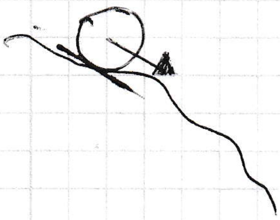
$\boxed{\text{Ex 2 / Blatt 03}}$

~~Gradient ≙ Steigung~~

$\boxed{8}$ a)



Gradient ≙ Steigung

$\mathrm{grad}(W_{pot}) = -\vec{F}$ $\qquad \vec{F} \sim \vec{a}$

Kraft wirkt entlang eines Potentialgefälles.

Kraft ist proportional zur Beschleunigung (Newton)

b) $\underbrace{f(\vec{r}) = \frac{1}{3+r^2}}_{\text{Skalarfeld}} \quad , \quad r^2 = x^2+y^2+z^2$

$$\vec{\nabla} f = \begin{pmatrix} \partial_x f(\vec{r}) \\ \partial_y f(\vec{r}) \\ \partial_z f(\vec{r}) \end{pmatrix} = \begin{pmatrix} \frac{-1}{(3+x^2+y^2+z^2)^2} \cdot 2x \\ \frac{-1}{(3+x^2+y^2+z^2)^2} \cdot 2y \\ \frac{-1}{(3+x^2+y^2+z^2)^2} \cdot 2z \end{pmatrix} = \frac{-2}{(3+r^2)^2} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

c) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\omega y \\ \omega x \\ 0 \end{pmatrix}$

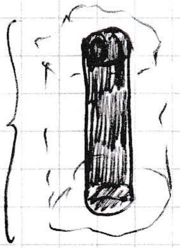
$\cancel{\mathrm{div}\, \vec{v} = \begin{pmatrix} -\partial_x & \omega y \\ \partial_y & \omega x \\ \partial_z & 0 \end{pmatrix}}$

$$\vec{\nabla} \vec{v} = (-\partial_x \omega y + \partial_y \omega \cdot x + \partial_z 0)$$
$$= 0 + 0 + 0 = \underline{\underline{0}}$$

d) $\mathrm{rot}(\vec{v}) = \vec{\nabla} \times \vec{v} = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} -\omega y \\ \omega x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2\omega \end{pmatrix} = 2\omega \hat{e}_z$

e)
a) $\mathrm{div}\, \vec{F} = 0$, $\mathrm{rot}\, \vec{F} \neq 0$
b) $\mathrm{div}\, \vec{F} \neq 0$, $\mathrm{rot}\, \vec{F} = 0$
c) $\mathrm{div}\, \vec{F} = 0$, $\mathrm{rot}\, \vec{F} = 0$
d) $\mathrm{div}\, \vec{F} = 0$, $\mathrm{rot}\, \vec{F} = 0$
e) $\mathrm{div}\, \vec{F} = 0$, $\mathrm{rot}\, \vec{F} \neq 0$
f) $\mathrm{div}\, \vec{F} \neq 0$, $\mathrm{rot}\, \vec{F} \neq 0$

$\boxed{9}$ a) $\int \vec{E}\, d\vec{s} = \frac{\cancel{4\pi} Q}{\varepsilon_0}$



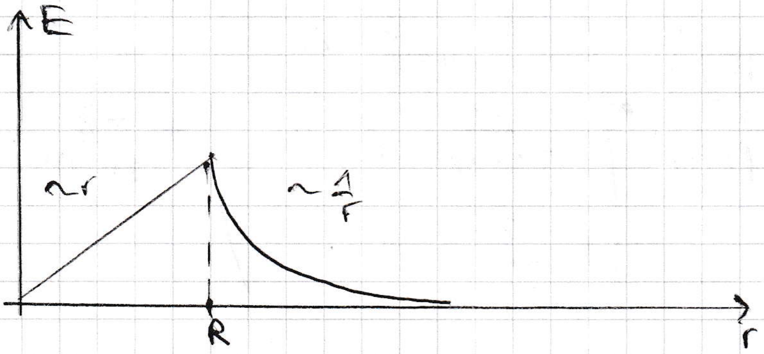
Zylindersymmetrisch!

$\vec{E} \underbrace{\int d\vec{s}}_{O_{Zyl}} = \frac{\cancel{4\pi} Q}{\varepsilon_0}$

$E_i \cdot 2\pi r L = \cancel{4\pi}\, \underbrace{Q(r)}_{} \cdot \frac{1}{\varepsilon_0} \Rightarrow E_i = \frac{2\pi \varrho r}{4\pi\varepsilon_0} = \frac{\varrho r}{2\varepsilon_0}$

$Q(r) = \cancel{4\pi}\, L \cdot r^2 \cdot \pi \varrho$

$E_a = \frac{\cancel{4\pi} Q}{\varepsilon_0 2\pi r L} = \frac{2\pi R^2 \varrho}{4\pi\varepsilon_0 r} = \frac{2\lambda}{r 4\pi\varepsilon_0} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}$



~~Potential:~~ $\cancel{\varphi =} \ \varphi = -\int E\, dr$ $\qquad \varphi_i = -\pi \varrho r^2 + C_1$

$\varphi_a = \cancel{\frac{2\pi R^2 \varrho}{r^2}} + C_2$

$2\pi R^2 \varrho \ln(r) + C_2$

da $\varphi(\infty) \stackrel{!}{=} 0 \Rightarrow C_2 = 0$

$\varphi_i(R) = \varphi_a(R) \Rightarrow C_1 - \pi \varrho R^2 = 2\pi R^2 \varrho \ln(r)$

