Aufgabe 1

a)

$$v_{Ph} = \frac{\omega}{k} \qquad k = \frac{2\pi}{\lambda_{Ph}} \qquad \omega = \frac{2\pi}{T}$$

$$v_{Gr} = \frac{d\omega}{dk}$$

$$\omega = v_p \cdot k$$

$$\frac{d\omega}{dk} = v_{ph} + \frac{dv_p}{dk} k \checkmark \qquad \frac{dk}{d\lambda} = -\frac{1}{\lambda^2} 2\pi \checkmark \longrightarrow dk = -\frac{1}{\lambda^2} 2\pi d\lambda \checkmark$$

$$\frac{d\omega}{dk} = v_{ph} - \frac{dv_{ph} \lambda^2 k}{2\pi d\lambda}$$

$$= v_{ph} - \frac{dv_{ph} \cancel{2\pi} \lambda^{\cancel{2}}}{\cancel{2\pi} d\lambda \cancel{\lambda}}$$

$$\underline{\underline{= v_{ph} - \lambda \frac{dv_{ph}}{d\lambda}}} \checkmark$$

1,5 Pkt

$\frac{4}{4}$

b)

Normale Dispersion

$$\frac{dn}{d\lambda} < 0 \qquad v_{ph} = \frac{c}{n} \qquad \frac{dv_p}{dn} = -\frac{1}{n^2}c < 0$$

$$v_{Gr} = v_{ph} - \lambda \underbrace{\underbrace{\frac{dv_{ph}}{dn}}_{<0} \underbrace{\frac{dn}{d\lambda}}_{<0}}_{>0}$$

$$\underline{\underline{v_{gr} < v_{ph}}}$$

✓ 1 Pkt

c)

$$n \approx 1 - \frac{a^2}{\omega^2} \qquad \frac{a^2}{\omega^2} < 1, \; a = \text{const.}$$

$$v_{ph} = \frac{\omega}{k} = \frac{c}{\underbrace{n}_{}} \longrightarrow v_{ph} > c$$

$$\frac{\omega}{k} = \frac{c}{n} \qquad \frac{\omega n}{c} = k \qquad \frac{dk}{d\omega} = \frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}$$

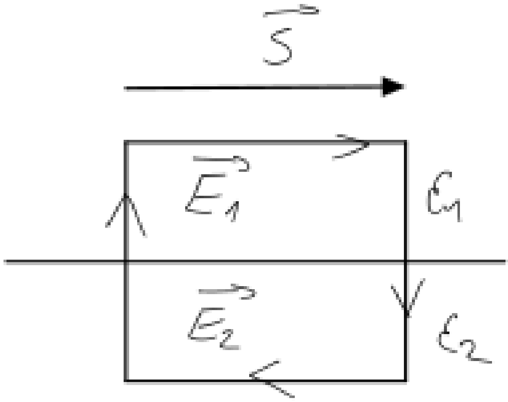
$$v_{gr} = \frac{d\omega}{dk} = \frac{1}{\frac{dk}{d\omega}} = \frac{c}{n + \omega\frac{dn}{d\omega}} = \frac{c}{1 - \frac{a^2}{\omega^2} + 2\omega\frac{a^2}{\omega^3}}$$

$$v_{gr} = \frac{c}{\underbrace{1 + \frac{a^2}{\omega^2}}_{>1}} < c$$

✓ 1,5 Pkt

Aufgabe 2

a) + b)



$$\oint \vec{E}\, d\vec{l} = 0$$

$$\vec{E}_{1\parallel}\vec{S} - \vec{E}_{2\parallel}\vec{S} = 0$$

$$\vec{S}\left(\vec{E}_{1\parallel} - \vec{E}_{2\parallel}\right) = 0$$

$$\vec{E}_{1\parallel} = \vec{E}_{2\parallel} \checkmark$$

$\Rightarrow D_\parallel$ unstetig

$$\oint_A \vec{D}\, d\vec{A} = -D_{1\perp}\Delta A + D_{2\perp}\Delta A = 0$$

$$= \Delta A(-D_{1\perp} + D_{2\perp}) = 0$$

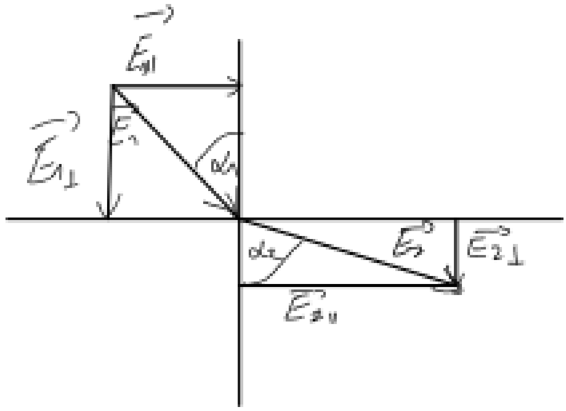
$$\rightarrow D_{1\perp} = D_{2\perp} \checkmark \text{ stetig}$$

$\vec{D} = \varepsilon_0 \varepsilon_r \cdot \vec{E}$ $\Rightarrow E_\perp$ unstetig

a) 2,5

b) 2,0

$$\varepsilon_0 \varepsilon_{r1} \cdot \vec{E}_{1\perp} = \varepsilon_0 \varepsilon_{r2} \vec{E}_{2\perp}$$



$$|\vec{E}_1| \sin(\alpha_1) = |\vec{E}_2| \sin(\alpha_2)$$

$$|\vec{E}_1| \cos(\alpha_1) = |\vec{E}_{1\perp}| \qquad |\vec{E}_2| \cos(\alpha_2) = |\vec{E}_{2\perp}|$$

$$\varepsilon_{r1}|\vec{E}_{1\perp}| = \varepsilon_{r2}|\vec{E}_{2\perp}|$$ unstetig

A3 a) adiabatisch: kein Wärmeaustausch (Reifen gut isoliert) aber Austausch von Arbeit (pumpen) ✓

b) geg.: $p_1 = 100$ kPa

$p_2 = 300$ kPa

$V_2 = 2{,}05\,l$

$\gamma = \frac{7}{5}$

$p \cdot V^\gamma = \text{const.}$

$\Rightarrow p_2 \cdot V_2^\gamma = p_1 \cdot V_1^\gamma$

$V_1 = \left(\frac{p_2}{p_1}\right)^{1/\gamma} \cdot V_2 = 3^{5/7} \cdot 2{,}05\,l$

$V_1 = 4{,}49\,l$ ✓ $\Rightarrow \Delta V = V_1 - V_2 = 2{,}44\,l$

$V_{pump} = \pi\ (0{,}032\,m)^2 \cdot 0{,}38\,m = 0{,}00122\,m^3 = 1{,}22\,l$

0,3056 l

d ≠ r !

$\Rightarrow$ man muss ~~zwei~~ 8 mal pumpen

1,5 Pkt

d) V = const. (begrenzt durch Reifen)

$\Rightarrow p \sim T\ ,\ \frac{p}{T} = \text{const.}$

$\frac{p_2}{T_2} = \frac{p_3}{288{,}15\,K}$ $\qquad p_3 = 300\,000\ Pa \cdot \frac{288{,}15\,K}{334{,}29\,K} = 275\,242\ Pa \approx 279\ kPa$ ✓

1 Pkt

für $T = 25°C = 298{,}15\,K$: $p_4 = 226\,850\ Pa \approx 227\ kPa$ ✓

Aufg. 4. a)

ⓐ $p_0 V_0 = nRT_0$

① $p_1 V_1 = nRT_1$

$$\left.\begin{aligned} p_0 V_0 &= nRT_0 \\ p_1 V_1 &= nRT_1 \end{aligned}\right\} \Rightarrow V_1 = \frac{V_0 \cdot T_1}{T_0} \quad (*)$$

$$V_0 = \frac{\pi b^2}{4} \cdot (h - f - d) \quad , V_1 = \frac{\pi b^2}{4} (h - f - d - \Delta s)$$

Mit $(*)$ folgt Δs!

ⓑ

Es gilt: $W = \int_{s_0}^{s_1} F \, ds = \int_{s_0}^{s_1} \Delta p \cdot A \, ds$

$$V = As \Rightarrow ds = \frac{1}{A} dV$$

$$\Rightarrow W = \int_{V_1}^{V_0} \Delta p \, dV \quad ; \quad \Delta p = p_0 \cdot \left(1 - \frac{V_1}{V}\right)$$

$$= \int_{V_1}^{V_0} p_0 \left(1 - \frac{V_1}{V}\right) dV = p_0 \Big[V - \ln(V) \cdot V_1 \Big]_{V_1}^{V_0}$$

$$= p_0 \left(V_0 - \ln(V_0) \cdot V_1 - V_1 + \ln(V_1) \cdot V_1 \right)$$

$$= p_0 \cdot \left(V_1 \cdot \ln\left(\frac{V_1}{V_0}\right) + V_0 - V_1 \right)$$