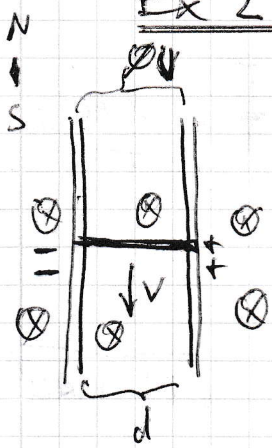
Ex 2 / Blatt 10

37

a)



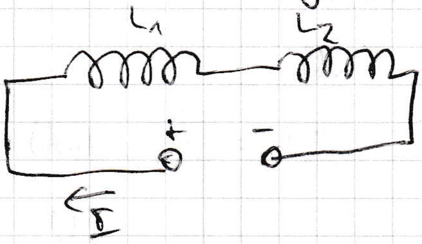
$$U_{ind} = -\frac{d\Phi}{dt} = -\frac{dA}{dt} \cdot B$$

$$= -dvB = -6{,}04 \cdot 10^{-3}\,V$$

b) $R = \rho_{Fe} \cdot \frac{l}{A} = 0{,}2\,\Omega$

$$I = \frac{U_{ind}}{R} = \underline{\underline{30{,}2\,mA}}$$

38 a) Reihenschaltung



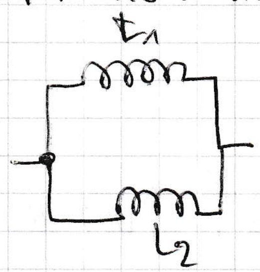
Generell: $U = L \cdot \dot{I}$

Strom I erzeugt ein Magnetfeld, dessen Änderung induziert eine Spannung

$$U = U_1 + U_2$$

$$= L_1 \dot{I} + L_2 \cdot \dot{I} = (L_1 + L_2) \cdot \dot{I} \Rightarrow \underline{\underline{L_{ges} = L_1 + L_2}}$$

b) Parallelschaltung



$U = const$

$I_{ges} = I_1 + I_2$

$$\dot{I}_{ges} = \dot{I}_1 + \dot{I}_2 = -\frac{U}{L_1} - \frac{U}{L_2} := -\frac{U}{L_{ges}}$$

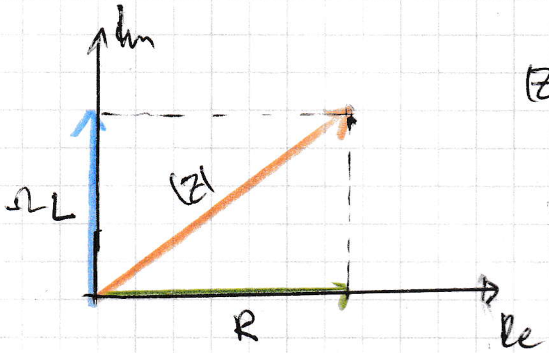
$$\Rightarrow \underline{\underline{\frac{1}{L_{ges}} = \frac{1}{L_1} + \frac{1}{L_2}}}$$

39 a) ohm'scher Widerstand $U = R \cdot I$

- bei Gleichspannung keine Induktion $R = \frac{\bar{U}}{\bar{I}} = 1{,}64\ \Omega$

b) $|Z| = \frac{U_{eff}}{I_{eff}} = \frac{\tilde{U}}{\tilde{I}} = 5{,}03\ \Omega$

c)

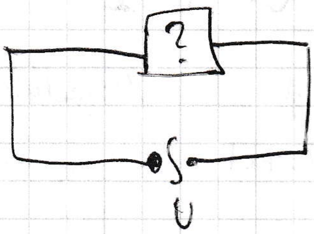


$|Z| = \cancel{\sqrt{R^2 + \omega L}}$ $\sqrt{R^2 + (\Omega L)^2}$

$\Rightarrow L = \sqrt{|Z|^2 - R^2} \cdot \frac{1}{\Omega}$ $\Omega = 2\pi f$

$= 15{,}1\ mH$

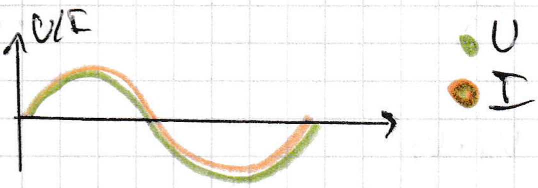
40 a) Wechselspannung



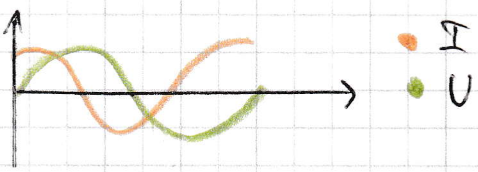
i) Widerstand R

$U(t) = U_0 \sin(\omega t)$

$I = \frac{U}{R} \Rightarrow I(t) = \frac{U_0}{R} \sin(\omega t)$

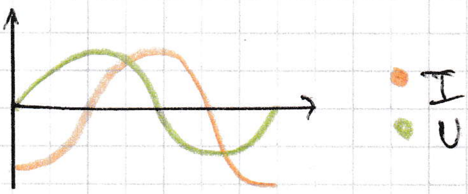


ii) Kapazität C $U(t) = \frac{Q(t)}{C}$ $I = \frac{dQ}{dt} = C \cdot \frac{dU(t)}{dt} = C \cdot U_0 \omega \cos(\omega t)$



iii) Induktivität L $U(t) = L \cdot \dot{I}(t) \Rightarrow I(t) = \frac{1}{L} \cdot \int U(t)\, dt$

