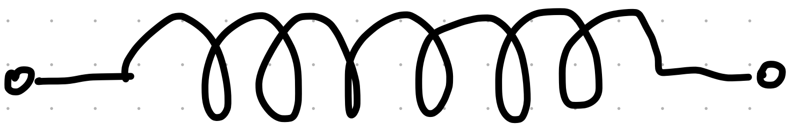
Vorlesung 13

Physik II

A. Ustinov

SS 2020



[SI] $\quad L = \mu_0 n_w^2 V$.

Energiedichte : $\quad w_{mag} = \frac{W_{mag}}{V} = \frac{\mu_0 n_w^2 I^2}{2} = \frac{B^2}{2\mu_0}$;

elektr. Feld $\qquad\qquad$ magn. Feld

Energie : $\quad \boxed{W_{el} = \frac{1}{2} C U^2 \; ; \quad W_{mag} = \frac{1}{2} L I^2}$

[SI]

Energiedichte : $\quad \boxed{w_{el} = \frac{1}{2} \varepsilon_0 E^2 \; ; \quad w_{mag} = \frac{B^2}{2\mu_0} = \frac{1}{2}\mu_0 H^2}$

elektromagnetische Feld : $\quad w_{em} = \frac{\varepsilon_0}{2} \left(E^2 + c^2 B^2 \right)$

[SI] $\quad$ mit $\quad \varepsilon_0 \mu_0 = \frac{1}{c^2}$; $\qquad$ Vakuum

Materie :

[SI]

$$w_{em} = \frac{1}{2}\left(E \cdot D + H \cdot B\right);$$

mit $D = \varepsilon \varepsilon_0 E$; $B = \mu \mu_0 H$;

Energie :

[cgs]

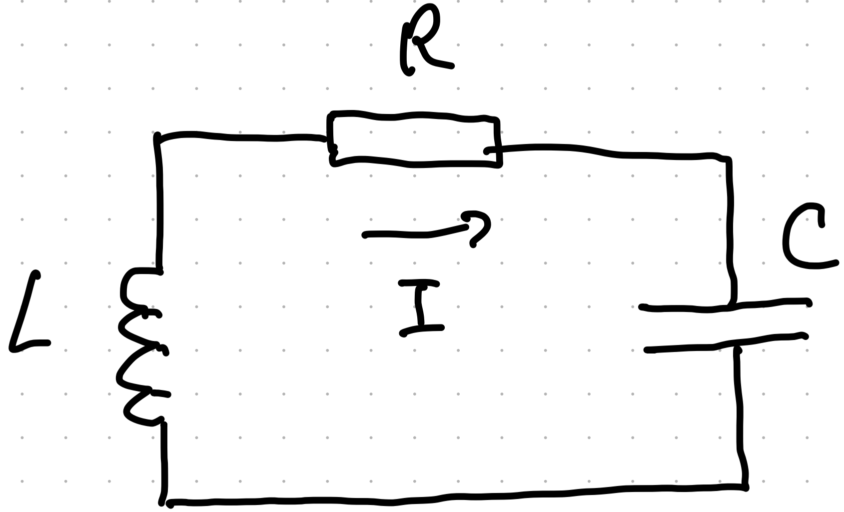
Energiedichte :

elektr. Feld	Magn. Feld
$W_{el} = \frac{1}{2} C U^2$;	$W_{mag} = \frac{\Phi^2}{2L} = \frac{L I^2}{2c^2}$
$w_{el} = \frac{E^2}{8\pi}$;	$w_{mag} = \frac{H^2}{8\pi}$

→ Materie : $w_{em} = \frac{1}{8\pi}\left(ED + HB\right)$;

4.4. Wechselstromkreise

Gedämfte elektromagnetische Schwingungen



[cgs] $IR + \frac{q}{C} = -\frac{L}{c^2} \cdot \frac{dI}{dt}$;

$I \hat{=} \frac{dq}{dt} \equiv \dot{q}$;

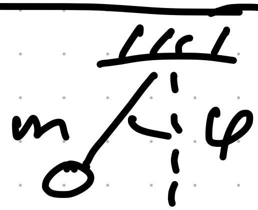
?

