Vorlesung 15

Physik II

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SS 2020

5. Die Maxwellschen Gleichungen

5.1. Integral- und Differential Form

[cgs]

integral:

$$
\begin{cases}
\oint_S D_n \, dS = 4\pi q; \\
\oint_S B_n \, dS = 0; \\
\oint_L E_l \, dl = -\frac{1}{c} \oint_S \frac{\partial B_n}{dt} \, ds; \\
\oint H_l \, dl = \frac{4\pi}{c} \oint_S j_n \, ds + \frac{1}{c} \oint_S \frac{\partial D_n}{\partial t} \, dS;
\end{cases}
$$

Verschiebungsstrom

differential:

$$\boxed{\begin{aligned} \operatorname{div} \vec{D} &= 4\pi \rho_{el} \, ; \\ \operatorname{div} \vec{B} &= 0 \, ; \\ \operatorname{rot} \vec{E} &= -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \, ; \\ \operatorname{rot} \vec{H} &= \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t} \, ; \end{aligned}}$$

[cgs]

[SI] differential:

$$\operatorname{div}\vec{E} = \frac{\rho_{el}}{\varepsilon_0};$$

$$\operatorname{div}\vec{B} = 0;$$

$$\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t};$$

$$\operatorname{rot}\vec{B} = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t};$$

$$\boxed{\begin{aligned}
\operatorname{div}\vec{D} &= \rho_{el}; \\
\operatorname{div}\vec{B} &= 0; \\
\operatorname{rot}\vec{E} &= -\frac{\partial\vec{B}}{\partial t}; \\
\operatorname{rot}\vec{H} &= \vec{j} + \frac{\partial\vec{D}}{\partial t};
\end{aligned}}$$

$$\frac{1}{c^2} = \varepsilon_0\mu_0; \quad \vec{D} = \varepsilon\varepsilon_0\vec{E}; \quad \vec{B} = \mu\mu_0\vec{H};$$

M.G. zusammen mit Lorentzkraft

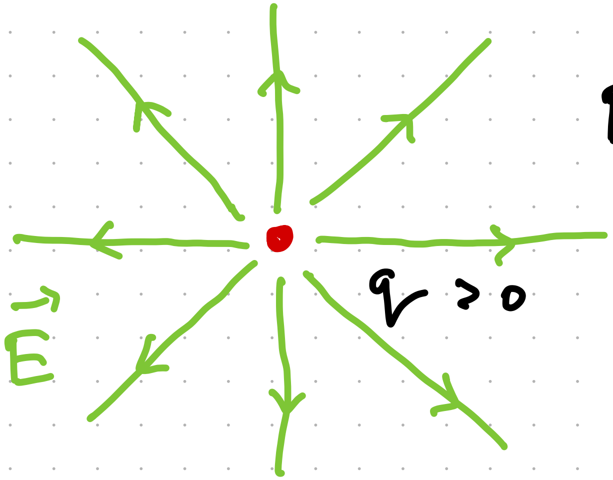
$$\vec{F}_L = q\left(\vec{E} + [\vec{v} \times \vec{B}]\right)$$

und Newtonschen Bewegungsgleichung

$$\vec{F} = \dot{\vec{P}}$$

beschreiben alle elektromagnetische Phenomäne.

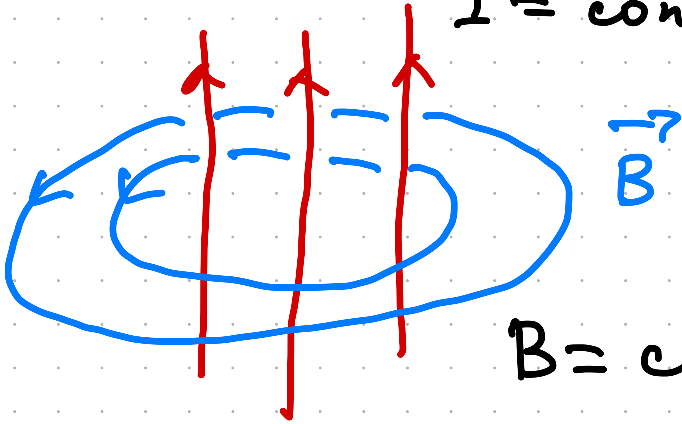
1)



