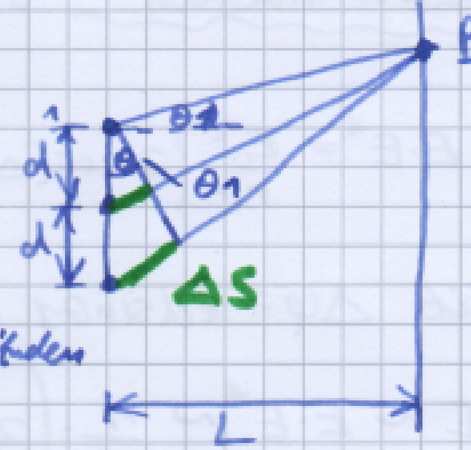
Aufg. 1.

Es gilt: $L >> d$



$\Rightarrow \theta_1 \cong \theta_2 \cong \theta_3$

3,5 / 4

Kugelwellen mit gleicher Amplituden

Also ist

$$\sin\theta_1 = \frac{\Delta s_{12}}{d} \quad \text{und} \quad \sin\theta_1 = \frac{\Delta s_{13}}{2d}$$

+1

$$\Rightarrow \underline{\underline{\Delta s_{12} = d\sin\theta_1}} \quad \text{und} \quad \underline{\underline{\Delta s_{13} = 2d\sin\theta_1}}$$

b) Es gilt: $E_1 = \frac{E_0}{r_1} e^0 = \frac{E_0}{r_1}$, $E_2 = \frac{E_0}{r_2} e^{ikd\sin\theta_1}$, $E_3 = \frac{E_0}{r_3} e^{i2kd\sin\theta_1}$

am Punkt P ?

da $$E(P) = \sum_n E_n e^{i\Delta\varphi_n} \quad \text{und} \quad \Delta\varphi = k\cdot\Delta s$$

$$E_1 = \frac{E_0}{r} e^{i(kr-\omega t)}$$

Also folgt $$E(P) = E_1 + E_2 + E_3$$

muss überall dahinter

$$\Rightarrow E(P) = E_0 \left(\frac{1}{r_1} + \frac{1}{r_2} e^{ikd\sin\theta_1} + \frac{1}{r_3} e^{2ikd\sin\theta_1} \right)$$

Wegen $L >> d$ gilt $r_1 \cong r_2 \cong r_3 \cong r_1$:

+1

$$\Rightarrow E(P) = \frac{E_0}{r_1} \left(1 + e^{ikd\sin\theta_1} + e^{i2kd\sin\theta_1} \right)$$

c)

$$\Rightarrow E\cdot E^* = \frac{E_0^2}{r_1^2} \left(1 + e^{ikd\sin\theta_1} + e^{i2kd\sin\theta_1}\right) \cdot \left(1 + e^{-ikd\sin\theta_1} + e^{-i2kd\sin\theta_1}\right)$$

$\Delta\varphi = kd\sin\theta_1$

$$= \frac{E_0^2}{r_1^2} \left(3 + 2\left(e^{i\Delta\varphi} + e^{-i\Delta\varphi}\right) + \left(e^{i2\Delta\varphi} + e^{-i2\Delta\varphi}\right)\right)$$

Euler: $e^{ix} = \cos x + i\sin x$

→

$$\Rightarrow E \cdot E^* = \frac{E_0^2}{r_1^2}\left(3 + 4\cos(\Delta\varphi) + 2\cdot\cos(2\Delta\varphi)\right)$$

Additionstheorem: $3 + 4\cdot\cos(x) + 2\cos(2x) = (2\cos(x)+1)^2$

"Wolfram"

$$\Rightarrow E \cdot E^* = \frac{E_0^2}{r_1^2}\left(2\cdot\cos(\Delta\varphi)+1\right)^2 = \frac{E_0^2}{r_1^2}\left[2\cdot\left(\cos(\Delta\varphi)+\frac{1}{2}\right)\right]^2$$

Mit $\Delta\varphi = kd\sin\theta_1$ folgt nun

$$I \sim E\cdot E^* \sim \frac{1}{r_1^2}\cdot\left(2\cdot\left(\cos(kd\sin\theta_1)+\frac{1}{2}\right)\right)^2$$

