\Delta\Phi\,;$$

Faradaysches Induktionsgesetz

Faraday : 1830 – 1840

Induktionsspannung entlang eines Leiters in einem zeitlich veränderlichen Magnetfeld.

$$\boxed{U_{ind} = -\frac{1}{c}\frac{\partial\Phi}{\partial t}} \quad [cgs] \qquad \boxed{U_{ind} = -\frac{\partial\Phi}{\partial t}} \quad [SI]$$

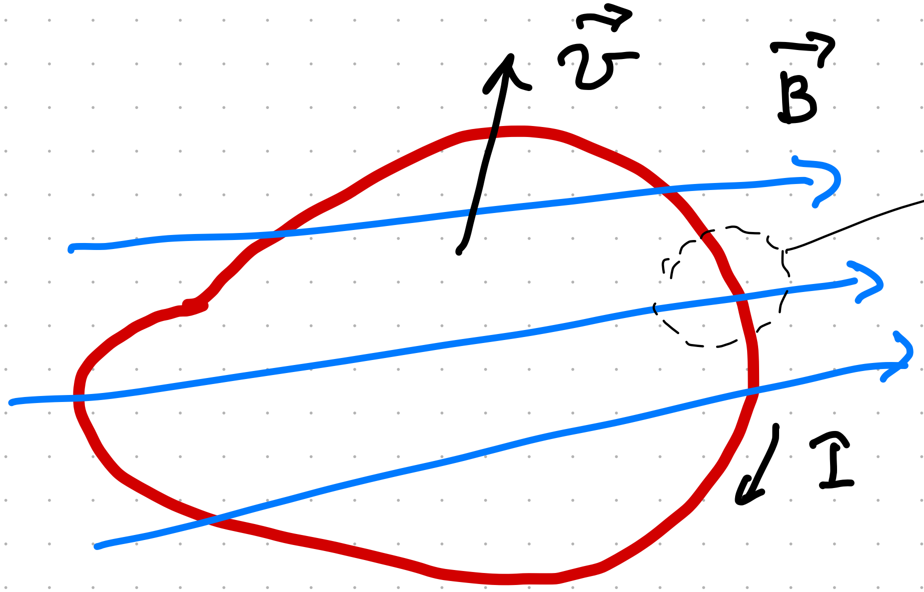
wichtig! (Energieerhaltungsatz)

$$d\Phi = B \cdot \underbrace{dS} + S \cdot \underbrace{dB} \;;$$

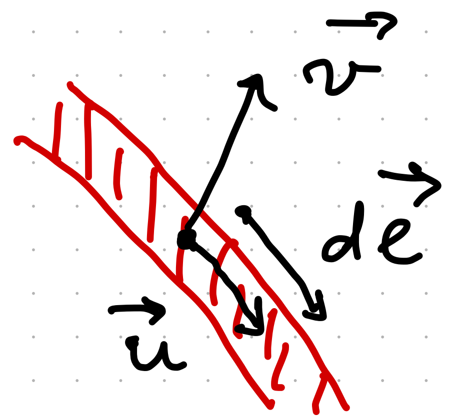
2 Möglichkeiten, S oder B zu ändern.

<u>Faraday</u>: $\vec{B}(t) = \text{const}$;

Erklärung durch Lorentzkraft.



beweg. Stromkreis



$$\vec{v}_\Sigma = \underbrace{\vec{v}}_{\text{Bewegung d. Leiter}} + \underbrace{\vec{u}}_{\text{Strom}} \;;$$

$$\vec{F} = \frac{q}{c}\left[\vec{v}\times\vec{B}\right] + \underbrace{\frac{q}{c}\left[\vec{u}\times\vec{B}\right]}_{\perp\, d\vec{l}} \;; \quad , \Rightarrow \Delta W = 0$$

$$\vec{E}_{ind} = \frac{1}{c}\left[\vec{v}\times\vec{B}\right] \;;$$

$$U_{ind} = \oint_L \left(\vec{E}_{ind}\cdot d\vec{l}\right) = \frac{1}{c}\oint_L \left(\left[\vec{v}\times\vec{B}\right]\cdot dl\right)$$

zykl. drehen
gemisch. Vektorprodukt

$$= \frac{1}{c}\oint_L \left(\left[d\vec{l}\times\vec{v}\right]\cdot\vec{B}\right);$$

$$U_{ind} = -\frac{1}{c}\oint_L \left(\vec{B}\cdot\left[\vec{v}\times d\vec{l}\right]\right) \;;$$