[SI] $L\ddot{q} + R\dot{q} + \frac{q}{C} = 0$; freier Schwingungen

$$\Rightarrow \boxed{\ddot{q} + 2\delta\dot{q} + \omega_0^2 q = 0} \; ; \; \delta = \frac{R}{2L} \; ; \; \omega_0^2 = \frac{1}{LC}$$

z.B.

oder



Ansatz : $q = A e^{\alpha t}$;

$\Rightarrow \alpha^2 + 2\delta\alpha + \omega_0^2 = 0$; $\underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim}{\alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}}$;

algem. Lösung : $q = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$;

1) $\underset{\sim\sim\sim\sim}{\omega_0^2 > \delta^2}$; $\alpha_{1,2} = -\delta \pm i\omega$;

gedämfte Schwingung $\omega = \sqrt{\omega_0^2 - \delta^2}$;

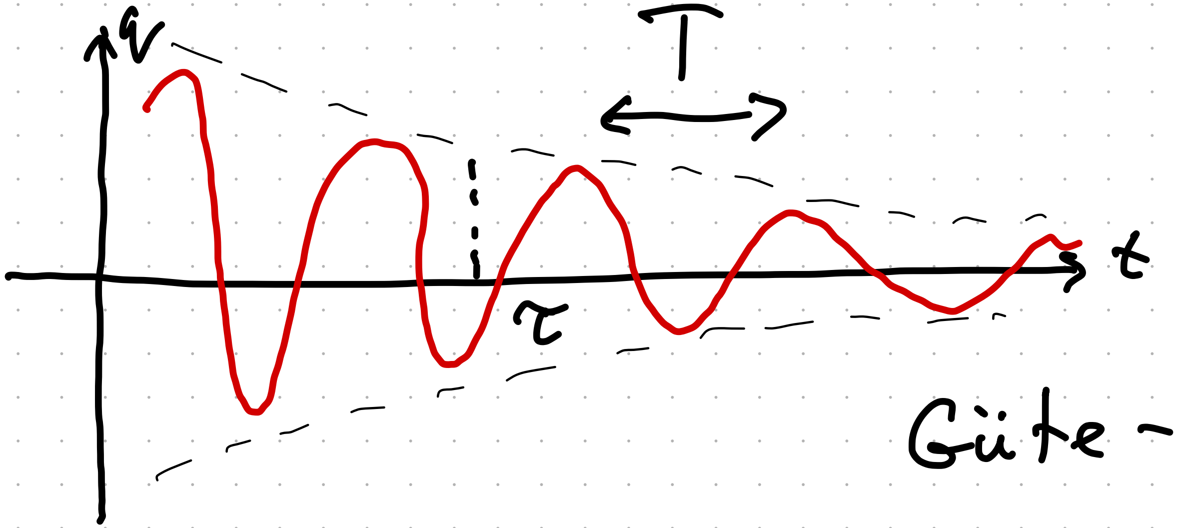
$$\underset{\text{reel}}{q(t)} = e^{-\delta t}\Big(\underbrace{A_1 e^{i\omega t} + A_2 e^{-i\omega t}}_{\text{komplex}}\Big);$$

Eulersche Formel : $e^{i\omega t} = \cos\omega t + i\sin\omega t$;

$$\underset{\text{reel}}{q(t)} = e^{-\delta t}\left(a\cdot\cos\omega t + b\cdot\sin\omega t\right) ;$$

2 Konstanten

$$q(t) = A\, e^{-\delta t}\cos(\omega t + \varphi) ;$$



$\tau = 1/\delta$;

Zeit Konstante

Güte-Faktor: $Q = \dfrac{\omega}{2\delta} \approx \pi N$;

$Q \gg 1$; $\omega \approx \omega_0$

$\underset{\text{Periode}}{N T} = \tau$;

2) $\underbrace{\omega_0^2 < \delta^2}_{\text{Kriechfall}}$; $\Rightarrow$ $\alpha_{1,2} = -\delta \pm \varkappa$;

$\varkappa = \sqrt{\delta^2 - \omega_0^2}$;

keine Schwingungen!

$$q(t) = A_1 e^{-\delta t} \cdot e^{\varkappa t} + A_2 e^{-\delta t} e^{-\varkappa t};$$

2 Exponente $\rightarrow$ nicht periodisch;

3) $\omega_0 = \delta$; $\Rightarrow$ aperiodischer Grenzfall!

