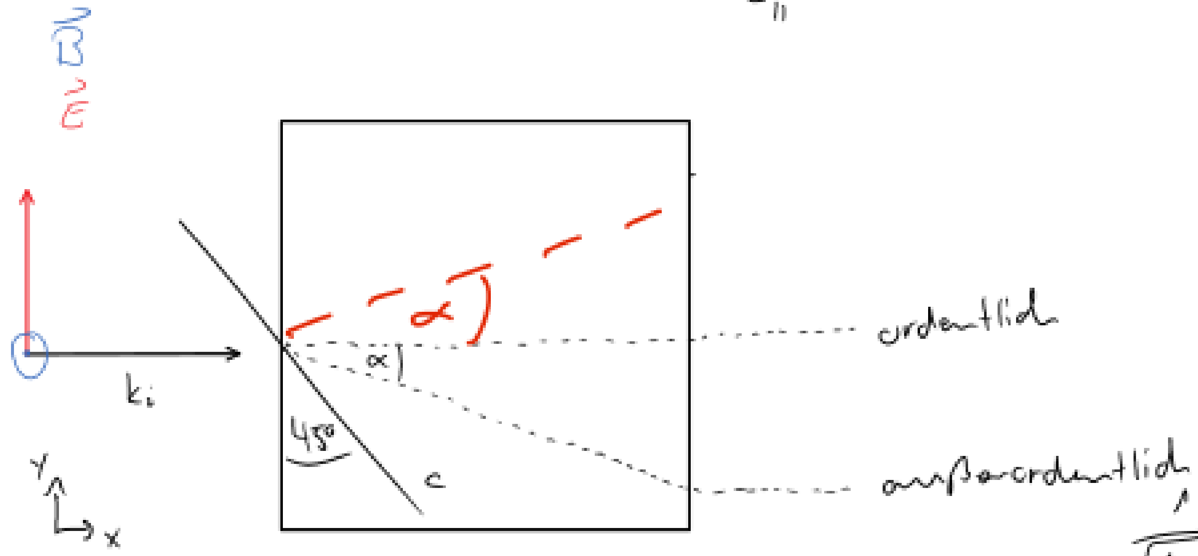
Aufgabe 1

$n_{\parallel} = 1{,}486 \quad , \quad n_{\perp} = 1{,}658$

$E_{\parallel} = \sin(45°)\, E \quad , \quad E_{\perp} = \cos(45°)\, E$



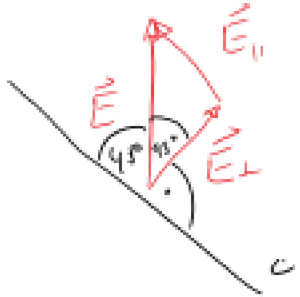
$4{,}5 / 5$

$n = \frac{c_0}{c_M} = \frac{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}}{\frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}} = \sqrt{\mu_r \varepsilon_r} \quad \Big| \text{ hier: } \mu_r = 1$

$\Rightarrow \quad n = \sqrt{\varepsilon_r}$

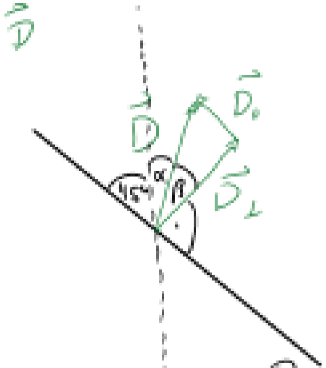
$D_{\parallel} = \varepsilon_0 \varepsilon_{\parallel} E_{\parallel}$

$D_{\perp} = \varepsilon_0 \varepsilon_{\perp} E_{\perp}$



$D_{\parallel} = \varepsilon_0 n_{\parallel}^2 E_{\parallel} = \varepsilon_0 n_{\parallel}^2 \sin(45°) \cdot E$

$D_{\perp} = \varepsilon_0 n_{\perp}^2 E_{\perp} = \varepsilon_0 n_{\perp}^2 \cos 45° \cdot E$



$|\vec{D}_{\perp}| > |\vec{D}_{\parallel}|$

$\alpha = \beta - 45°$

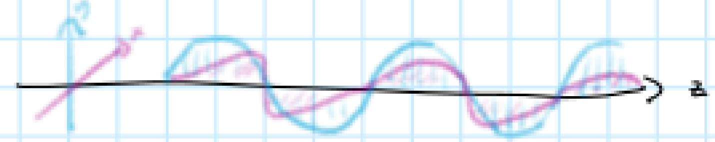
$\tan(\beta) = \frac{D_{\parallel}}{D_{\perp}} = \frac{\varepsilon_0 n_{\parallel}^2 \sin(45°) \cdot E}{\cancel{\varepsilon_0} n_{\perp}^2 \cancel{\sin(45°)} \cdot \cancel{E}} = \frac{n_{\parallel}^2}{n_{\perp}^2}$

$\beta = \arctan\left(\left(\frac{n_{\parallel}}{n_{\perp}}\right)^2\right) = 38{,}8°$

$\alpha = 45° - \beta = 45° - 38{,}8° = \underline{\underline{6{,}2°}}$ ✓

Der außerordentliche Strahl wird in ~~negative~~ positiver y-Richtung abgelenkt.

$\boxed{2}$ $\vec{E}(t,z) = E_0 \begin{pmatrix} \cos(\omega t + kz) \\ \cos(\omega t + kz) \\ 0 \end{pmatrix}$

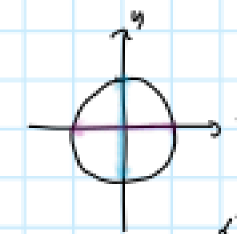


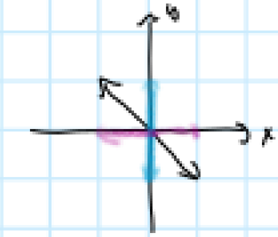
5/5

a) linear polarization ↗ ✓ 0,5

b) $\lambda/4$ Plättle, optical axis $\parallel$ to x $\Rightarrow$ E_x shifted by $\lambda/4$ $\Rightarrow$ circularly polarized ✓ 1

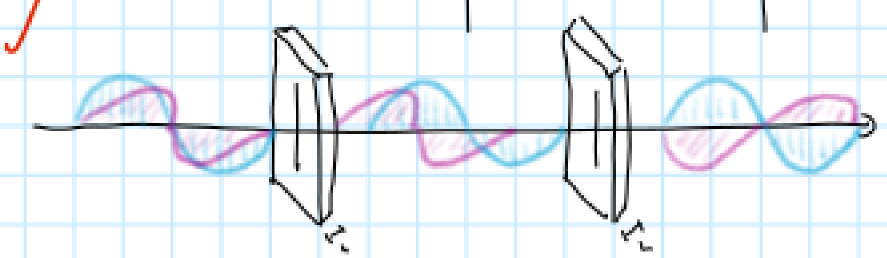
$\vec{E}_1 = E_0 \begin{pmatrix} \cos(\omega t + kz) + \frac{\pi}{2k} \\ \cos(\omega t + kz) \\ 0 \end{pmatrix}$ ✓ $\Delta\varphi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$ $\lambda = \frac{2\pi}{k} \Rightarrow \frac{\lambda}{4} = \frac{\pi}{2k}$





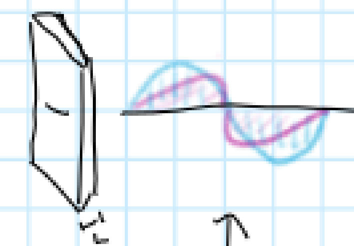
c) 2^{nd} $\lambda/4$ Plättle, $\Rightarrow$ shift E_x by $\lambda/4$ again $\Rightarrow$ linearly polarized in opposite direction ✓

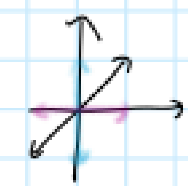
$\vec{E}_2 = E_0 \begin{pmatrix} \cos(\omega t + kz) + \frac{\pi}{\cancel{2}k} \\ \cos(\omega t + kz) \\ 0 \end{pmatrix} = E_0 \begin{pmatrix} -\cos(\omega t - kz) \\ \cos(\omega t - kz) \\ 0 \end{pmatrix}$ 1



d) 2^{nd} Plättle turned $90°$ $\Rightarrow$ E_y shifted by $\lambda/4$ $\Rightarrow$ linearly polarized in original direction (same as a)

$\vec{E}_2 = E_0 \begin{pmatrix} \cos(\omega t + kz) + \frac{\pi}{\cancel{2k}} \\ \cos(\omega t + kz) + \frac{\pi}{\cancel{2k}} \\ 0 \end{pmatrix}$ ✓ 1





e) The wave plate needs to be made of a birefringent material, ✓

$\Rightarrow L = \frac{\lambda}{4\Delta n}$ ✓ $= \frac{\lambda}{4} \frac{1}{n_\perp - n_\parallel}$ 1,5

5/5

3 a) mit Gleichverteilungssatz:

$$KE = \frac{3}{2} kT = \frac{1}{2} m v^2$$

$$\Rightarrow \overline{v^2} = \frac{\cancel{2}\, 3 kT}{m} = \frac{3kT}{m}$$

mit Maxwell verteilung: $f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \, e^{-\frac{mv^2}{2kT}}$

$$\overline{v^2} = \int_0^\infty v^2 f(v)\, dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^\infty v^4 e^{-\frac{mv^2}{2kT}} dv$$

from table:

$$\int_0^\infty x^{2n} e^{-\frac{x^2}{a^2}} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{a}{2}\right)^{2n+1}$$

$n=2$, $x=v$, $a^2 = \left(\frac{2kT}{m}\right)$

$$= \left(\frac{m}{2\cancel{\pi} kT}\right)^{3/2} 4\cancel{\pi}\sqrt{\pi} \left(\frac{4!}{2!}\right) \left(\frac{\sqrt{\frac{2kT}{m}}}{2}\right)^5$$

$$= \frac{4 \cdot 4!}{2! \cdot 2^5} \left(\frac{m^3}{(2kT)^3} \quad \frac{(2kT)^5}{m^5}\right)^{1/2}$$

$$= \frac{3}{2} \frac{2kT}{m}$$

$$= \frac{3kT}{m}$$

3,5

b) most common speed $\Rightarrow$ max of maxwell dist.

$$\frac{df(v)}{dv} = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \left(\left(-\frac{2mv}{2kT}\right) e^{-\frac{mv^2}{2kT}} v^2 + 2v e^{-\frac{mv^2}{2kT}}\right) = 0$$

$$\Rightarrow v e^{-\frac{mv^2}{2kT}} \left(2 - \frac{mv^2}{kT}\right) = 0$$ disregard $v=0$ solution

$$\Rightarrow 2 = \frac{mv^2}{kT}$$

$$\Rightarrow v^2 = \frac{2kT}{m} \Rightarrow v = \sqrt{\frac{2kT}{m}}$$

1,5

Aufg. 4. Geg.: $T = 10°C$, $p_0 = 1013\,hPa$,
$V_0 = 10\,dm^3$, $m_B = 1g$

a) Auftriebskraft und Gewichtskraft müssen im GG sein

$\Rightarrow F_A = F_G \Leftrightarrow \rho_{Luft} \cdot g \cdot V_0 = m_{Ges} \cdot g$

$\Leftrightarrow m_B + m_{Brief} + \rho_{He} \cdot V_0 = \rho_{Luft} \cdot V_0$

$\Rightarrow m_{Brief} = (\rho_{Luft} - \rho_{He}) \cdot V_0 - m_B$

$\Rightarrow m_{Brief} = \left(1{,}25 \frac{kg}{m^3} - 0{,}1785 \frac{kg}{m^3}\right) \cdot V_0 - m_B$

$\Rightarrow m_{Brief} \cong \underline{\underline{9{,}72\,g}}$ ✓ 2,5

b) Zustände: ① $p_0 V_0 = nRT_0$, $T_0 = const$

② $p(h) \cdot V = nRT_0$

$\overset{①}{\Rightarrow} nR = \frac{p_0 V_0}{T_0}$

$\Rightarrow V = \frac{nRT_0}{p(h)} = V_0 \cdot \frac{p_0}{p(h)}$

$\Rightarrow \frac{V}{V_0} = \frac{p_0}{p(h)} = e^{\frac{\rho_{Luft} \cdot g \cdot h}{p_0}} \Rightarrow h = \frac{p_0}{\rho_{Luft} \cdot g} \cdot \ln\left(\frac{V}{V_0}\right)$

Mit $V = 1{,}20 \cdot V_0$

2,5

$\Rightarrow \underline{\underline{h \cong 1506{,}2\,m}}$ ✓