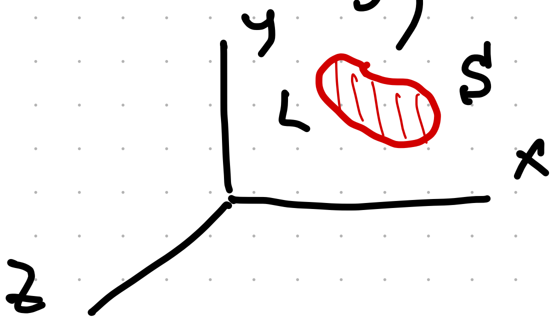
$$U_{ind} = -\frac{1}{c\,\Delta t}\oint_L \left(\vec{B}\cdot\underbrace{[\Delta\vec{r}\times d\vec{\ell}\,]}_{\Delta\vec{S}}\right);$$

$$\Rightarrow \quad U_{ind} = -\frac{1}{c}\frac{\partial\Phi}{\partial t}\Bigg|_{B=\text{const}};$$

"gefrorene feld"

Maxwell !

(vs Faraday)



$\vec{B}(t) \neq \text{const}$; $\vec{B}(x,y,z,\underline{\underline{t}})$;

Zirkulation:

$$\underbrace{\oint_L \vec{E}\,d\vec{\ell}}_{\text{in statik}\,=\,0} = -\frac{1}{c}\int_S \frac{d}{dt}\left(\vec{B}\,d\vec{s}\right) \overset{\text{Maxwell}}{=} -\frac{1}{c}\frac{\partial\Phi}{\partial t};$$

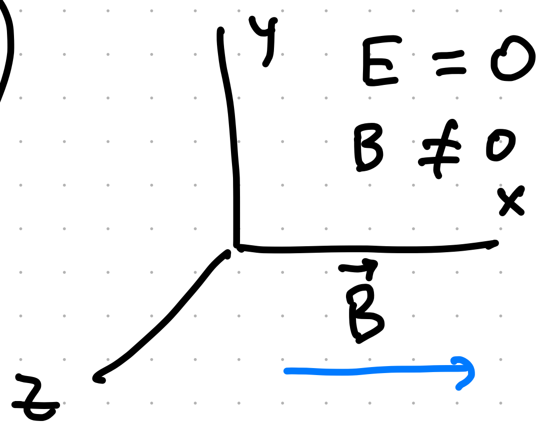
NB || Elektromagnetische Feld ($\vec{B}$ vs $\vec{E}$)

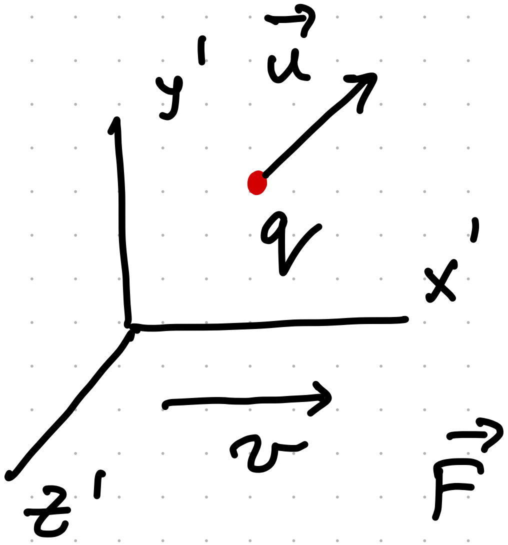
relative nach Bezugssystem,

Transformation nach Bezugssystem:

(nichtrelativistische Fall) $v^2 \ll c^2$

1)





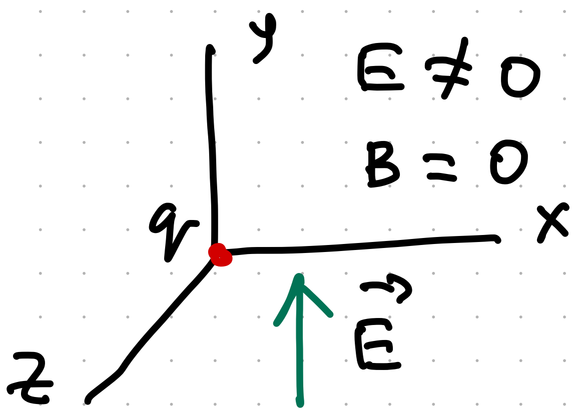
$$\vec{F} = \frac{q}{c}\left[(\vec{u}+\vec{v})\times\vec{B}\right];$$