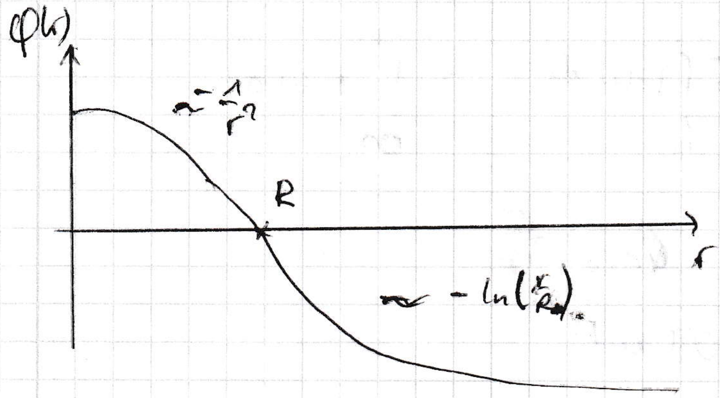
$\varphi = -\int E\, dr$ $\qquad \varphi_i = -\int_R^r \frac{\varrho r}{2\varepsilon_0}\, dr = \pi R^2 \varrho - \pi \varrho r^2 \cdot \frac{1}{4\pi\varepsilon_0}$

$= \frac{1}{4\pi\varepsilon_0} \pi \varrho (R^2 - r^2) = \frac{\lambda}{4\pi\varepsilon_0}\left(1 - \frac{r^2}{R^2}\right)$

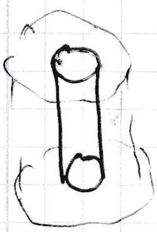
$\varphi_a = -\int_R^r \frac{2\pi R^2 \varrho}{4\pi\varepsilon_0 r}\, dr = -2\pi R^2 \varrho \ln\left(\frac{r}{R}\right) \cdot \frac{1}{4\pi\varepsilon_0}$

$= -\frac{2\lambda}{4\pi\varepsilon_0} \ln\left(\frac{r}{R}\right)$

$\varphi_a(R) = \varphi_i(R) = 0$ ✓



b) <u>hohler Stab</u> $\quad Q = \lambda \cdot L$



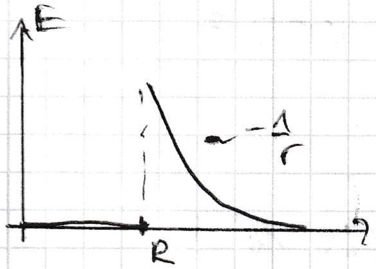
$E \cdot 2\pi r L = \frac{4\pi Q}{\varepsilon_0} \qquad Q_i = 0$

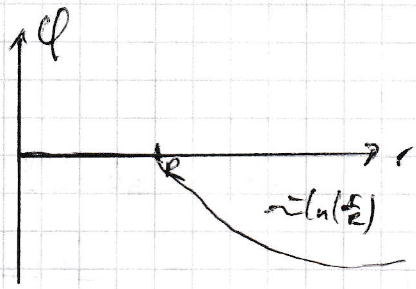
$\Rightarrow E_i = 0 \Rightarrow \varphi_i = 0 + C$

$Q_a = Q \Rightarrow E_a = \frac{4\pi Q}{\varepsilon_0 2\pi r L} = \frac{2\lambda}{r\, 4\pi\varepsilon_0}$

$\varphi_a = -\int E_a\, dr = -\int_R^r \frac{2\lambda}{r}\, dr = -\frac{2\lambda}{4\pi\varepsilon_0} \ln\left(\frac{r}{R}\right)$

$\varphi_a(R) = 0 \Rightarrow C = 0$





$\boxed{10}$ a) $C = \frac{Q}{U}$ $\qquad U = \int_{r_1}^{r_2} E(r)\,dr = \int_{\infty}^{r_E} E(r)\,dr$

$E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

$U = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_E}$

$\Rightarrow C = 4\pi\varepsilon_0 r_E = 709\,\mu F$

b) $E(r_E) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{(r_E)^2} \Rightarrow Q = E \cdot 4\pi\varepsilon_0 \cdot (r_E)^2$

$= \underline{\underline{5{,}42 \cdot 10^5\,C}}$

$U(r_E) = \frac{Q}{r_E} \frac{1}{4\pi\varepsilon_0} = r_E \cdot E(r_E) = \underline{\underline{7{,}65 \cdot 10^8\,V}}$

c) $C = \frac{Q}{U} \qquad U = -\int_{r_E}^{r_A} E(r)\,dr$

$r_A = 6391\,km \qquad r_E = 6371\,km$

$U = \frac{1}{4\pi\varepsilon_0} \left(\frac{Q}{r_E} - \frac{Q}{r_A} \right) \qquad C = \frac{Q}{U} = 4\pi\varepsilon_0 \cdot \frac{1}{\frac{1}{r_E} - \frac{1}{r_A}}$

$= 0{,}23\,F$

Q ... bleibt gleich!

$U = \frac{Q}{C} = \frac{5{,}42 \cdot 10^5\,C}{0{,}23\,F} = \underline{\underline{2{,}39 \cdot 10^6\,V}}$