Schwingkreis: Energie

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = 0 \quad \Big\}\ \text{multipl.}\ \dot{q} \quad ;$$

$$\Rightarrow \quad - d\Big(\underbrace{\frac{L\dot{q}^2}{2} + \frac{q^2}{2C}}\Big) = \underbrace{R\dot{q}^2 dt}_{\text{Verluste}}$$

"kinetische" Energie "potentiel." Energie

Integrieren : $\int_t^{t+T} dt \ldots$;

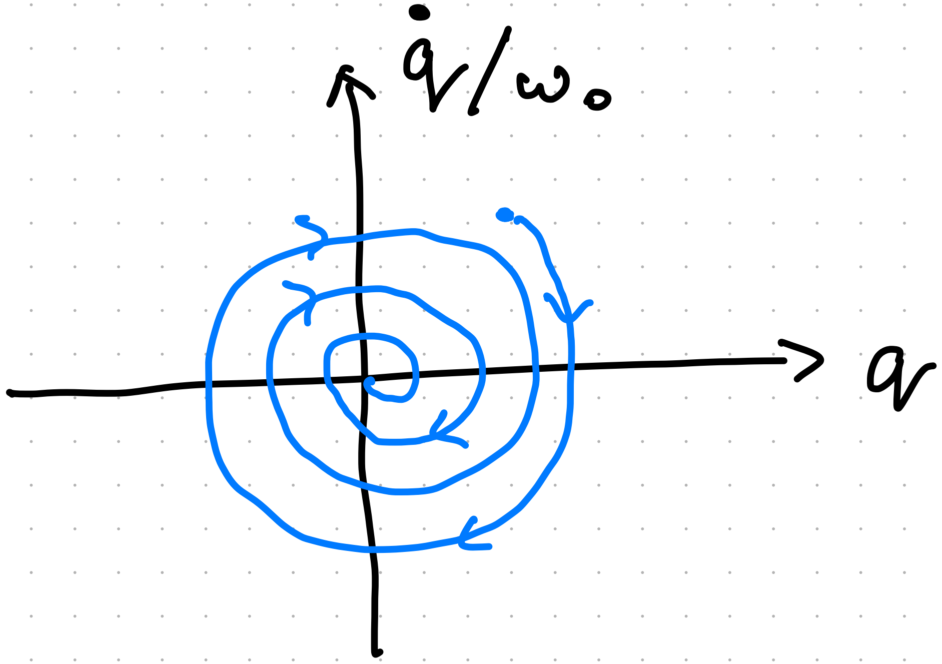
Energie : $W(t) - W(t+T) = \frac{2T}{L} R \underbrace{\int_t^{t+T} \frac{1}{T} \frac{Lq^2}{2} dt}_{\langle W_{mag} \rangle \text{ kinet.}}$

$\langle \text{kin. E} \rangle = \langle \text{poten. E} \rangle$;

$$\frac{W(t)}{W(t) - W(t+T)} = \frac{L}{TR} = \frac{2L}{2TR} = \frac{1}{2\delta T} = \frac{Q}{2\pi} ;$$

