Blatt 3

9)

a)

Balmer : $n=2$ (sichtbar, deshalb zuerst entdeckt)

Brackett : $n=4$

Lyman : $n=1$

Paschen : $n=3$

Pfund : $n=5$

Wasserstoff : Bindungsenergie $n=1$: $13{,}6\ eV$

b)

Bohrsches Atommodell : $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r}$ (1)

und : $mvr = \frac{nh}{2\pi}$ $(L = n\hbar)$ (2)

$$\Longrightarrow \frac{1}{4\pi\varepsilon_0}\frac{me^2}{r} = (mv)^2 \quad \text{aus (1) : (1a)}$$

$$(mv)^2(2\pi)^2r^2 = (nh)^2 = \frac{1}{4\pi\varepsilon_0}\frac{me^2}{r}(2\pi)^2r^2$$

$$r_n = \frac{n^2h^2 4\pi\varepsilon_0}{(2\pi)^2me^2} = n^2\hbar^2\frac{4\pi\varepsilon_0}{me^2}$$

$$r_n = n^2\cdot 0{,}53\,\text{Å} = n^2a_0 \quad \text{mit} \quad a_0 = \frac{\hbar^2 4\pi\varepsilon_0}{me^2} \quad (3)$$

$$E_n = E_{pot} + E_{kin} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} + \frac{1}{2}mv_n^2$$

$$\underset{\text{aus (2)}}{v_n =} \frac{n\hbar}{mr_n} = \frac{\hbar}{mna_0}$$

$$E_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{n^2a_0} + \frac{1}{2}\frac{\hbar^2}{mn^2a_0^2} \qquad \text{aus (3)}: \frac{ma_0}{\hbar^2} = \frac{4\pi\varepsilon_0}{e^2}$$

$$= -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{1}{n^2} = -2{,}18\cdot 10^{-18}\,J\cdot\frac{1}{n^2} = -13{,}6\,eV\,\frac{1}{n^2}$$

n	r_n	E_n
1	0,53 Å	$-13{,}6$ eV
2	2,12 Å	$-3{,}4$ eV
3	4,77 Å	$-1{,}5$ eV

c)

$$m_\mu = 207\, m_e \mathrel{\hat{=}} \frac{1}{9}\, m_p \qquad m_p = 1836\, m_0$$

$\longrightarrow$ Mitbewegung des Kerns berücksichtigen, reduzierte Masse

$$m^* = \frac{207\, m_e \cdot m_p}{207\, m_e + m_p} = 186\, m_e$$

$$r_n = \frac{4\pi\varepsilon_0 \hbar^2}{e^2 m_e}\, n^2 = a_0\, n^2 \qquad \text{aus b)}$$

$$r_n^\mu = \frac{4\pi\varepsilon_0 \hbar^2}{e^2 m^*}\, n^2 = a_0 \frac{m_e}{m^*}\, n^2 = a_0 \frac{n^2}{186}$$

$$r_1^\mu = 2{,}84 \cdot 10^{-3}\ \text{Å}$$

$$r_2^\mu = 1{,}14 \cdot 10^{-2}\ \text{Å}$$

$$E_n^\mu = -\frac{1}{2}\frac{e^2}{\left(4\pi\varepsilon_0 \frac{a_0}{186}\right)}\ \frac{1}{n^2}$$

$$E_m^\mu - E_n^\mu = -13{,}6\ eV \cdot 186 \left(\frac{1}{m^2} - \frac{1}{n^2}\right) \Rightarrow E_2^\mu - E_1^\mu = -13{,}6\, eV \cdot 186 \cdot \left(\frac{1}{4} - 1\right)$$

$$= 1{,}90\ keV$$

d)

$$e^2 \rightarrow Z e^2$$

$$m_e \rightarrow m^* = \frac{207\, m_e \cdot \overset{A(Ni)}{59} \cdot 1836\, m_e}{207\, m_e + 59 \cdot 1836\, m_e} \approx m_\mu$$

$$\Rightarrow r_1^\mu = \hbar^2 \frac{4\pi\varepsilon_0}{m_\mu Z e^2} = a_0 \frac{1}{Z \frac{m_\mu}{m_0}} = a_0 \cdot \frac{1}{28 \cdot 207} = 9{,}14 \cdot 10^{-15}\, m$$

$$E_1^\mu = -13{,}6\, eV \cdot Z^2 \frac{m_\mu}{m_0} = -2{,}2\, MeV$$

$$(\text{Auf } 4): \; E_n = -\frac{1}{2} \left(\frac{e^2}{4\pi\varepsilon_0 a_0} \right) \frac{1}{n^2} \longrightarrow E_n^\mu = -\frac{1}{2} \left(\frac{Z e^2}{4\pi\varepsilon_0 a_0 \left(\frac{m_0}{m_\mu} \frac{1}{Z} \right)} \right) \frac{1}{n^2}$$