$E = \text{const}$

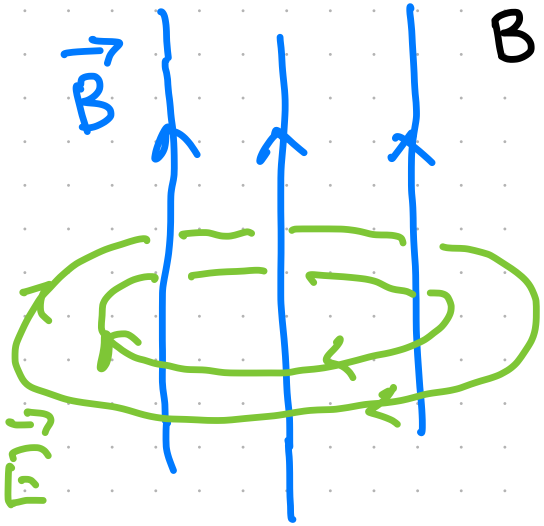
2)

$I = \text{const}$



$B = \text{const}$

3)



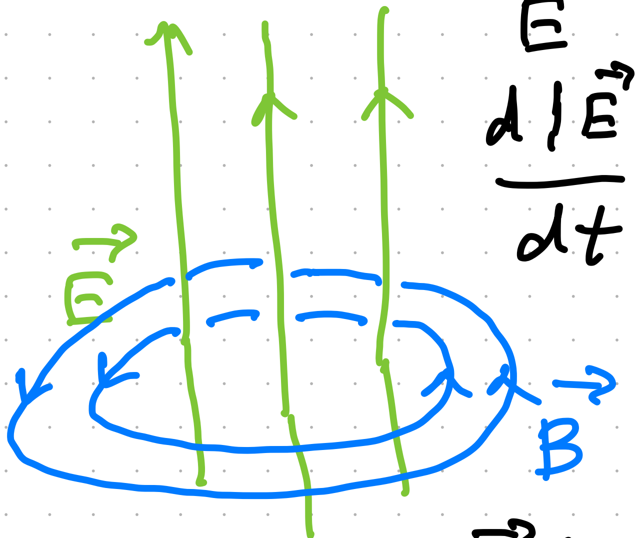
$\vec{B}$; $\dfrac{d|\vec{B}|}{dt} > 0$

$\vec{E}(t)$

4)

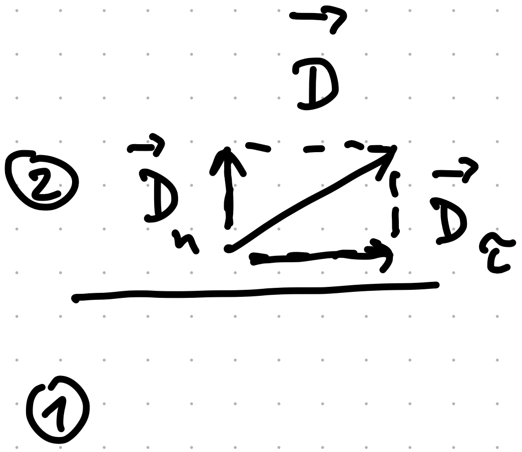
$\vec{E}$

$\dfrac{d|\vec{E}|}{dt} > 0$



$\vec{B}(t)$

Randbedingungen zu M.G. [cgs]



1) $D_{2n} - D_{1n} = 4\pi\sigma$;

2) $B_{2n} - B_{1n} = 0$;

3) $E_{2\tau} - E_{1\tau} = 0$;

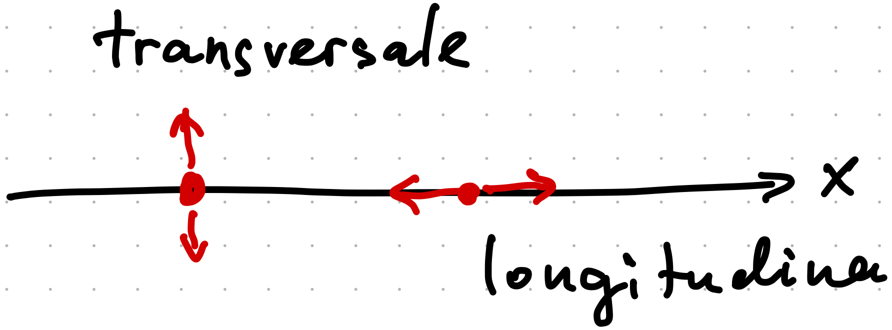
4) $H_{2\tau} - H_{1\tau} = \frac{4\pi}{c} j$;

$$\Downarrow$$

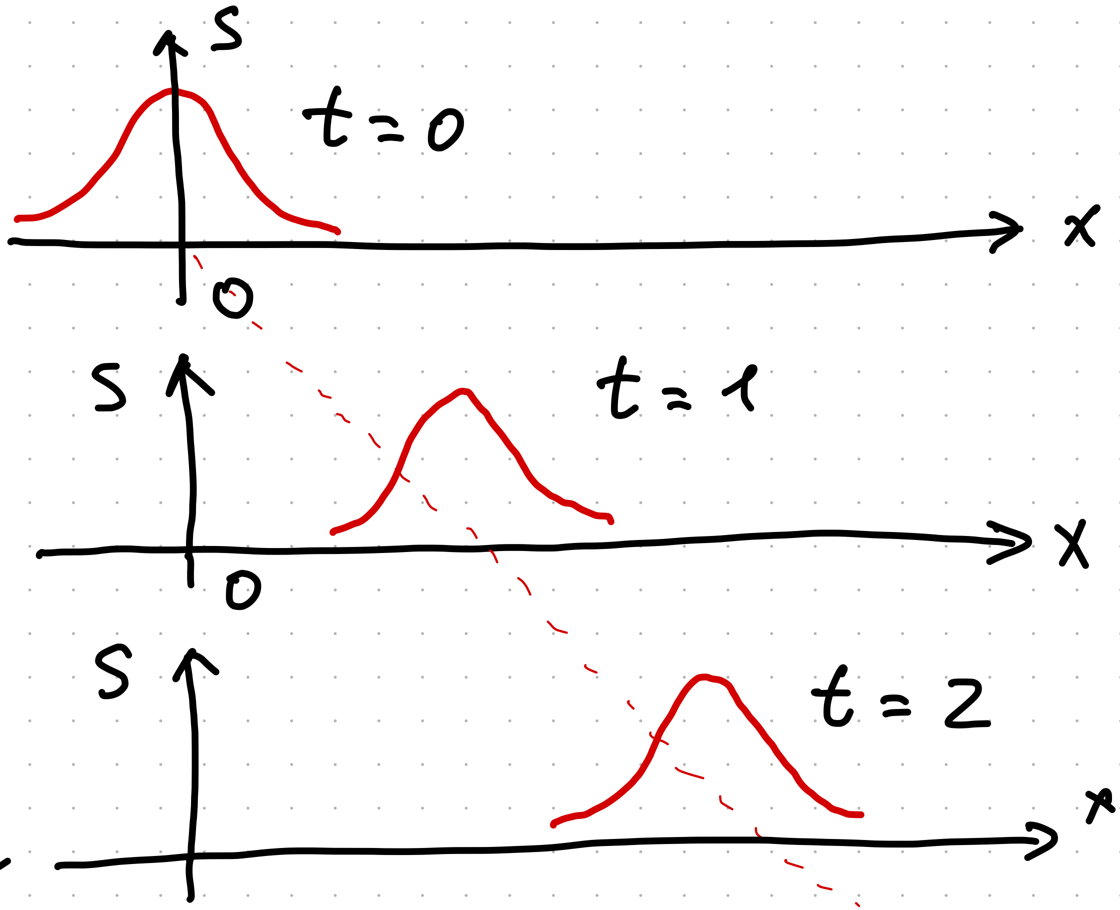
$$[\vec{n} \times \vec{H}_2] - [\vec{n} \times \vec{H}_1] = \frac{4\pi}{c} \vec{j} .$$

5.2. Elektromagnetische Wellen

Wellen (Einführ.)



z. B.



