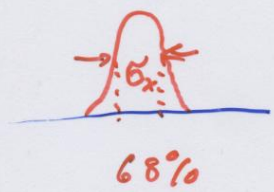
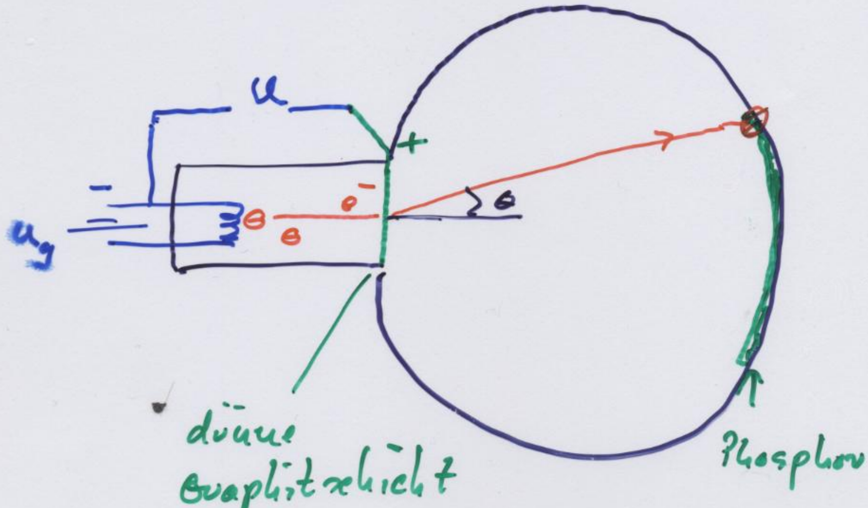
- Zur Unschärferelation:

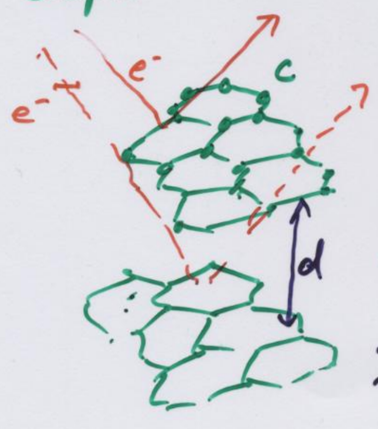
$$\Delta p \cdot \Delta x \gtrsim h \qquad \text{Heisenberg}$$

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2} \qquad \text{Gaußsches Paket}$$



Versuch: Wellennatur von Elektronen





Laufstreckendifferenz:

$\lambda = 2d\sin\theta$ Interferenz.

Wir erwarten:

$$\lambda = \frac{h}{m \cdot v};$$

$$E_{kin} = e \cdot U = \frac{1}{2} m_e v^2$$

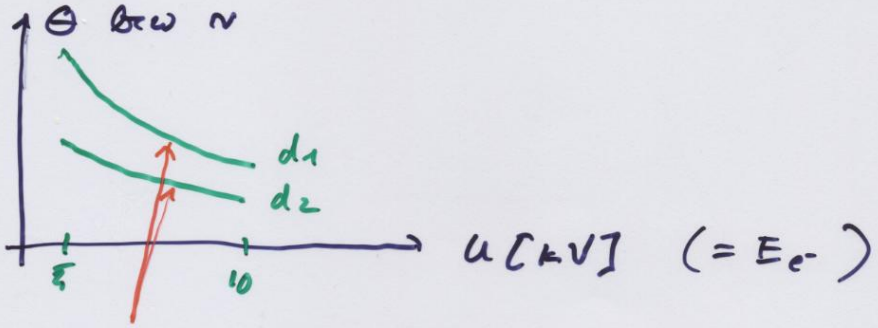
$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{e \cdot U \cdot 2m_e}}$$

Für $U = 12$ keV :

$$\lambda = 0{,}011 \text{ nm} <$$

$$d_1 = 0{,}21 \text{ nm} ,$$
$$d_2 = 0{,}12 \text{ nm}$$

Beobachtung:



Abnahme der Wellenlänge λ .

---

zu 6.1.4. Nicht-relativist. Materiewellen im Potential :

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r},t) = \underbrace{\left(-\frac{\hbar^2}{2m} \Delta + V(\vec{r}) \right)}_{\text{Hamilton-Operator}} \psi(\vec{r},t)$$

$$= \underbrace{E \cdot \psi(\vec{r},t)}_{\text{Eigenwert}}$$

A Brief Chronology of Quantum Theory Development

1900 Explanation of blackbody radiation by energy quantization.
Max Planck (*Nobel Prize 1918*).

1900 Discovery that the energy of electrons emitted by the photoelectric effect was independent of the light intensity.
Philip von Lenard (*Nobel Prize 1905*).