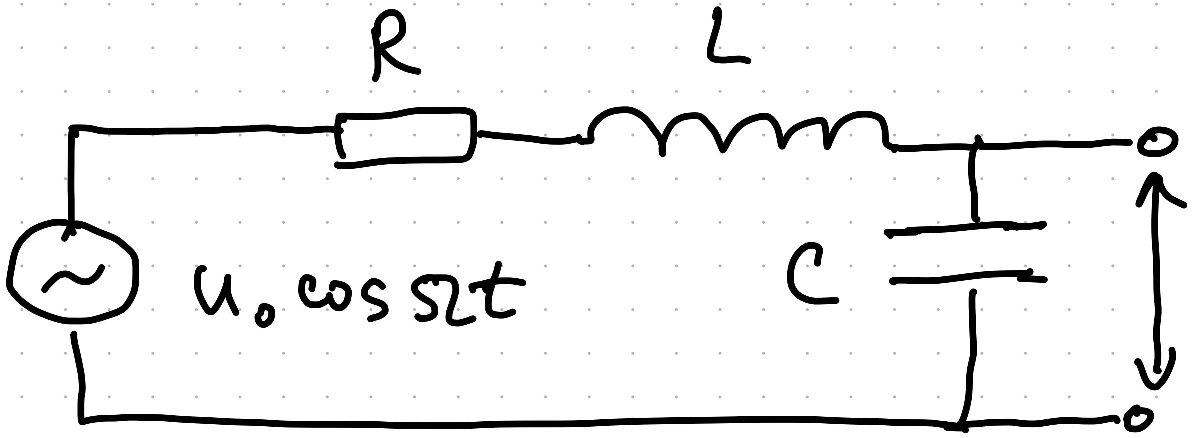
$\forall$ Schwingungen : $Q = 2\pi \dfrac{E_{\text{gespeichert}}}{\Delta E_T}$

Phasenraum :



die Menge aller möglichen Zustände eines dynamischen Systems.

Erzwungene Schwingungen



$u_c = v \;;\; q \sim v$

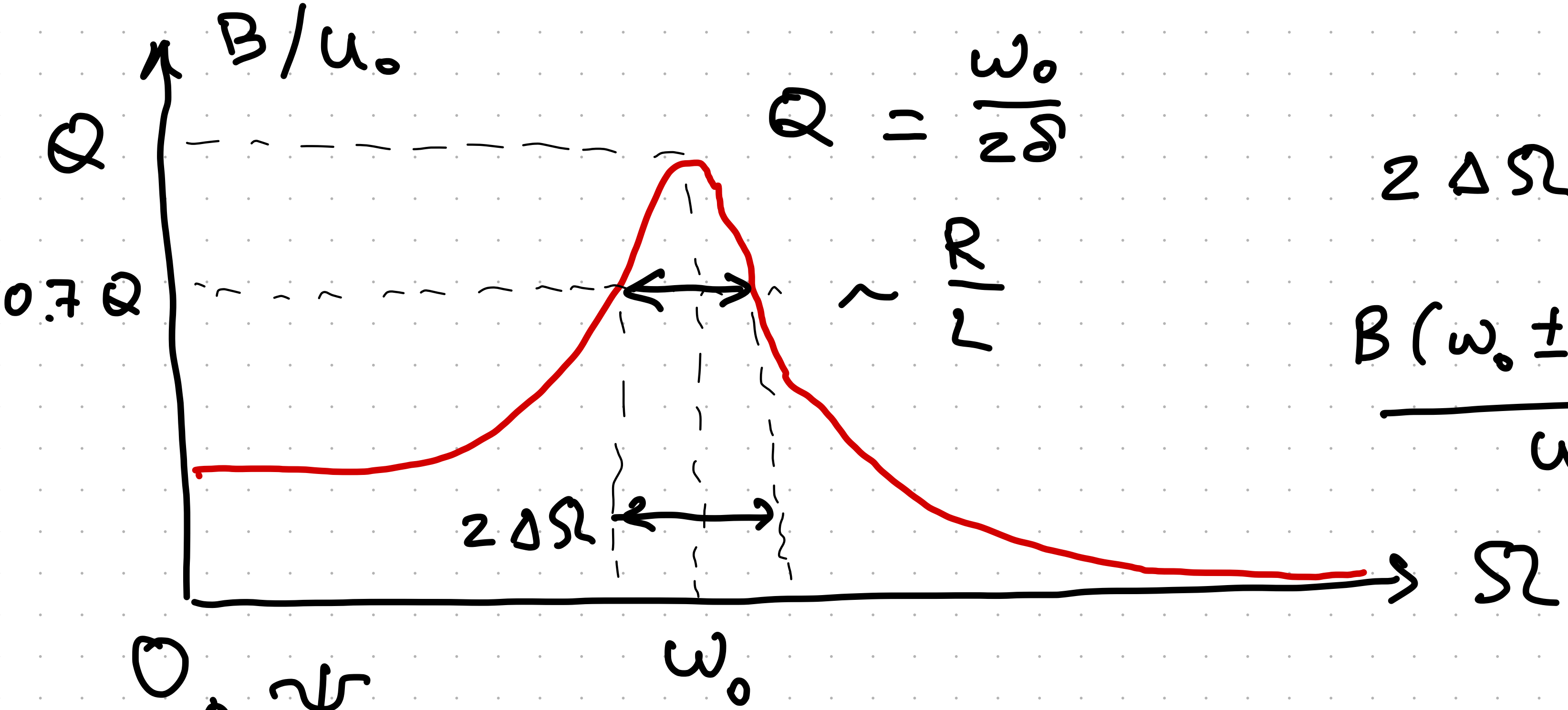
$$\ddot{v} + 2\delta \dot{v} + \omega_0^2 v = \omega_0^2 u_0 \cos \Omega t \;;$$

