3.4 Brechung in dispersiven Medien

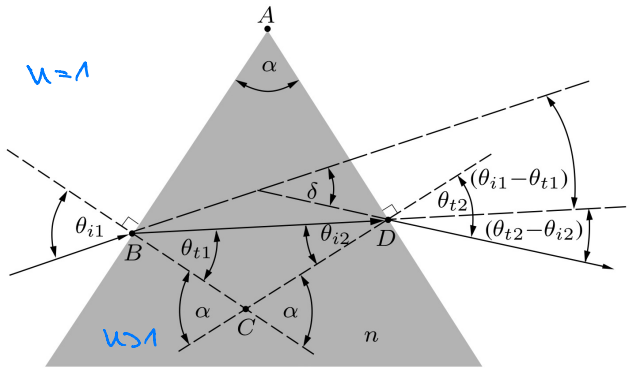
Pink Floyd. Dark side of the moon
1973

normale Dispersion

$$\frac{\partial n}{\partial \omega} > 0$$

$$\frac{\partial n}{\partial \lambda} < 0$$

Brechungsindex nimmt zu mit Frequenz,
blaues Licht wird stärker gebrochen als rotes.



Prisma mit Öffnungswinkel
α

δ : Ablenkwinkel

Herleitung

$$\delta = (\theta_{i1} - \theta_{t1}) + (\theta_{t2} - \theta_{i2})$$

$$\alpha = \theta_{t1} + \theta_{i2}$$

$$\delta = \theta_{i1} + \theta_{t2} - \alpha$$

$$\theta_{t2} = \sin^{-1}(n \sin\theta_{i2}) = \sin^{-1}(n \sin(\alpha - \theta_{t1}))$$

$$\delta \approx \theta_{i1} + \sin^{-1}\left[\sin\alpha\,(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha\right] - \alpha$$

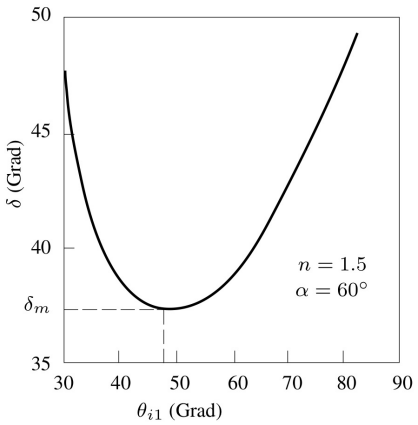


Bild 5.67: Ablenkungswinkel als Funktion des Einfallswinkels.

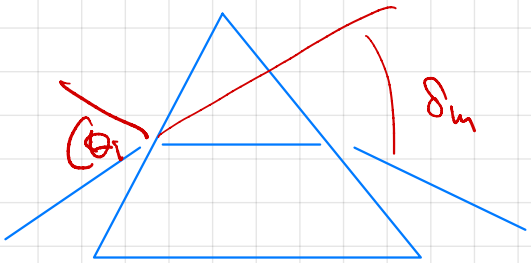
δ_m ist das minimale Ablenkungswinkel

Es gilt $\delta = \delta_m$ $\theta_{i1} \neq \theta_{t2}$

Diese Situation gilt ein ganz kleines für δ_m ↓

$\Rightarrow$ $\theta_{i1} = \theta_{t2}$

$\Rightarrow$ δ_m ist realisiert für den symmetrischen Fall



$\theta_{i1} = \theta_{t2}$

$\Rightarrow \delta = \delta_m$

Für δ_m bekommen wir

$$n = \frac{\sin((\delta_m + \alpha)/2)}{\sin \alpha/2}$$

→ Genaue Methode zur Bestimmung von Brechungsindizes.

Kronleuchter

Wikipedia

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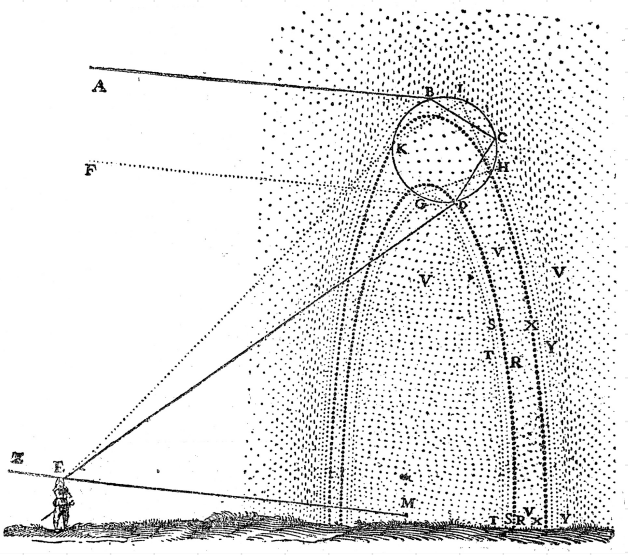
$n_{Wasser} \approx 1.33$