1905 Explanation of the photoelectric effect.
Albert Einstein (*Nobel Prize 1921*).

1905 The theory of special relativity.
Albert Einstein (*Nobel Prize 1921*).

1907–1911 Explanation of the specific heats of solids by energy quantization.
Albert Einstein (*Nobel Prize 1921*).

1911 Observation of the nuclear atom.
Ernest Rutherford (*Nobel Prize, Chemistry, 1908*)

1913 First quantized model of the hydrogen atom.
Niels Bohr (*Nobel Prize 1922*).

1916 Experimental studies of the photoelectric effect.
Robert Millikan (*Nobel Prize 1923*).

1923 Discovery and explanation of the collisions between light quanta and electrons.
Arthur Compton (*Nobel Prize, with C. T. Wilson, 1927*).

1924 Proposal that electrons have an associated wavelength $\lambda = h/p$.
Prince Louis Victor de Broglie (*Nobel Prize 1929*).

1925 Mathematical theory of wave mechanics.
Erwin Schrödinger (*Nobel Prize, with P. Dirac, 1933*).

1925 Mathematical theory of matrix mechanics.
Werner Heisenberg (*Nobel Prize 1932*).

1925 The Exclusion Principle.
Wolfgang Pauli (*Nobel Prize 1945*).

1926 Statistical interpretation of the wave function.
Max Born (*Nobel Prize 1954*).

1927 The Uncertainty Principle.
Werner Heisenberg (*Nobel Prize 1932*).

1927 Observation of electron-wave diffraction by crystals.
Clinton Davisson (*Nobel Prize, with G. P. Thompson, 1937*).

1928 Relativistic theory of quantum mechanics and the prediction of the positron.
Paul Dirac (*Nobel Prize, with E. Schrödinger, 1933*).

1932 Observation of the positron.
Carl Anderson (*Nobel Prize, with Victor Hess, 1936*).

1948 Completion of the theory of quantum electrodynamics.
Sin-Itiro Tomanaga, Julian Schwinger, and Richard Feynman (*Nobel Prize 1965*).

A. PICCARD E. HENRIOT P. EHRENFEST Ed. HERZEN Th. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H. FOWLER L. BRILLOUIN

P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR

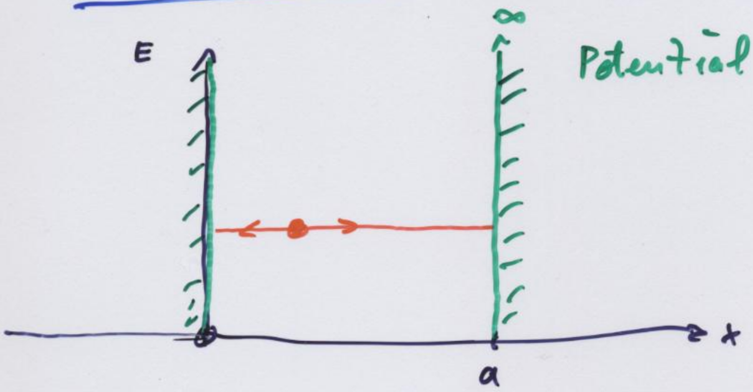
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

THE PARTICIPANTS of the Fifth Solvay Congress, 1927. (International Institute of Physics and Chemistry photo; courtesy of AIP Emilio Segrè Visual Archives.)

6.2 Einfache Quantenmechanische Systeme

6.2.1 Teilchen im Kasten (1d)



Potential ∞ hoch

a. Im Kasten: Freies Teilchen

Ansatz: $\psi(x,t) = a_1 e^{i(kx-\omega t)} + a_2 e^{-i(kx+\omega t)}$

2 Wellen: $E = \frac{p^2}{2m}$: $k = \pm\frac{\sqrt{2mE}}{\hbar}$

Auch: stationärer Fall: $t = 0$

$u(x) = a_1 e^{ikx} + a_2 e^{-ikx}$

Randbedingungen:

$u(x) = 0$ für $x \geq a$, ≤ 0

$\Rightarrow u(0) = a_1 + a_2 = 0$

$\Rightarrow u(x) = a_1 (e^{ikx} - e^{-ikx})$

$u(a) = 0$

$= 2i\, a_1 \sin k \cdot a$

$\Rightarrow k \cdot a = n \cdot \pi$

$k = n \cdot \frac{\pi}{a}$

Lösung:

$$\psi(x,t) = a_1 \cdot \left(e^{in\frac{\pi}{a}x} - e^{-in\frac{\pi}{a}x}\right) e^{-i\omega t}$$

