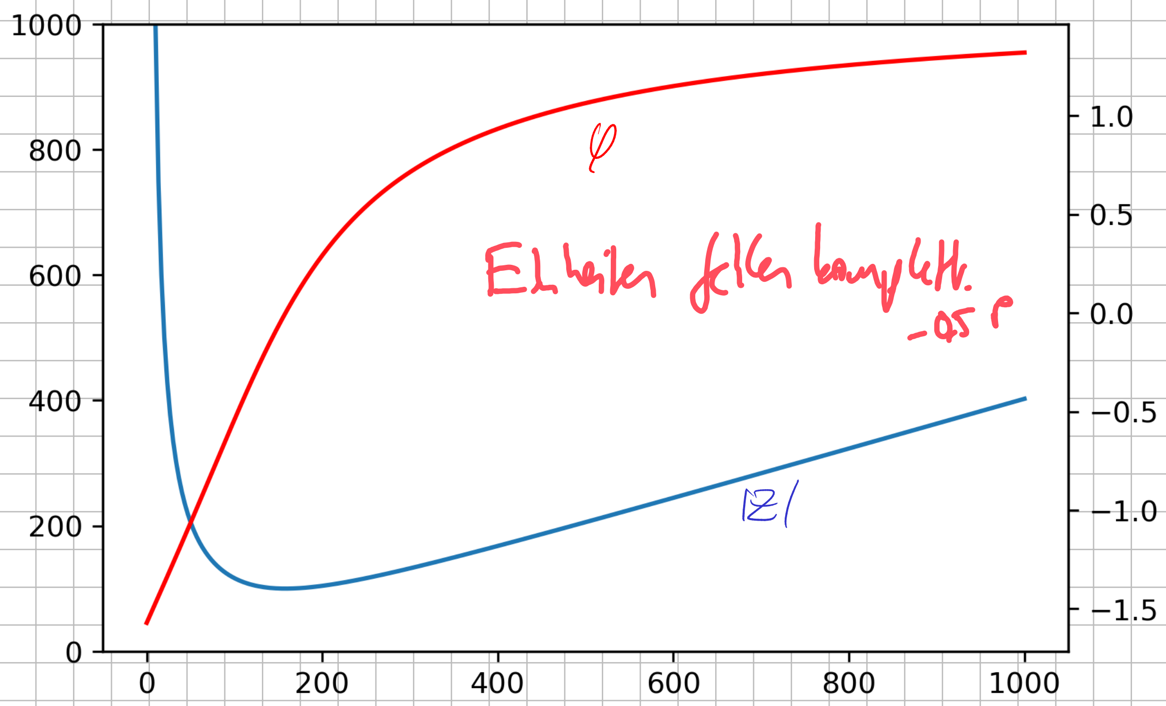
Aufgabe 1

a) $Z_{ges} = R + i\omega L + \frac{1}{i\omega C} = R + i\left(\omega L - \frac{1}{\omega C}\right)$

$|Z_{ges}| = \sqrt{R^2 + \omega^2 L^2 - 2\frac{L}{C} + \frac{1}{\omega^2 C^2}}$ ✓ $\varphi = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) =$ ✓



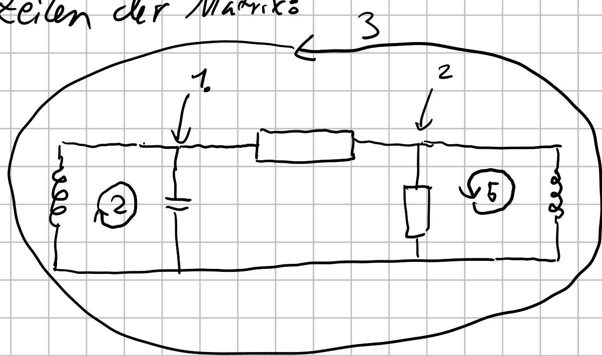
(ω)

$\langle P \rangle = \frac{U_{eff}^2}{|Z|}\cos(\varphi) = \frac{U_0^2}{2|Z|}\cos(\varphi) = 140{,}657\ W$ f

2,5

Aufgabe 2

Zeilen der Matrix:



$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ iL\omega & 0 & -iL\omega & 0 & Z \\ iL\omega & -\frac{i}{\omega C} & 0 & 0 & 0 \\ 0 & 0 & i\omega L & -\frac{i}{\omega C} & 0 \end{bmatrix}$$

Mit $Z = \frac{1}{i\omega C_K}$

$0 \stackrel{!}{=} \det(A) = \frac{1 - 2CL\omega^2 - 2C_K L\omega^2 + C^2L^2\omega^4 + 2CC_KL^2\omega^4}{iC^2C_K\omega^3}$

$\Rightarrow \omega_{1/2} = \pm\frac{1}{\sqrt{CL}} = \omega_0$ ✓

$\Rightarrow \omega_{3/4} = \pm\frac{1}{\sqrt{CL + 2C_KL}}$ ✓

b) Mit $Z = i\omega L_K$ $\quad 0 \stackrel{!}{=} \det(A) = \frac{-2L - L_K + 2CL^2\omega^2 + 2CLL_K\omega^2 - C^2L^2L_K\omega^4}{iC^2\omega}$

$\Rightarrow \omega_{1/2} = \pm\omega_0$ ✓ $\quad \omega_{3/4} = \pm\sqrt{\frac{2L + L_K}{CLL_K}}$ ✓

5

Aufgabe 3

a) $Q(t) = Q_0 \, e^{-\frac{t}{RC}}$ ✓ $\quad I(t) = \dot{Q} = -\frac{Q_0}{RC} e^{-\frac{t}{RC}}$ ✓ $\qquad A = \pi r_0^2$

b) $E(t) = \frac{Q(t)}{Cd}$

$j_V(t) = \dot{\vec{E}} \varepsilon_0 = \frac{d\,Q(t)}{dt} \cdot \frac{1}{Cd} \varepsilon_0 = I(t) \frac{\varepsilon_0}{Cd}$

$I_V(t) = \underbrace{I(t) \overbrace{\frac{\varepsilon_0}{Cd} \cdot A}^{=1}}_{\text{da Feld homogen ist.}} = I(t)$ => Der Verschiebungstrom ist gleich dem Strom $I(t)$ ✓

c) Kreisintegral mit Radius r:

Für $r \geq r_0$: $H = \frac{1}{2\pi r} \int_A j_V \, da = \frac{1}{2\pi r} \int_0^{r_0} 2\pi r \, j_V \, dr = \frac{1}{2\pi r} \cdot \pi r_0^2 \, j_V = \frac{1}{2\pi r} I(t)$ ✓

Für $r \leq r_0$: $H = \frac{1}{2\pi r} \int_0^{r} 2\pi r \, j_V \, dr = \frac{1}{2\pi r} \cdot \pi r^2 \, j_V = \frac{r}{2} \frac{r_0^2}{r_0^2} j_V = \frac{r}{2\pi r_0^2} I(t)$ ✓

6

Ex_11

July 6, 2023

[2]:

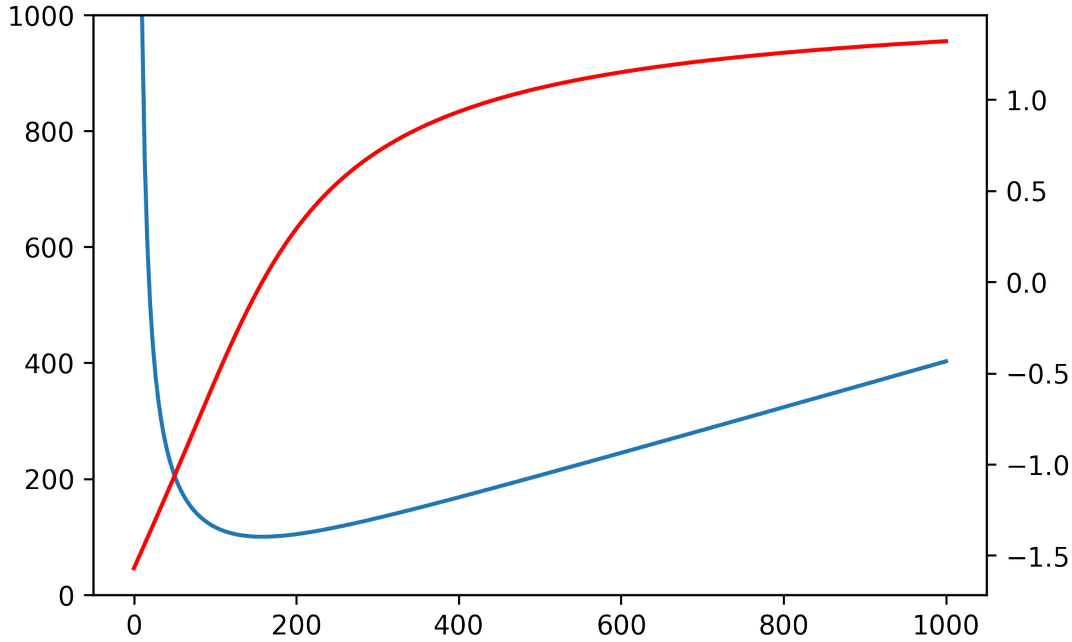
```
import matplotlib.pyplot as plt
import matplotlib as mpl
mpl.rcParams['figure.dpi'] = 300
import numpy as np
import warnings
warnings.filterwarnings('ignore')

L = 0.4
C = 100e-6
R = 100

w = np.linspace(0,1000, 300)

ax1 = plt.gca()
ax2 = ax1.twinx()
Z = np.sqrt(R**2 + w**2*L**2-2*L/C + 1/(w**2*C**2))
phi = np.arctan2(w*L-1/(w*C),R)

ax1.plot(w, Z)
ax2.plot(w, phi, color="red")
ax1.set_ylim([0,1000])
None
```



[7]:
```
U0 = 230
w = 50 * 2 * np.pi

Z = np.sqrt(R**2 + w**2*L**2-2*L/C + 1/(w**2*C**2))
phi = np.arctan2(w*L-1/(w*C),R)
P = U0**2/(Z)*np.cos(phi)
P
```

[7]: 281.31446251860075

[ ]:

In[ ]:= $A = \begin{pmatrix} 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ I\,L\,w & 0 & -I\,L\,w & 0 & Z \\ I\,L\,w & \frac{-I}{w\,c} & 0 & 0 & 0 \\ 0 & 0 & I\,w\,L & \frac{-I}{w\,c} & 0 \end{pmatrix};$

$\text{Det}\left[A\ /.\ Z \to \frac{-I}{w\,Ck}\right]$

$\text{Solve}\left[\text{Det}\left[A\ /.\ Z \to \frac{-I}{w\,Ck}\right] == 0,\ w\right]$

Out[ ]= $-\frac{i\left(1 - 2\,c\,L\,w^2 - 2\,Ck\,L\,w^2 + c^2\,L^2\,w^4 + 2\,c\,Ck\,L^2\,w^4\right)}{c^2\,Ck\,w^3}$

Out[ ]= $\left\{\left\{w \to -\frac{1}{\sqrt{c}\ \sqrt{L}}\right\}, \left\{w \to \frac{1}{\sqrt{c}\ \sqrt{L}}\right\}, \left\{w \to -\frac{1}{\sqrt{c\,L + 2\,Ck\,L}}\right\}, \left\{w \to \frac{1}{\sqrt{c\,L + 2\,Ck\,L}}\right\}\right\}$

In[ ]:= Det[A /. Z → I w Lk]

Solve[Det[A /. Z → I w Lk] == 0, w]

Out[ ]= $-\frac{i\left(-2\,L - Lk + 2\,c\,L^2\,w^2 + 2\,c\,L\,Lk\,w^2 - c^2\,L^2\,Lk\,w^4\right)}{c^2\,w}$

Out[ ]= $\left\{\left\{w \to -\frac{1}{\sqrt{c}\ \sqrt{L}}\right\}, \left\{w \to \frac{1}{\sqrt{c}\ \sqrt{L}}\right\}, \left\{w \to -\frac{\sqrt{2\,L + Lk}}{\sqrt{c}\ \sqrt{L}\ \sqrt{Lk}}\right\}, \left\{w \to \frac{\sqrt{2\,L + Lk}}{\sqrt{c}\ \sqrt{L}\ \sqrt{Lk}}\right\}\right\}$