$$S = \exp\left[-\frac{x^2}{a^2}\right]$$

$$S = \exp\left[-\frac{(x-vt)^2}{a^2}\right]$$

Solitary waves
↓
solitonen

$$s(x,t) = s\left(t - \frac{x}{v}\right); \quad t - \frac{x}{v} = \xi;$$

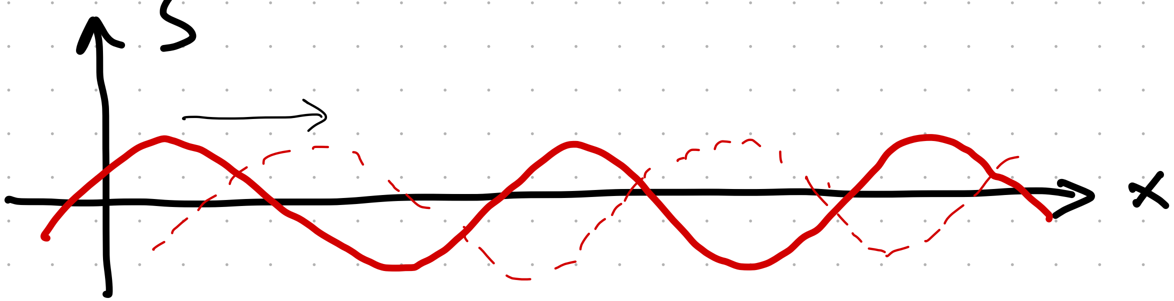
$$\frac{\partial s}{\partial t} = \frac{\partial s}{\partial \xi} \cdot \underbrace{\frac{\partial \xi}{\partial t}}_{=1} = \frac{\partial s}{\partial \xi}; \quad \frac{\partial^2 s}{\partial t^2} = \frac{\partial^2 s}{\partial \xi\, \partial t} = \frac{\partial^2 s}{\partial \xi^2} \cdot \underbrace{\frac{\partial \xi}{\partial t}}_{=1};$$

$$\Rightarrow \frac{\partial^2 s}{\partial t^2} = \frac{\partial^2 s}{\partial \xi^2}; \quad \frac{\partial s}{\partial x} = \frac{\partial s}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} = \frac{\partial s}{\partial \xi} \cdot \left(-\frac{1}{v}\right);$$

$$\frac{\partial^2 s}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 s}{\partial \xi^2};$$

$$\boxed{\frac{\partial^2 s}{\partial t^2} = v^2 \frac{\partial^2 s}{\partial x^2}};$$

Wellengleichung



Lineare Welle: $S(x,t) = S_0 \cos\left[\omega\left(t - \frac{x}{v}\right)\right]$;

$$(*) \quad S(x,t) = S_0 \cos(\omega t - kx);$$

"Zeit-Frequenz" "Räumliche Frequenz"

Wellenzahl: $k = \frac{\omega}{v}$;

$k = \frac{2\pi}{\lambda}$; λ ist Wellelänge;

$$\lambda = \frac{2\pi}{k} = \frac{2\pi v}{\omega} = \frac{v}{f}; \quad [f] = Hz;$$

$$f = \frac{v}{\lambda}; \quad f = \frac{\omega}{2\pi};$$

Allgem. Form:

$$(**) \quad S(x,t) = S_0 e^{i(\omega t - kx)}; \quad Re(**) \Rightarrow (*)$$

Elektromagnetische Wellen:

$$\left\{\begin{aligned} \operatorname{div}\vec{E} &= \cancel{\frac{\rho_{el}}{\varepsilon_0}} = 0\,; \\ \operatorname{div}\vec{B} &= 0\,; \\ \operatorname{rot}\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\,; \\ \operatorname{rot}\vec{B} &= \underset{0}{\underset{\shortparallel}{\cancel{\mu_0\vec{j}}}} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\,; \end{aligned}\right\}$$

Im Vakuum oder in einem homogenen, isotroper, neutralen und nichtleitenden Medium

