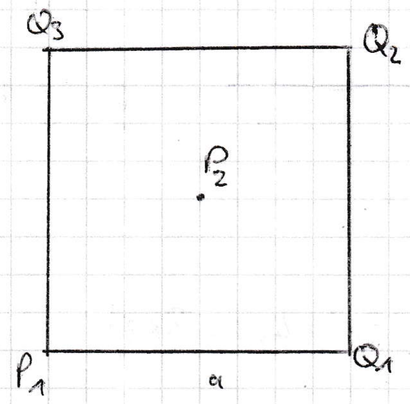
Ex 2 / Blatt 2

4 $\phi = \frac{Q}{4\pi\varepsilon_0 r}$



a) Potential an P_1 & P_2 ?

P_1: Summe der Potentiale $\overline{P_1Q_1} = a$ $\overline{P_1Q_2} = \sqrt{2}a$

$\overline{P_1Q_3} = a$

$$\Rightarrow \phi_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{a} + \frac{Q_3}{a} + \frac{Q_2}{\sqrt{2}a}\right)$$

nach einsetzen: $\underline{\phi_1 = 58{,}1\ V}$

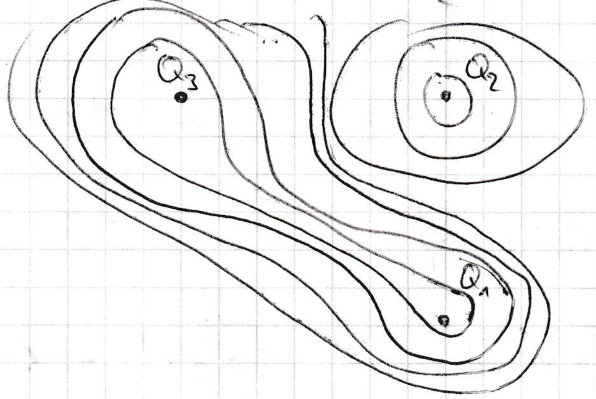
$\overline{P_2Q_i} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

$$P_2: \phi_2 = \frac{\sqrt{2}}{4\pi\varepsilon_0 a}(Q_1 + Q_3 + Q_2)$$

$$= \underline{63{,}6\ V}$$

U: $U = \phi_2 - \phi_1 = \underline{5{,}5\ V}$

b)



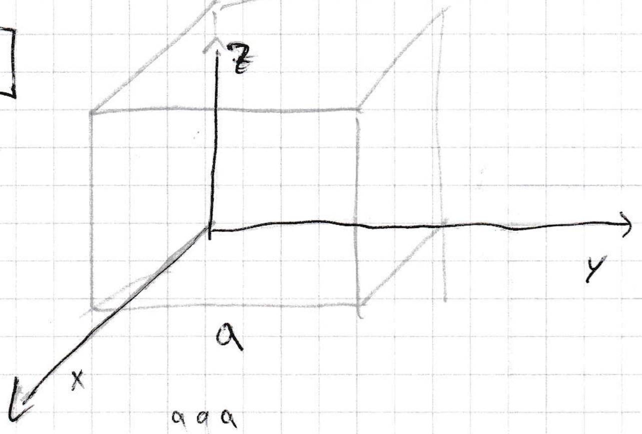
$F = -\text{grad}\,\phi$

$\Rightarrow$ Änderung der Feldlinien

$\rightarrow$ zwischen Q_2 & Q_3 ist diese Änderung am stärksten

Oder ganz nah an Q_3

$\boxed{5}$



$$Q = \int_V \rho \, dV$$

$$\rho = \rho_0 \left(2x^2 + 4yz - 3xz\right)$$

$$Q = \int_0^a\int_0^a\int_0^a dx\,dy\,dz \;\; \rho_0 \left(2x^2 + 4yz - 3xz\right)$$

$$Q = \rho_0 \int_0^a dx \int_0^a dz \left[2x^2 y + 2y^2 z - 3xzy\right]_0^a$$

$$= \rho_0 \int_0^a dx \int_0^a dz \left(2x^2 a + 2a^2 z - 3xza\right)$$

$$= \rho_0 \int_0^a dx \left(2x^2 a^2 + a^4 - \frac{3}{2} a^3 x\right)$$

$$= \rho_0 \;\; \frac{2}{3} a^5 + a^5 - \frac{3}{4} a^5 \quad = \quad \rho_0 \, a^5 \left(\frac{8}{12} + \frac{12}{12} - \frac{9}{12}\right) = \underline{\underline{\frac{11}{12} a^5 \rho_0}}$$