Lösung = Allgem. Lösung (zu homogene Gl.) + Partikulärlös. (zu inhomog. Gl.)

$$A\, e^{-\delta t} \cos(\omega t + \varphi) \qquad B \cos(\Omega t + \Psi)$$

$\omega_0^2 \gg \delta^2 \;; \quad \omega \simeq \omega_0$

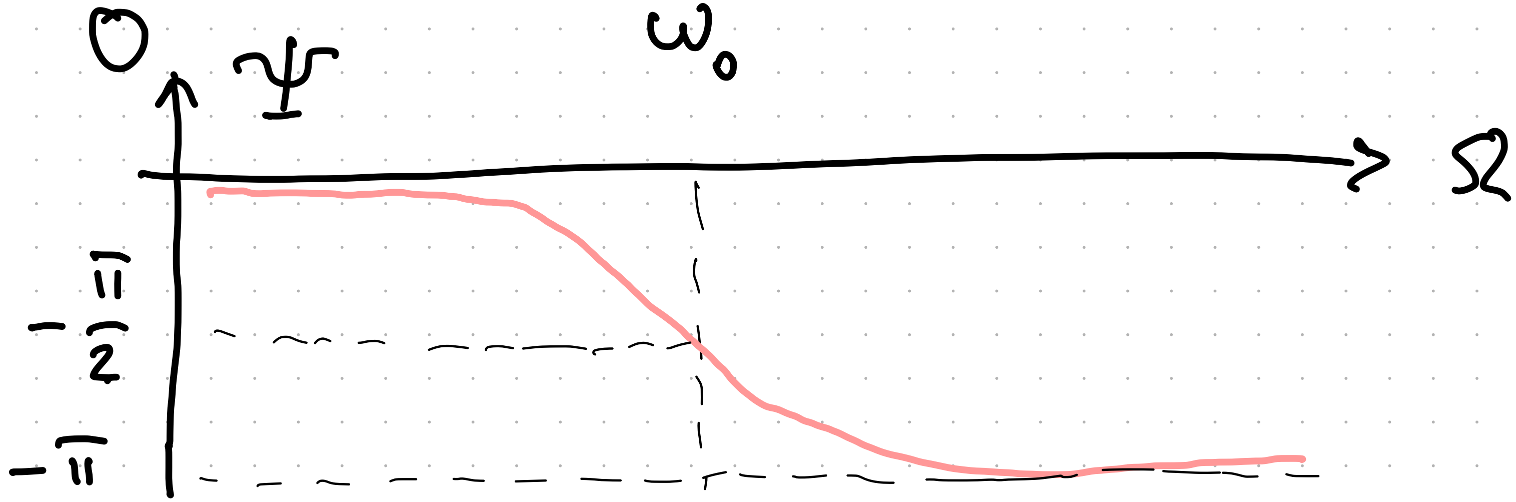
$$B = \frac{\omega_0^2 u_0}{\sqrt{(\omega_0^2 - \Omega^2)^2 + 4\delta^2\Omega^2}}; \quad \Psi = -\operatorname{arctg}\frac{2\delta\Omega}{\omega_0^2 - \Omega^2}$$



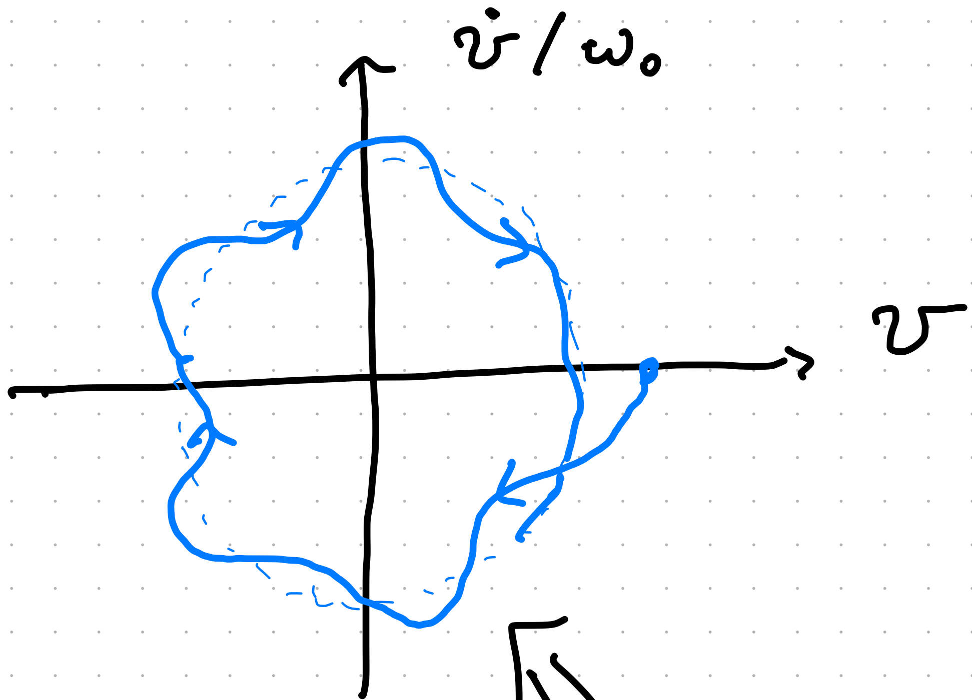
$$Q = \frac{\omega_0}{2\delta}$$

$$2\Delta\Omega = 2\delta = \frac{R}{L};$$

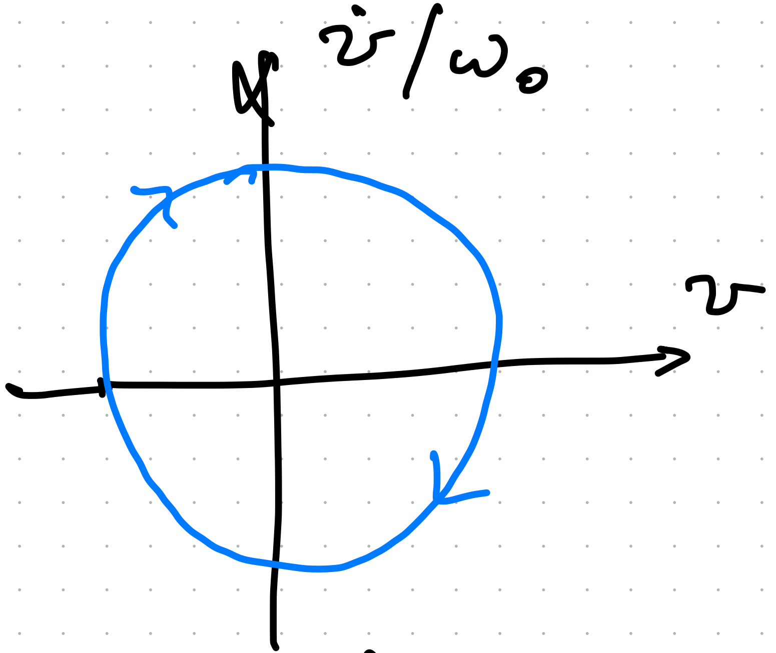
$$\frac{B(\omega_0 \pm \Delta\Omega)}{u_0} = \frac{B(\omega_0)}{\sqrt{2}\, u_0}$$