ist invariant

$$\vec{F} = q\,\vec{E}' + \frac{e}{c}\left[\vec{u}\times\vec{B}'\right];$$

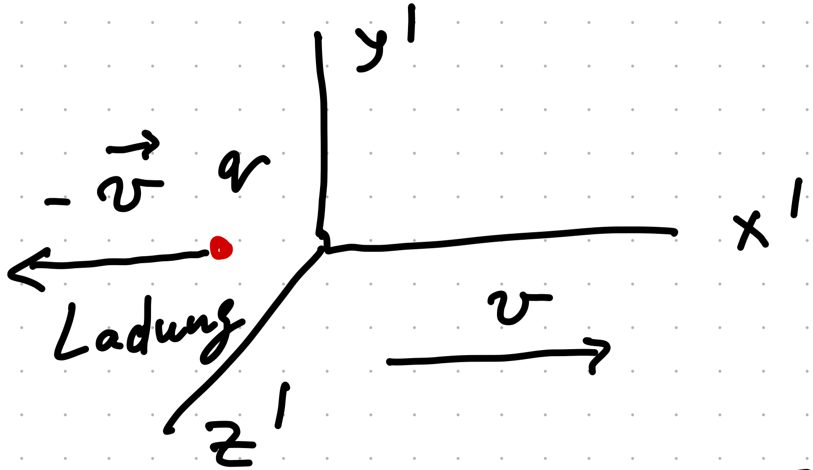
$$\vec{E}' = \frac{1}{c}\left[\vec{v}\times\vec{B}\right]; \quad \vec{B}' = \vec{B};$$

2)



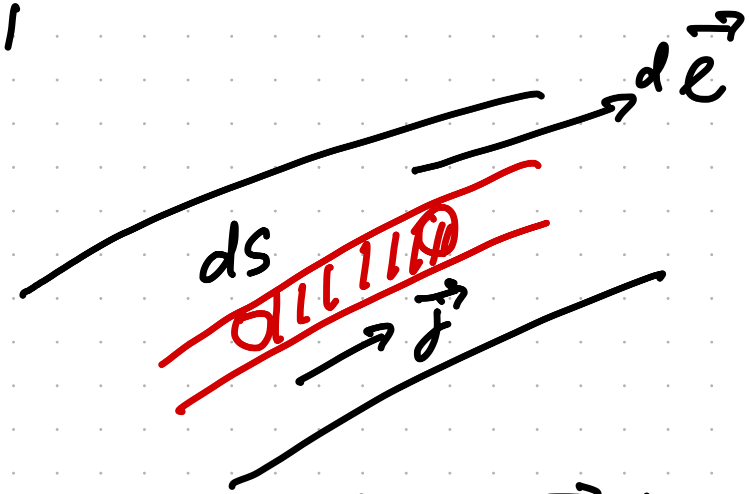
$E \neq 0$

$B = 0$



B' – ?

E' – ?



Biot – Savart!

$$d\vec B = \frac{I}{c}\,\frac{[d\vec l \times \vec r]}{r^3};$$

$$I = j\,ds; \qquad \vec j = ne\vec v; \qquad \vec j \parallel \vec v \parallel d\vec l$$

$$d\vec B = \frac{j\,ds}{c}\,\frac{[d\vec l \times \vec r]}{r^3} = \frac{ds \cdot dl}{c}\,\frac{[\vec j \times \vec r]}{r^3}$$

$$= \frac{ds\,dl\,ne}{c} \cdot \frac{[\vec v \times \vec r]}{r^3};$$

$\rho_{el} \cdot dV$

pro Elementarladung:

$$d\vec{B}_e = \frac{e}{c}\frac{[\vec{v}\times\vec{r}]}{r^3} = \frac{1}{c}\left[\vec{v}\times\frac{e\vec{r}}{r^3}\right] = \frac{1}{c}[\vec{v}\times\vec{E}];$$

$\vec{E}$ Coulomb. Feld

$$d\vec{B}_e = \frac{1}{c}[\vec{v}\times\vec{E}_e];$$

$$\vec{B}' = -\frac{1}{c}[\vec{v}\times\vec{E}]; \quad \vec{E}' = \vec{E};$$

Relativitätsprinzip ($v^2 \ll c^2$):