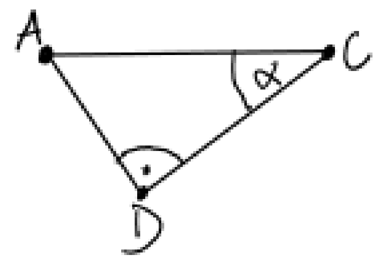
+1,5

A2

6/6

a) $\Delta S = n(\underbrace{\overline{AB} + \overline{BC}}_{\text{Weg von Strahl 2}}) - \underbrace{\overline{AD}}_{\text{Weg von Strahl 1}} \cdot \overset{=1}{n_{Luft}}$

$\cos\beta = \frac{d}{\overline{AB}}$; $\overline{AB} = \overline{BC} = \frac{d}{\cos\beta}$



$\sin\alpha = \frac{\overline{AD}}{\overline{AC}}$; $\overline{AD} = \overline{AC}\sin\alpha$

$\sin\alpha = n\sin\beta$

$\tan\beta = \frac{\frac{1}{2}\overline{AC}}{d}$; $\overline{AC} = 2d\tan\beta$

$\Rightarrow \overline{AD} = 2dn\frac{\sin^2\beta}{\cos\beta}$

$\Delta S = \frac{2nd}{\cos\beta} - 2nd\frac{\sin^2\beta}{\cos\beta} = 2nd\frac{1-\sin^2\beta}{\cos\beta} = 2nd\cos\beta$

$= 2nd\sqrt{1-\sin^2\beta} = 2d\sqrt{n^2-\sin^2\alpha}$ ✓ +2

b) t: Transmissionskoeffizient beim Einfallen

t': Transmissionskoeffizient beim Ausfallen

r: Reflexionskoeffizient innerhalb der Platte

Strahl 1: 2 mal transmittiert → $E_1 = tt'E_0$

Strahl 2: 2 mal transmittiert und 2 mal reflektiert → $E_2 = tt'r^2E_0\underbrace{e^{i\Delta\varphi}}_{\text{Phasenverschiebung durch längeren zurückgelegten Weg}}$ ✓

Strahl 3: $E_3 = tt'r^2 \cdot r^2 \cdot E_0 \cdot e^{2i\Delta\varphi}$ ✓

⋮

$E_t = E_1 + E_2 + \dots = tt'E_0(1 + r^2e^{i\Delta\varphi} + r^4e^{2i\Delta\varphi} + \dots)$

$= 1 + r^2e^{i\Delta\varphi} + (r^2e^{i\Delta\varphi})^2 + \dots$

$= \frac{1}{1-r^2e^{i\Delta\varphi}}$ (geometrische Reihe)

$E_t = tt'E_0\frac{1}{1-r^2e^{i\Delta\varphi}}$

+2

$$E_t = tt' E_0 \frac{1}{1 - r^2 e^{i\Delta\varphi}}$$

c) $$E_t E_t^* = t^2 t'^2 E_0^2 \frac{1}{1-r^2 e^{i\Delta\varphi}} \cdot \frac{1}{1-r^2 e^{-i\Delta\varphi}} = \frac{t^2 t'^2 E_0^2}{1 - r^2 e^{i\Delta\varphi} - r^2 e^{-i\Delta\varphi} + r^4 e^{i\Delta\varphi - i\Delta\varphi}}$$

$$= \frac{t^2 t'^2 E_0^2}{1 + r^4 - 2r^2 \cos\Delta\varphi} = \frac{t^2 t'^2 E_0^2}{(1-r^2)^2 + 2r^2 - 2r^2 \cos\Delta\varphi}$$

mit $t \cdot t' = 1 - r^2$ (Stokes Relation) und $1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$ gilt:

$$I_t \sim E_t E_t^* = \frac{E_0^2 (1-r^2)^2}{(1-r^2)^2 + 4r^2 \sin^2\left(\frac{\Delta\varphi}{2}\right)}$$