$\boxed{6}$ ~~Gaußscher Satz~~

$$\phi = -\int E(r)\, dr \qquad \text{bzw.} \qquad \phi(r) = -\int_\infty^r E(r')\, dr'$$

$E_{außen}$ aus VL: $\quad E(r) = \frac{Q}{r^2}$

$$\phi = -\int E(r)\, dr \quad \Rightarrow \quad \phi_a(r) = -\int \frac{Q}{r^2}\, dr$$

$$\phi_a(r) = \frac{Q}{r} + C \qquad \text{da } \phi(\infty) \overset{!}{=} 0$$

$$\Rightarrow C = 0$$

$$\phi_a(r) = \frac{Q}{r}$$

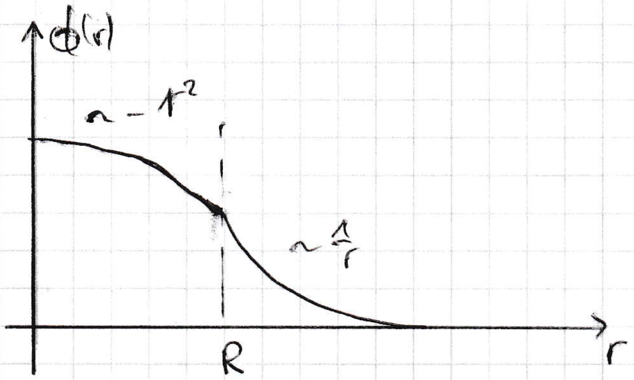
Innen:

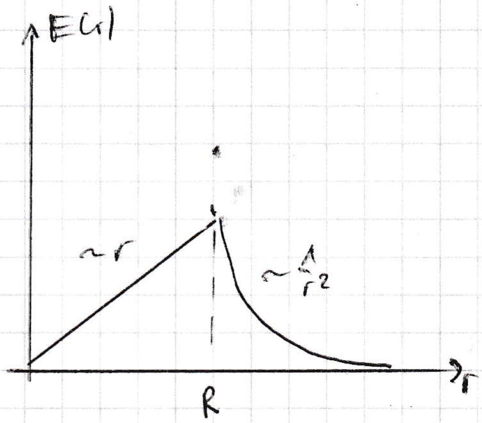
$$E_i = \frac{Q}{R^3} \cdot r \qquad \phi = -\int \frac{Q}{R^3} \cdot r \, dr \quad = \quad -\frac{Q}{2R^3} r^2 + C'$$

$\phi(R)$ muss für beide Lösungen gleich sein

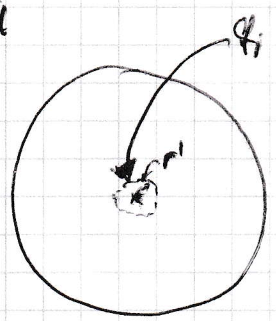
$$\frac{Q}{R} = -\frac{Q}{2R^3} R^2 + C' \quad \Rightarrow \quad C' = \frac{3Q}{2R}$$

$$\Phi_i(r) = \frac{3Q}{2R} - \frac{Q}{2R^3} r^2$$





b) $W = q\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l}$ $W_{ges} = \sum_i q_i \int_{P_1}^{P_2} \vec{E}\cdot d\vec{l}$