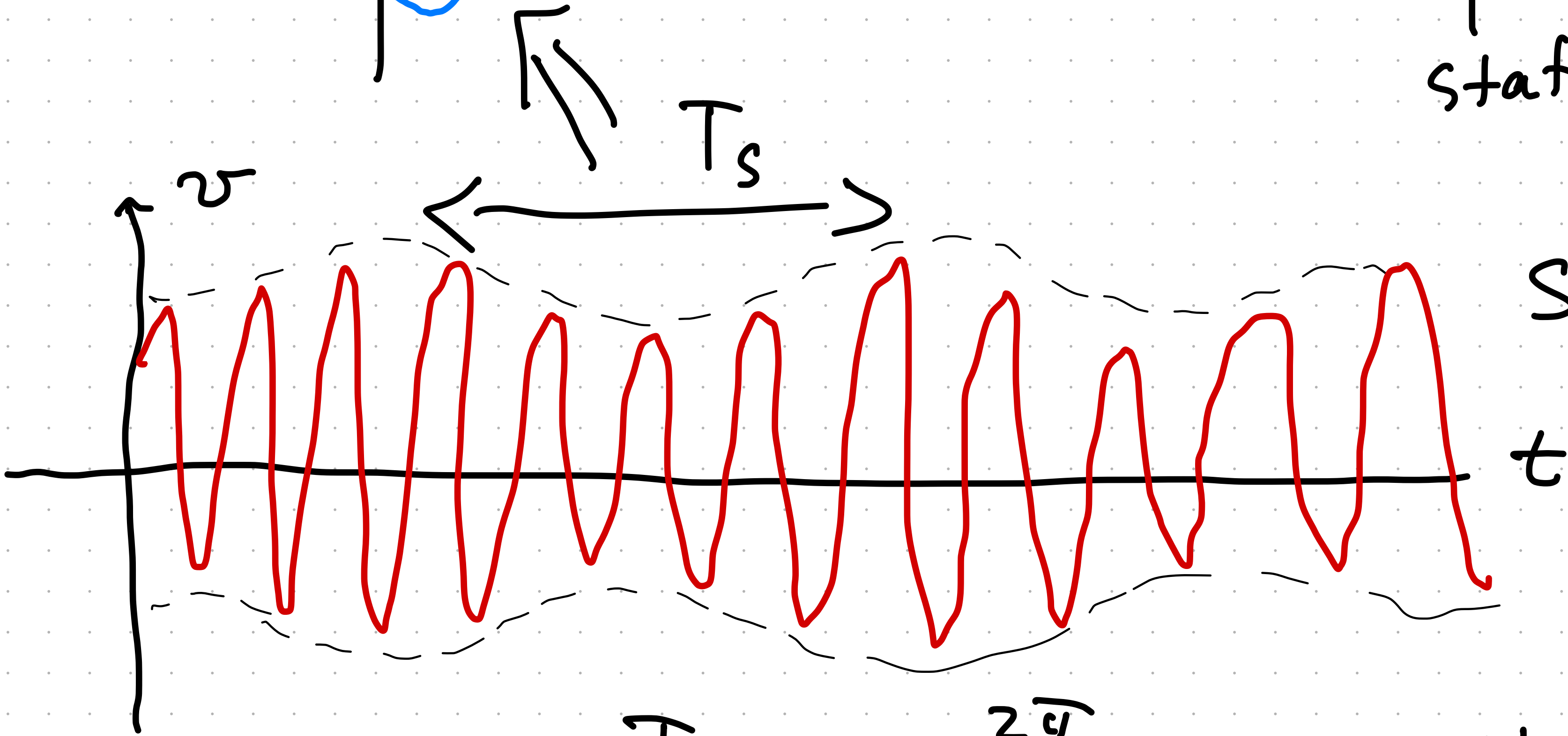
$$Q = \frac{\omega_0}{2\delta} = \frac{\omega_0}{2\Delta\Omega};$$