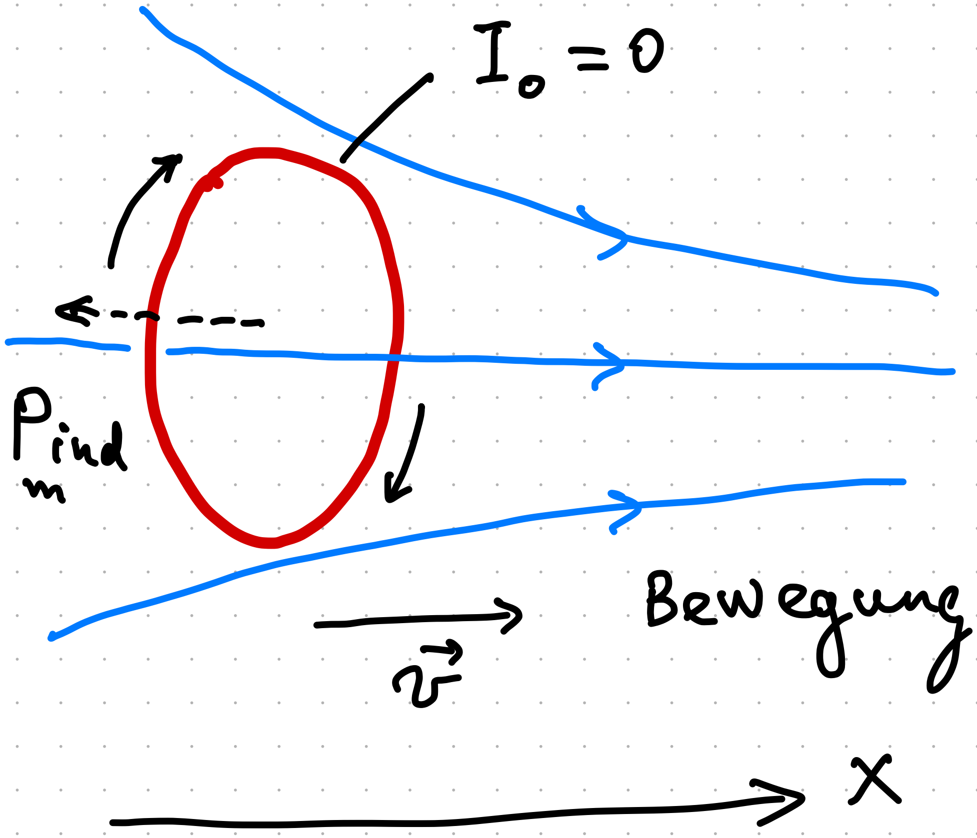
$$\boxed{\begin{aligned}\vec{E}' &= \frac{1}{c}[\vec{v}\times\vec{B}] + \vec{E}\\ \vec{B}' &= -\frac{1}{c}[\vec{v}\times\vec{E}] + \vec{B}\end{aligned}}$$

[cgs]

$$\boxed{\begin{aligned}\vec{E}' &= \vec{E} + [\vec{v}\times\vec{B}]\\ \vec{B}' &= \vec{B} - \varepsilon_0\mu_0[\vec{v}\times\vec{E}]\end{aligned}}$$

[SI] $\quad \varepsilon_0\mu_0 = \frac{1}{c^2}$

$$U_{ind} = -\frac{1}{c}\frac{d\Phi}{dt} ;$$

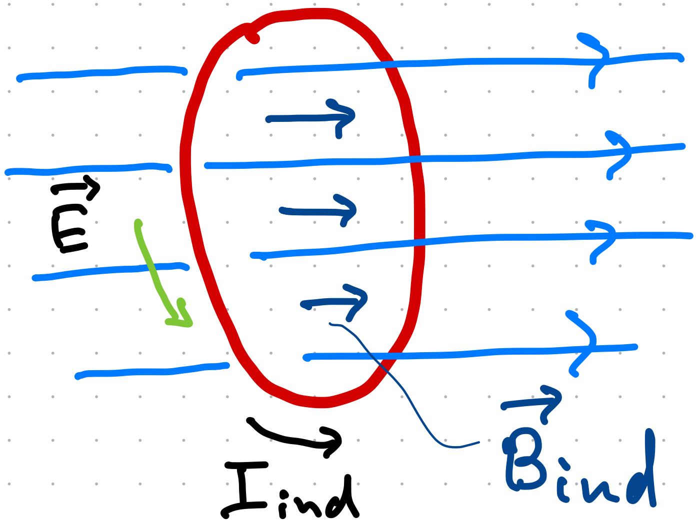


$$\frac{dB}{dx} > 0 ; \quad \frac{d\Phi}{dt} > 0$$

$$I_{ind} < 0$$

! Ein magnetisches Feld, welches sich zeitlich ändert, erzeugt ein elektrisches Wirbelfeld.

Lenzsche Regel:



Die durch Induktion entstehenden Ströme, Felder und Kräfte behindern stets den die Induktion einleitenden Vorgang.

$$\frac{d|\vec{B}_0|}{dt} < 0 \;;$$

← Verringerung des Feldes B_0.

! Bei $\frac{d|\vec{B}_0|}{dt} > 0$ kehren sich alle Felder $\vec{B}_{ind}$ und Ströme I_{ind}.