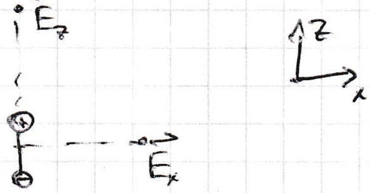
$$\Rightarrow W = \int_0^R \rho \underbrace{\int_0^{r'} \vec{E}\, d\vec{l}}_{\Phi(r')} \, dV$$

$$W = \rho \int \frac{Q}{r'}\, dV = \rho^2 \frac{4\pi}{3} \int r'^2\, dV = \rho^2 \frac{4\pi}{3} \cdot 4\pi \int_0^R r'^4\, dr'$$

$$= \rho^2 \frac{4\pi}{3} \frac{4\pi}{\cancel{3}} \frac{\cancel{3}}{5} R^5 = \underline{\underline{\frac{3}{5}\frac{Q^2}{R}}}$$

$\boxed{7}$ Aus Vorlesung

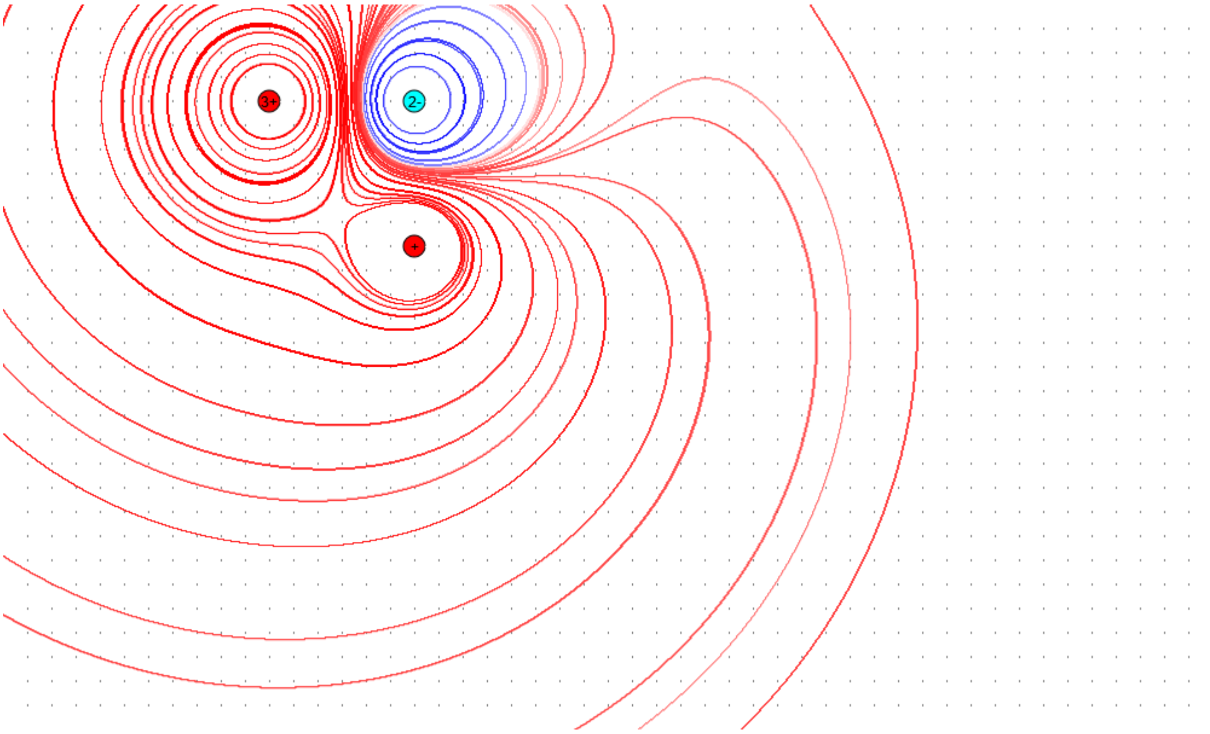
a) + b)



$|\vec{E}_z| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{a^3} = 6{,}11 \cdot 10^7 \frac{V}{m}$ in z

$|\vec{E}_x| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{a^3} = -3{,}056 \cdot 10^7 \frac{V}{m}$ entgegen z

Äquipotenziallinien:



Elektrisches Feld: