Aufgabe 1

$I = \frac{U}{R}$

Mit dem Satz von Stokes und $\text{rot}\, B = \mu_0 j$ fällt auf, dass der Strom des Außenleiters keinen Einfluss auf das innere B-Feld hat.

B-Feld um einen langen Leiter: $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{e}_\varphi$

$$\Phi = \int_{r_1}^{r_2}\int_0^L \vec{B}\, d\vec{a} = \frac{\mu_0 I}{2\pi}\int_{r_1}^{r_2}\int_0^L \frac{1}{r}\, dl\, dr = \frac{\mu_0 I L}{2\pi}\ln\left(\frac{r_2}{r_1}\right) ✓$$

Außen und Innenleiter bilden zusammen mit dem Widerstand eine Leiterschlaufe.

$$-L\cdot\dot{I} = U_{ind} = -\dot{\Phi} = -\frac{\mu_0 \dot{I} L}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow L = \frac{\mu_0 L}{2\pi}\ln\left(\frac{r_2}{r_1}\right) ✓$$

(4)

Aufgabe 2

$$W = \frac{1}{2}\vec{B}\vec{H} = \frac{1}{2\mu\mu_0}\vec{B}^2 = \frac{1}{2\mu\mu_0}B^2$$

$$\vec{B} = \frac{\mu_0\mu}{2\pi r}\underbrace{I\frac{r^2}{R^2}}\hat{e}_\varphi = \frac{\mu_0\mu I}{2\pi R^2}r\,\hat{e}_\varphi ✓$$

Strom im inneren des Kreises mit radius r

Volumenintegral (Zylinderintegral)

$$W = \int_0^L\int_0^R\int_0^{2\pi r} w\, d\varphi\, dr\, dl = \int_0^L\int_0^R 2\pi r\frac{1}{2\mu\mu_0}B^2\, dr\, dl = \frac{L\mu_0\mu I^2}{4\pi R^4}\int_0^R r^3 dr = \frac{L\mu_0\mu I^2}{16\pi R^4}R^4 = \frac{1}{2}L_i I^2 ✓$$

$$\Rightarrow \frac{L_i}{L} = \frac{\mu_0\mu}{8\pi} ✓$$

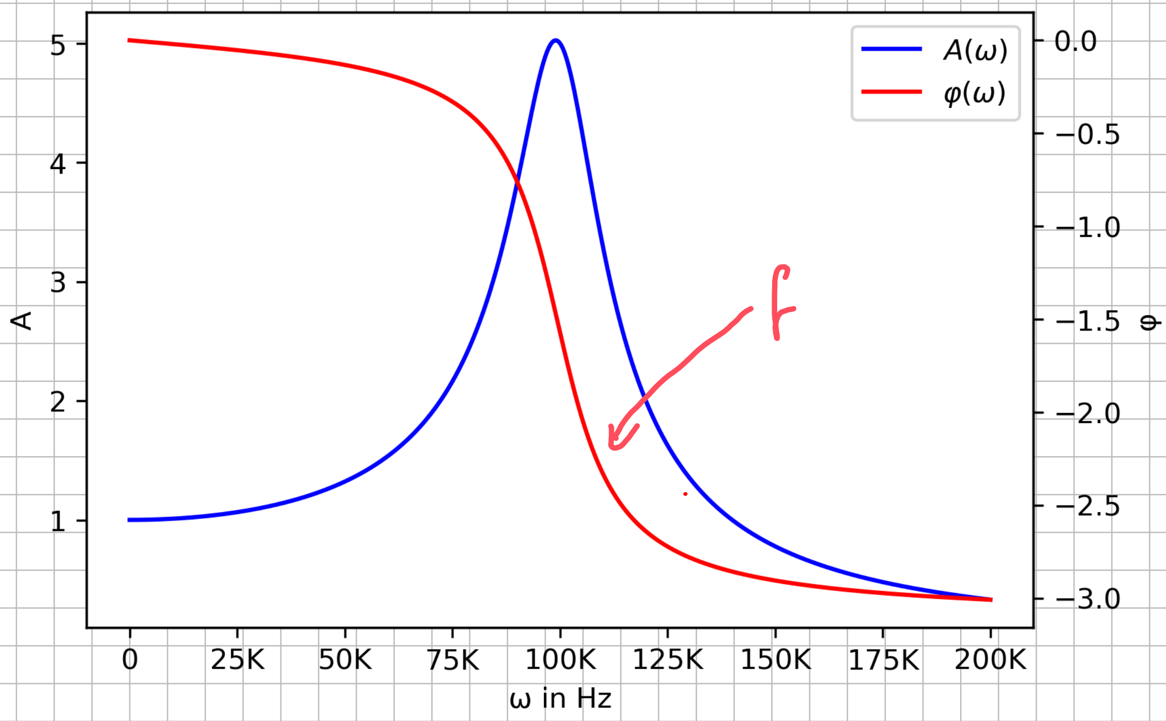
(3)

Aufgabe 3

a) Spannungsteiler aus (R, L) und C.

$$\hat{U}_a(t) = \frac{\frac{1}{i\omega C}}{R + i\omega L + \frac{1}{i\omega C}}\hat{U}_e(t) = \frac{\hat{U}_e(t)}{(R + i\omega L + \frac{1}{i\omega C})i\omega C}$$

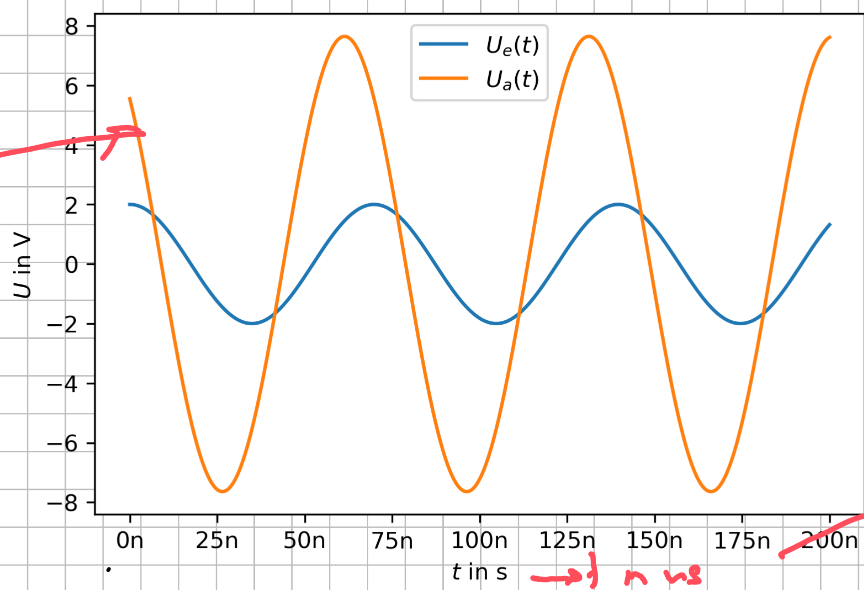
$$\Rightarrow \hat{A}(\omega) = \frac{\hat{U}_a}{\hat{U}_e} = \frac{1}{i\omega RC - \omega^2 LC + 1} ✓ \Rightarrow A = |\hat{A}(\omega)| = \frac{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2} \Rightarrow \varphi(\omega) = \tan^{-1}\left(-\frac{\omega RC}{1-\omega^2 LC}\right) ✓$$



2,5

b) $|\hat{U}_a(t)| = \left|\frac{\hat{U}_e(t)}{i\omega RC - \omega^2 LC + 1}\right| = \left|\frac{U_0 e^{i\omega t}}{i\omega RC - \omega^2 LC + 1}\right| = U_0 A(\omega) \cos(\omega t + \varphi(\omega))$ ✓

Falsche Phase



→ in ns

2

Warum reell?

c) $\hat{I}_C(t) = \frac{\hat{U}_a(t)}{\frac{1}{i\omega C}} = U_a(t)\, i\omega C = \frac{U_0 e^{i\omega t}}{(R + i\omega L - i\frac{1}{\omega C})} = U_0 e^{i\omega t} \frac{R - i\omega L + i\frac{1}{\omega C}}{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$\Rightarrow G_1 = \frac{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}{R^2 + (\omega L - \frac{1}{\omega C})^2}$ $\qquad \varphi_1 = \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$

$\Rightarrow I_C(t) = U_0 G_I \cos(\omega t + \varphi)$

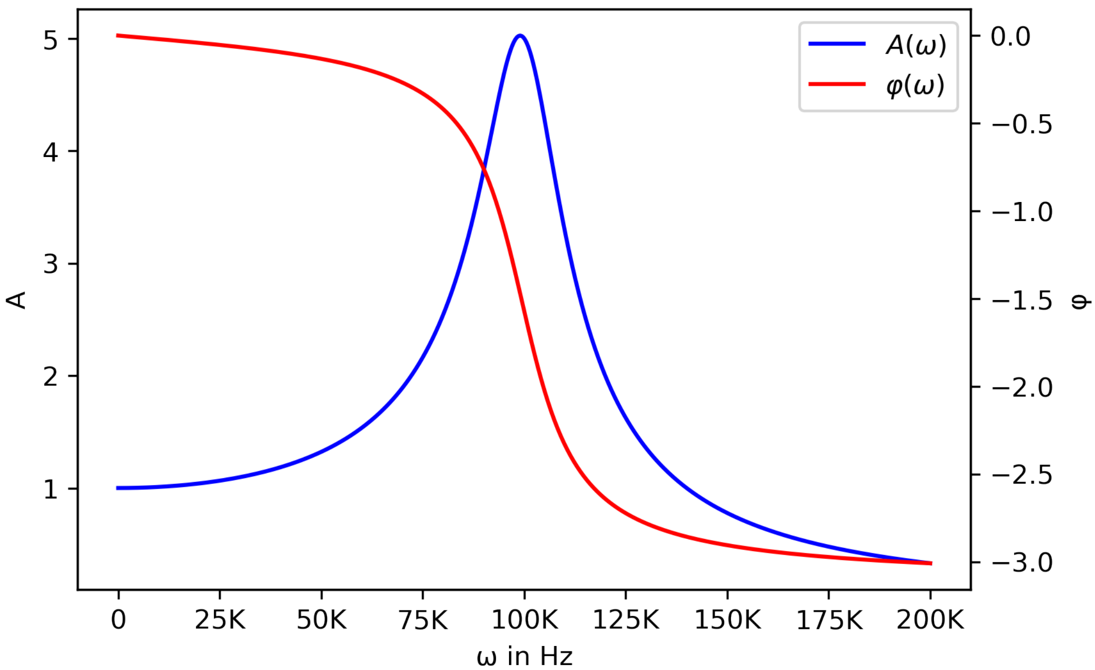
$\Rightarrow I_0 = U_0 G_I \approx 0{,}1027\,A$ f

0,5 / 5

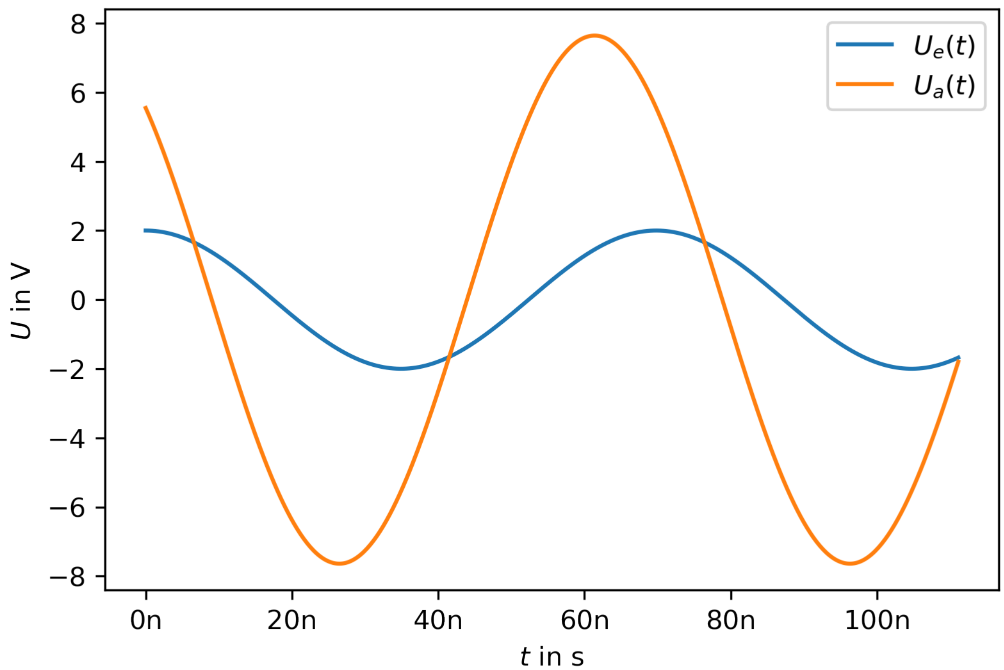
Ex_10

July 1, 2023

[8]:
```python
import numpy as np
import matplotlib as mpl
mpl.rcParams['figure.dpi'] = 300
import matplotlib.pyplot as plt

fig = plt.figure()
ax = fig.add_subplot(111)
ax2 = ax.twinx()
mkfunc = lambda x, pos: '%1.0fM' % (x * 1e-6) if x >= 1e6 else '%1.0fK' % (x *␣
↪1e-3) if x >= 1e3 else '%1.0f' % x
mkformatter = mpl.ticker.FuncFormatter(mkfunc)
ax.xaxis.set_major_formatter(mkformatter)

U0 = 2
C = 100e-9
L = 1e-3
R=20

ω = np.linspace(0,200e3, 500)
A = np.sqrt((1-ω**2*L*C)**2 + (ω*R*C)**2)/((1-ω**2*L*C)**2 + ω**2*R**2*C**2)
φ = np.arctan(-ω*R*C/(1-ω**2*L*C))
φ = φ - np.pi*np.int8(φ>0)

ax.plot(ω, A, c="blue", label="$A(ω)$")
ax.set_xlabel("ω in Hz")
ax.set_ylabel("A")

ax2.plot(ω, φ, c="red", label="$φ(ω)$")
ax2.set_ylabel("φ")

lines, labels = ax.get_legend_handles_labels()
lines2, labels2 = ax2.get_legend_handles_labels()
ax2.legend(lines + lines2, labels + labels2, loc=0)
None
```



[98]:
```
ω = 90e3
t = np.linspace(0,0.2e-3,500)
t = np.linspace(0,10/ω,500)
A = np.sqrt((1-ω**2*L*C)**2 + (ω*R*C)**2)/((1-ω**2*L*C)**2 + ω**2*R**2*C**2)
φ = np.arctan(ω*R*C/(1-ω**2*L*C))
φ = φ + np.pi*(φ<0)

ax = plt.gca()
mkfunc = lambda x, pos: '%1.0fn' % (x * 1e6)
mkformatter = mpl.ticker.FuncFormatter(mkfunc)
ax.xaxis.set_major_formatter(mkformatter)

plt.plot(t, U0*np.cos(ω*t), label="$U_e(t)$")
plt.plot(t, U0*A*np.cos(ω*t+φ), label="$U_a(t)$")
plt.legend()
plt.xlabel("$t$ in s")
plt.ylabel("$U$ in V")
None
```



[100]:
```
I0 = U0 * np.sqrt(R**2 + (ω*L - 1/(ω*C)))/(R**2 + (ω*L - 1/(ω*C)))
I0
```

[100]: 0.10274816178761126