$\Rightarrow$



stationäre

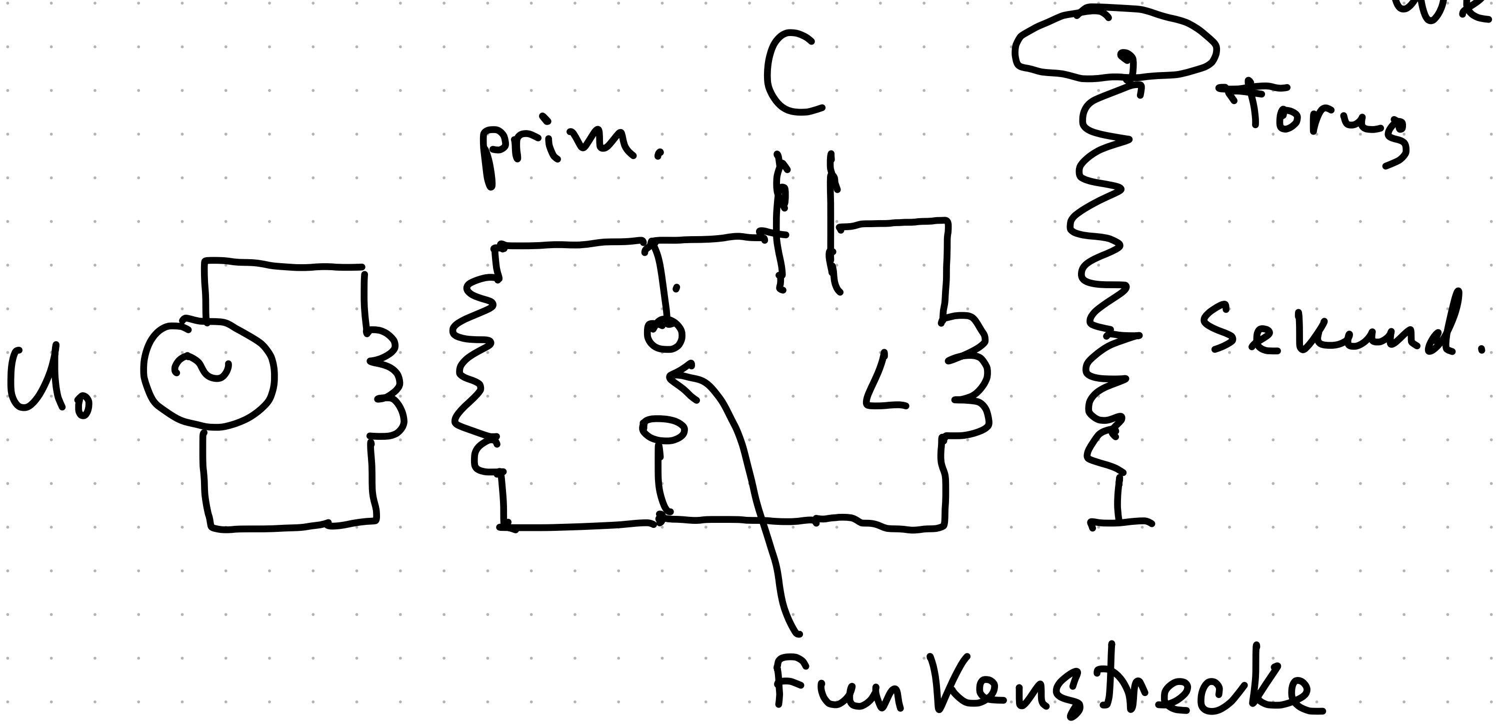


Schwebung

$$T_s \sim \frac{2\pi}{|\omega_0 - \Omega|} \quad ; \text{ mit } |\omega_0 - \Omega| > \delta$$

Tesla-Transformator

Resonanztranformator → hochfrequente Wechselspannung



$f \sim 100\ kHz$

SGTC = Spark Gap Tesla Coil