+2

3/3

Aufgabe 3

Annahme: alles gerechnet für 1 mol

K_1: $T_1 = 60°C = 333\,K$

$U_1 = CT_1 = 4{,}2\,\frac{kJ}{K} \cdot 333K = 1400\,kJ$

K_2: $T_2 = 20°C = 293\,K$

$U_2 = 4{,}2\,\frac{kJ}{K} \cdot 293\,K = 1230\,kJ$

Energieerhaltung, keine Arbeit wird geleistet:

$$T_0 = \frac{T_1 + T_2}{2} = 313\,K$$

$$dS = \frac{dU}{T} = \frac{C\,dT}{T}$$

$$\Delta S_1 = \int_{T_1}^{T_0} \frac{C}{T}\,dT = C \ln\left(\frac{T_0}{T_1}\right) = 4{,}2\,\frac{kJ}{K} \ln\left(\frac{313\,K}{333\,K}\right) = -0{,}260\,\frac{kJ}{K}$$

$$\Delta S_2 = \int_{T_2}^{T_0} \frac{C}{T}\,dT = C \ln\left(\frac{T_0}{T_2}\right) = 4{,}2\,\frac{kJ}{K} \ln\left(\frac{313\,K}{293\,K}\right) = 0{,}277\,\frac{kJ}{K}$$

$$S_{ges} = \Delta S_1 + \Delta S_2 = -0{,}26\tfrac{kJ}{K} + 0{,}277\tfrac{kJ}{K} = 0{,}017\,\frac{kJ}{K}$$

Aufgabe 4

2,5/3

(a) $dG = -S\,dT + \underbrace{V\,dp}_{=0,\text{ da } p \text{ const.}}$

$$\Delta G = -\int_{293K}^{303K} 70 \frac{J}{K\cdot mol} = -70 \frac{J}{K\cdot mol} (303K - 293K) = -700 \frac{J}{mol}$$

✓ +1,5

(b) $dG = \underbrace{-S\,dT}_{\substack{=0 \\ \text{da } T = \text{const.}}} + V\,dp$

+1

$$-\Delta G = \int_{p_0}^{p} V\,dp' = V(p - p_0) = V \cdot p - G_0$$

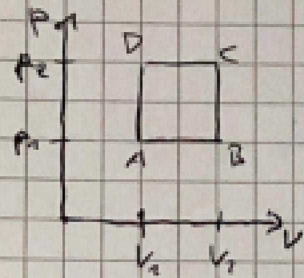
$$p = \frac{\Delta G - G_0}{V} = -\frac{-70 \frac{J}{K\cdot mol} \cdot 303K}{\frac{18 \frac{ml}{mol}}{1000000 \frac{ml}{m^3}}} = 1{,}18 \cdot 10^9\ hPa$$

$$p = \frac{\Delta G}{V} = \frac{700\ J/mol}{18 \cdot 10^{-6} \frac{m^3}{mol}} = 3{,}89 \cdot 10^7\ Pa$$

4/4

(5) isobarer Prozess: $dQ = C_p dT$

isochorer Prozess: $dQ = C_v dT$



$C_p = C_v + R$

$pV = RT$

$T_A = \frac{p_1 V_1}{R}$

$T_B = \frac{p_1 V_2}{R}$

$T_C = \frac{p_2 V_2}{R}$

$T_D = \frac{p_2 V_1}{R}$

$\Delta Q_{II} = \Delta Q_{p=konst.} + \Delta Q_{V=konst.} = (C_v + R)(T_B - T_A) + C_v (T_C - T_B)$

$= \frac{C_v}{R}(p_2 V_2 - p_1 V_2) + p_1 (V_2 - V_1)$

$\Delta Q_{I} = \Delta Q_{V=konst.} + \Delta Q_{p=konst.} = C_v (T_D - T_A) + (C_v + R)(T_C - T_D)$

$= \frac{C_v}{R}(p_2 V_1 - p_1 V_1) + p_2 (V_2 - V_1)$

✓ +3

$\Delta\Delta Q = p_1 \Delta V - p_2 \Delta V = \Delta p \Delta V > 0$

Wäre Q ein Zustandsgröße, dann müsste die Differenz null sein. ✓

b) $W = -\int p dV$

$W_{II} = -p_1 \Delta V$ ✓

$W_I = -p_2 \Delta V$ ✓ +1

$\Delta W = \Delta p \Delta V = \Delta\Delta Q$ ✓