$n = 1 \dots \infty$

Wahrscheinlichkeit, Teilchen im Kasten zu finden: 1

$$\int_0^a |u(x)|^2 dx = |a_1|^2 \cdot \int_0^a dx \left(2 - e^{i\frac{2\pi}{a}nx} - e^{-i\frac{2\pi}{a}nx}\right)$$

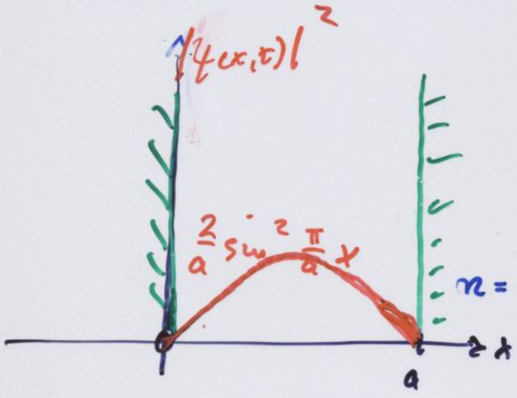
$$= |a_1|^2 \cdot 2a \overset{!}{=} 1$$

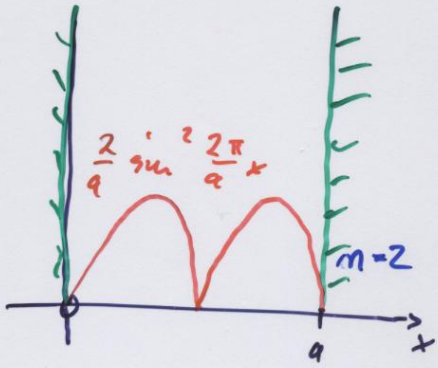
$$\Rightarrow a_1 = \frac{1}{\sqrt{2a}}$$

a. Wellengleichung

$$\Rightarrow \psi(x,t) = \frac{1}{\sqrt{2a}} \left(e^{in\frac{\pi}{a}x} - e^{-in\frac{\pi}{a}x}\right) e^{-i\omega t}$$

b. Wahrscheinlichkeitsdichte

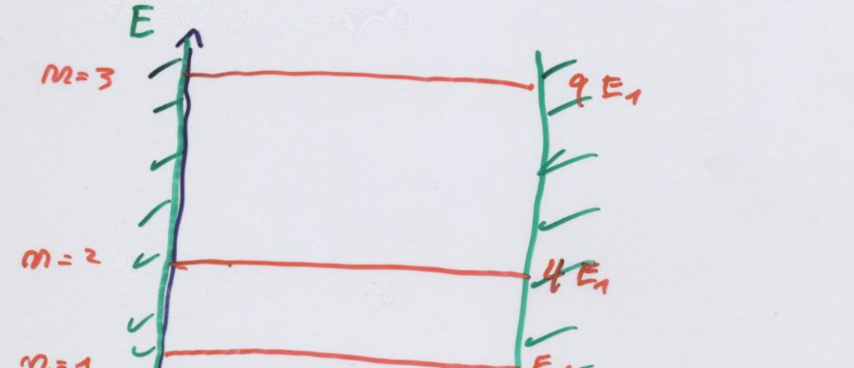




c Energiezustände

$$E = E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}n^2;$$

$n = 1 \ldots \infty$



Niedrigste Energie > 0 !

d) Erfüllt $\Psi(x,t)$ Schrödingergleichung?

$$i\hbar \frac{\partial}{\partial t} \Psi(x,t) = i\hbar \frac{\partial}{\partial t}\left(\frac{1}{\sqrt{2a}}\left(e^{i\frac{n\pi}{a}x} - e^{-i\frac{n\pi}{a}x}\right) \cdot e^{-i\omega t}\right)$$

$$= -i\hbar \cdot i\omega \; \Psi(x,t)$$

$$= +\hbar\omega \; \Psi(x,t) = E\,\Psi(x,t)$$

einerseits

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) = +\frac{\hbar^2}{2m}\frac{n^2\pi^2}{a^2}\Psi(x,t)$$

$$= +\frac{\hbar^2}{2m}k^2 \; \Psi(x,t)$$

$$= E \cdot \Psi(x,t)$$

andererseits

Beispiel:

a) e^- im Kasten von $a = 5\,\text{Å}$ (Atom)

$$E_1 = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2} = 2{,}4 \; 10^{-19}\,\text{J}$$

$$= 1{,}5 \; \text{eV}$$

b) Neutronen im Kern $a = 5\,\text{fm}$ $\quad E_1 = 8\,\text{MeV}$