Descard 1637
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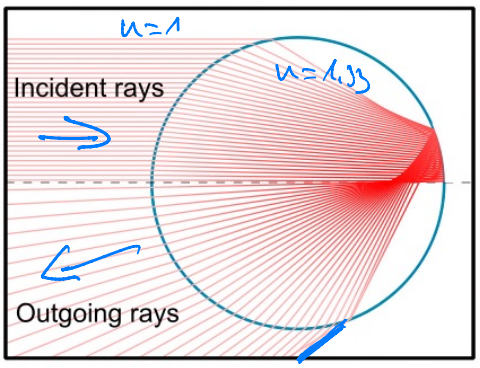

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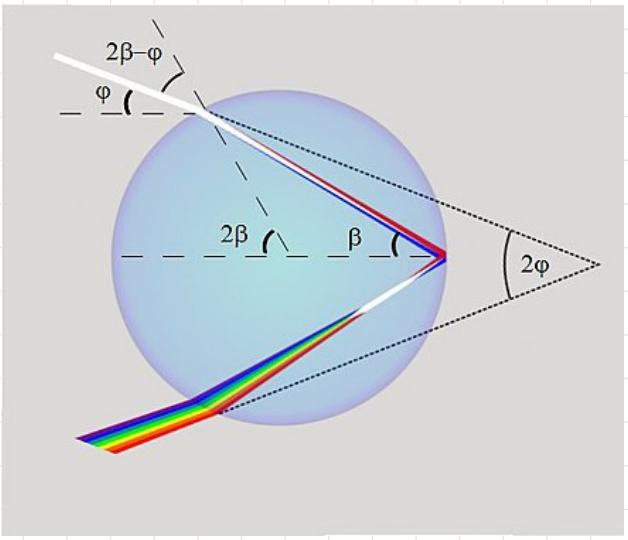


$$\sin(2\beta - \phi) = n \sin\beta$$

$$n = 1{,}333$$

$$\phi = 2\beta - \arcsin(n \sin\beta)$$

$$\frac{d\phi}{d\beta} = 0$$

$$0 = 2 - \frac{1}{\sqrt{1 - (n\sin\beta)^2}}\, n\cos\beta$$

$$\Leftrightarrow \quad 2 = \frac{1}{\sqrt{1 - n^2\sin^2\beta}}\, n\cos\beta$$

$$\Leftrightarrow \quad 2 = \frac{1}{\sqrt{1 - n^2(1-\cos^2\beta)}}\, n\cos\beta$$

$$\Leftrightarrow \quad 2 = \frac{1}{\sqrt{1 - n^2 + n^2\cos^2\beta}}\, n\cos\beta$$

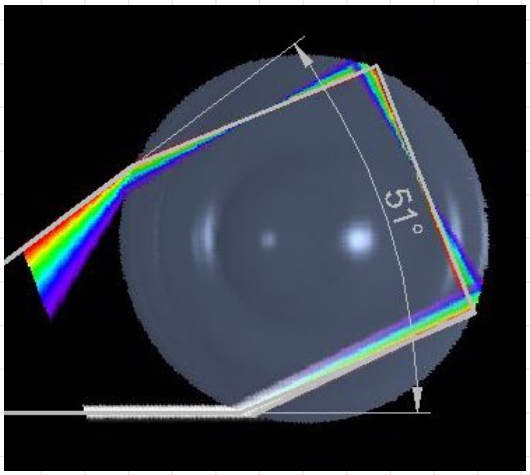
$$\hookrightarrow \quad 4\left(1-u^2+u^2\cos^2\beta\right) = u^2\cos^2\beta$$

$$\hookrightarrow \quad 2u^2\cos^2\beta = 4u^2-4$$

$$\hookrightarrow \quad \cos^2\beta = \frac{4}{3}\,\frac{4}{2u^2}$$

$$\hookrightarrow \quad \beta = \arccos\sqrt{\frac{4}{3}-\frac{4}{2u^2}} = 40.4^\circ \qquad u = 1.333$$

$$\Rightarrow \quad 2\phi_{max} = 42^\circ$$



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