$= -\frac{U_0}{L\omega} \cos(\omega t)$



b) I_0 , $I_{eff} = \frac{1}{\sqrt{2}} I_0$, $\langle P \rangle$

i) $I_0 = \frac{U_0}{R} = \underline{\underline{50\,mA}}$ $\Rightarrow$ $I_{eff} = \underline{\underline{35{,}4\,mA}}$

$\langle P \rangle = U_{eff} \cdot I_{eff} = \underline{\underline{125\,mW}}$

ii) $I_0 = C \cdot U_0 \cdot \omega = \underline{\underline{25\,mA}}$

$I_{eff} = \underline{\underline{17{,}7\,mA}}$ $\qquad$ $P = \underline{\underline{62{,}5\,mW}}$

iii) $I_0 = \frac{U_0}{L\omega} = \underline{\underline{50\,mA}}$

$I_{eff} = \underline{\underline{35{,}4\,mA}}$ $\qquad$ $P = \underline{\underline{125\,mW}}$

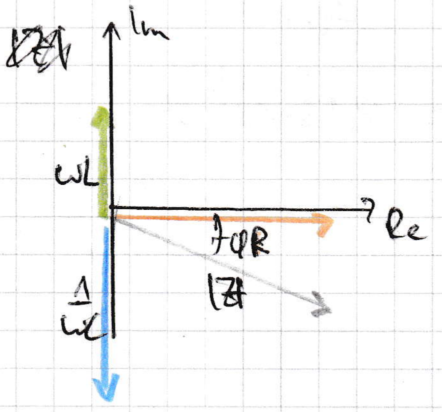
c) —[R]—[L]—[C]— $\qquad$ $I_R = I_C = I_L$

DGL: $U = U_1 + U_2 + U_3$ $\qquad$ $U \overset{!}{=} Z \cdot I$

$= R \cdot I + L \cdot \dot{I} + \frac{1}{C} \int I \, dt$ $\qquad$ mit $I = I_0 e^{i\omega t}$

$= \cancel{RI}\, I_0 \left[R e^{i\omega t} + i\omega L e^{i\omega t} + \frac{1}{i\omega C} e^{i\omega t} \right]$

$I_0 Z e^{i\omega t} = e^{i\omega t} I_0 \underbrace{\left[R + i\left(\omega L - \frac{1}{\omega C}\right) \right]}_{Z}$



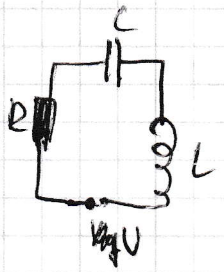
$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$= \underline{\underline{141{,}4\,\Omega}}$

$\varphi = \arctan\left[\frac{\frac{1}{\omega C} - \omega L}{R} \right] = \underline{\underline{45°}}$

$\boxed{41}$ Schwingkreis

a)



Spannung auf Kondensator C

$$U_C = \frac{Q}{C} \Rightarrow \frac{dU_C}{dt} = \frac{1}{C} I$$

$$U_R = R \cdot I = RC \frac{dU_C}{dt}$$

$$U_L = L \frac{dI}{dt} = LC \frac{d^2 U_C}{dt^2}$$

Ohne Anregung:

$$0 = U_R + U_L + U_C$$

$$\Rightarrow 0 = RC \frac{dU_C}{dt} + LC \frac{d^2 U_C}{dt^2} + U_C$$ ~~Ohne Anregung~~

mit Anregung $$U_q = RC \frac{dU_C}{dt} + LC \frac{d^2 U_C}{dt^2} + U_C$$

b) $$\frac{d^2 U_C}{dt^2} + \frac{R}{L} \frac{dU_C}{dt} + \frac{1}{LC} U_C = 0$$

Auf die Form des ~~ha~~ gedämpften harm Osci bringen.

$$\ddot{x}(t) + 2\gamma \dot{x}(t) + \omega_0^2 x(t) = 0 \Rightarrow x(t) = e^{-\gamma t}\left(c_1 e^{\sqrt{\gamma^2 - \omega_0^2}\, t} + c_2 e^{-\sqrt{\gamma^2 - \omega_0^2}\, t}\right)$$

$\Rightarrow \gamma = \frac{R}{2L}$ Dämpfung $\quad \omega_0 = \sqrt{\frac{1}{LC}}$ Eigenfrequenz

Für $R = 0$ $\quad 2\pi f = \sqrt{\frac{1}{LC}} \Rightarrow LC = \frac{1}{4\pi^2 f^2}$

$$= 2{,}53 \cdot 10^{-8}\, s^2$$

z.B. $C = 1\,\mu F$

$L = 25{,}3\, mH$

c) aperiodischer Grenzfall $\gamma = \omega_0$

$\frac{R}{2L} = \omega_0 \qquad \frac{R}{2L} = \sqrt{\frac{1}{LC}} \Rightarrow R = 2\sqrt{\frac{L}{C}}$ z.B. $\underline{318\,\Omega}$