$\rho_{el} = 0$; $\vec{j} = 0$

i) $$\operatorname{rot}(\operatorname{rot}\vec{E}) = \operatorname{rot}\left(-\frac{d\vec{B}}{dt}\right)$$

$$= -\frac{\partial}{\partial t}\operatorname{rot}\vec{B} = \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}\,;$$

ii) $$\operatorname{rot}(\operatorname{rot}\vec{E}) = \underset{\overset{\shortparallel}{0}\ (\rho_{el}=0)}{\operatorname{grad}(\operatorname{div}\vec{E})} - \operatorname{div}\underbrace{(\operatorname{grad}\vec{E})}_{\Delta\ \text{Laplace Operator}}\,;$$

$$\boxed{\Delta \vec{E} = \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$ Wellengleichung.

$$\frac{1}{c^2} = \varepsilon_0 \mu_0 \; ; \quad c \simeq 3 \cdot 10^8 \frac{m}{s} \; ;$$

Lichtgeschwindigkeit

$$\left\{ \begin{aligned}
&\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \frac{1}{c^2} \cdot \frac{\partial^2 E_x}{\partial t^2} \; ; \\
&\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \frac{1}{c^2} \cdot \frac{\partial^2 E_y}{\partial t^2} \; ; \\
&\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \frac{1}{c^2} \cdot \frac{\partial^2 E_z}{\partial t^2} \; ;
\end{aligned} \right.$$

Maxwell ~ 1862 (theorie)

Hertz ~ 1888 (experiment)

Lösung der Wellengleichung durch ebene Welle $\Rightarrow$ Ansatz : $\vec{E}(\vec{r},t) = \vec{E}_0 \sin(\omega t - \vec{k}\cdot\vec{r})$;

wobei $\omega = 2\pi f$; $k = 2\pi/\lambda$;

wegen $\vec{k}\cdot\vec{r} = k_x \cdot x + k_y \cdot y + k_z \cdot z$ und

$$\frac{\partial E_x}{\partial x} = -k_x E_{0x} \cos(\omega t - \vec{k}\cdot\vec{r}) \ ;$$

$$\frac{\partial E_y}{\partial y} = -k_y E_{0y} \cos(\omega t - \vec{k}\cdot\vec{r}) ;$$

$$\frac{\partial E_z}{\partial z} = -k_z E_{0z} \cos(\omega t - \vec{k}\cdot\vec{r}) ;$$

ist $0 = \operatorname{div} \vec{E} = \frac{\partial E}{\partial x} + \frac{\partial E}{\partial y} + \frac{\partial E}{\partial z} = \vec{k} \cdot \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r})$;

$\Rightarrow$ für alle $\omega, \vec{k}$ nur erfüllt wenn

$\boxed{\vec{k} \cdot \vec{E}_0 = 0}$; D. h. $\vec{k} \perp \vec{E}$

EM Wellen sind <u>transversale Wellen</u>!

$$\Rightarrow \left(\operatorname{rot} \vec{E}\right)_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$= -\left(k_y E_{0z} - k_z E_{0y}\right) \cos(\omega t - \vec{k} \cdot \vec{r});$$

$$\operatorname{rot} \vec{E} = -\left[\vec{k} \times \vec{E}_0\right] \cdot \cos(\omega t - \vec{k} \cdot \vec{r}) = -\frac{\partial \vec{B}}{\partial t};$$

$$\Rightarrow \quad \vec{B}(\vec{r},t) = \int_0^t \frac{\partial \vec{B}}{\partial t}\, dt + \underbrace{\vec{B}_c}_{\text{const} = 0} \quad ;$$

da ein zusätzliches statisches B-feld keine Auswirkung hat.

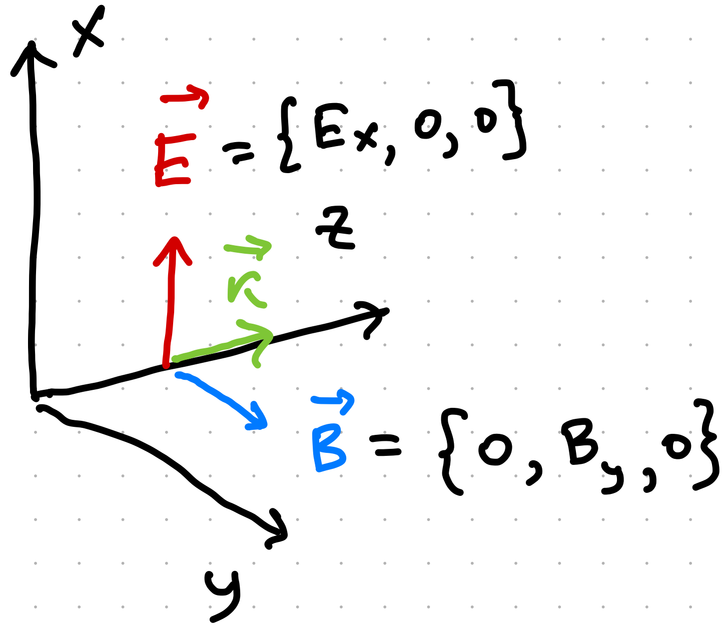
$$\vec{B}(\vec{r},t) = \vec{B}_0 \cdot \sin(\omega t - \vec{k}\cdot\vec{r}) \quad ;$$

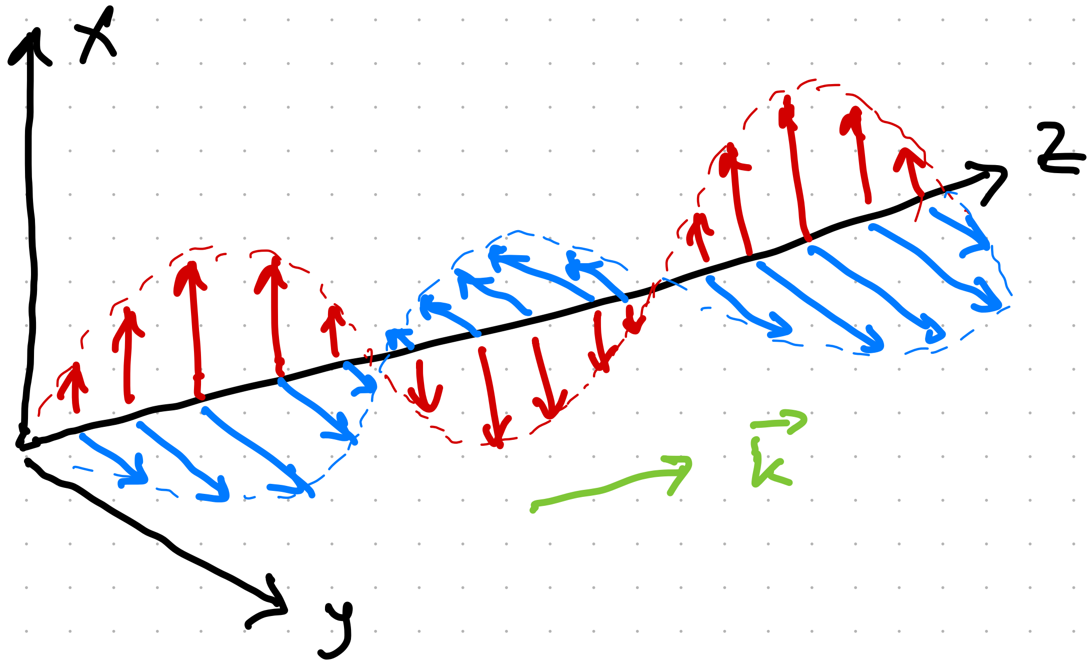
wobei $\vec{B}_0 = \frac{1}{\omega}\left[\vec{k} \times \vec{E}_0\right]$ ist.

$\vec{E}$, $\vec{B}$ schwingen gleichphasig.

Die Amplituden der EM-Welle $|\vec{E}_0|$ und $|\vec{B}_0|$ sind voneinderer abhängig :

$$|\vec{B}_0| = \frac{k}{\omega}|\vec{E}_0| = \frac{1}{c}|\vec{E}_0|\,; \quad |\vec{E}_0| = c\cdot|\vec{B}_0|\,;$$





Vakuum:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Medium $\varepsilon > 1$ und $\mu > 1$:

$$c' = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu \mu_0}} = \frac{c}{\sqrt{\varepsilon \mu}} < c\,;$$

$n = \sqrt{\varepsilon\mu}$ Brechungsindex