10) $\frac{E}{hc} = - \frac{R_\infty}{1 + \frac{m_e}{m_n}} \frac{Z^2}{n^2}$

1. Linie

$$\Delta \frac{E}{hc} = +4 R_\infty \left(\frac{1}{4^2} - \frac{1}{5^2} \right) \left(\frac{1}{1 + \frac{m_e}{m_{^4He}}} - \frac{1}{1 + \frac{m_e}{m_{^3He}}} \right)$$

$$= 9876{,}33 \ \frac{1}{cm} \cdot 4{,}483 \cdot 10^{-5} = 0{,}4428 \ \frac{1}{cm}$$

$$\Delta E = 5{,}5 \cdot 10^{-5} \ eV$$

3. Linie

$$\Delta \frac{E}{hc} = 4 R_\infty \left(\frac{1}{4^2} - \frac{1}{7^2} \right) \left(\frac{1}{1 + \frac{m_e}{m_{^4He}}} - \frac{1}{1 + \frac{m_e}{m_{^3He}}} \right)$$

$$= 0{,}828 \ \frac{1}{cm}$$

$$\Delta E = 1{,}03 \cdot 10^{-4} \ eV$$

11) a) $Z = 2$, berechne a_n und $b_{n,k}$ mit $k = 1, 2, \ldots n$

$n = 1$ $\quad a_1 = \frac{a_0}{2}$ $\quad b_{1,1} = \frac{a_0}{2} = a_1$

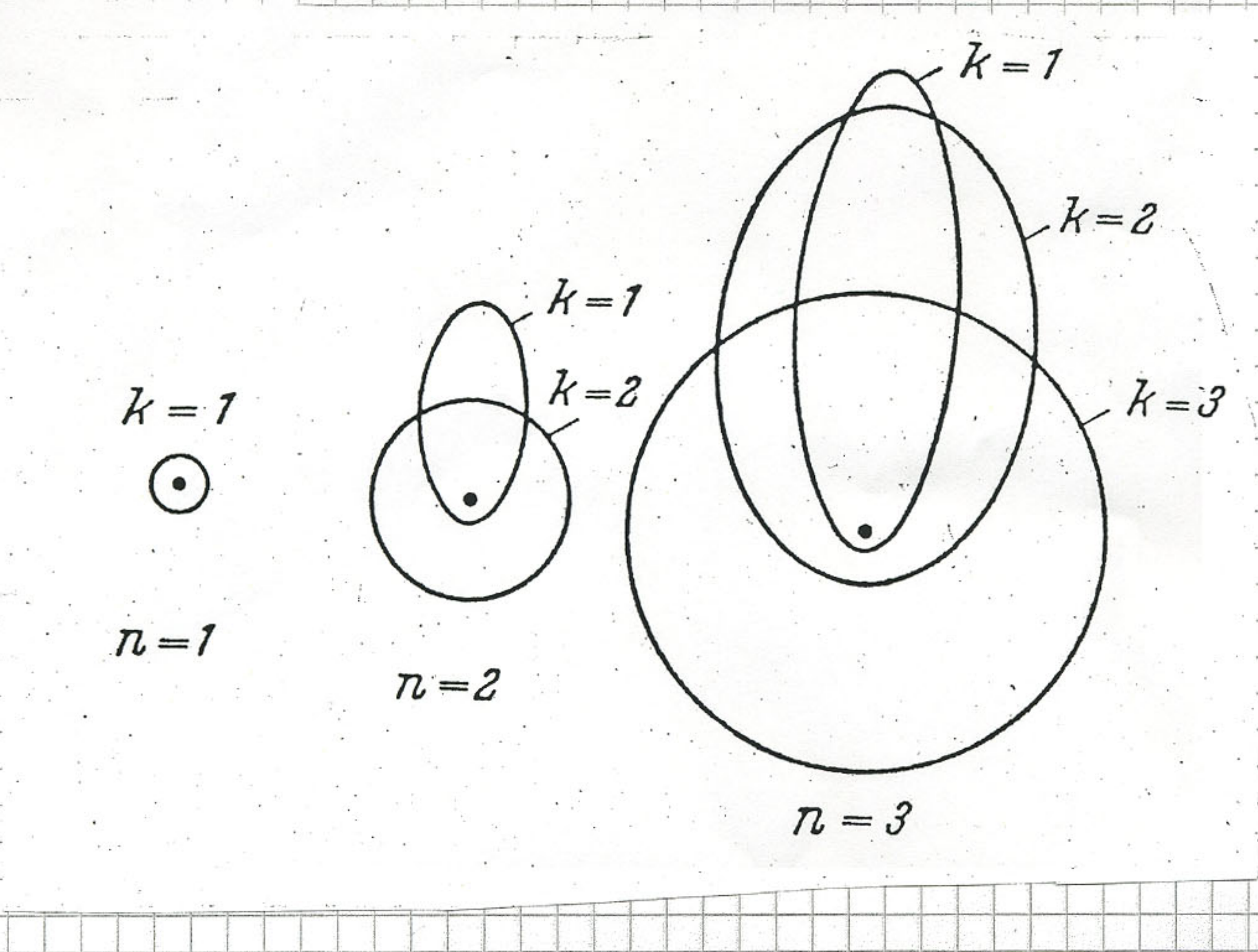
$n = 2$ $\quad a_2 = 2a_0$ $\quad b_{2,1} = a_0$

$b_{2,2} = 2a_0 = a_2$

$n = 3$ $\quad a_3 = \frac{9}{2} a_0$ $\quad b_{3,1} = \frac{3}{2} a_0$

$b_{3,2} = 3a_0$

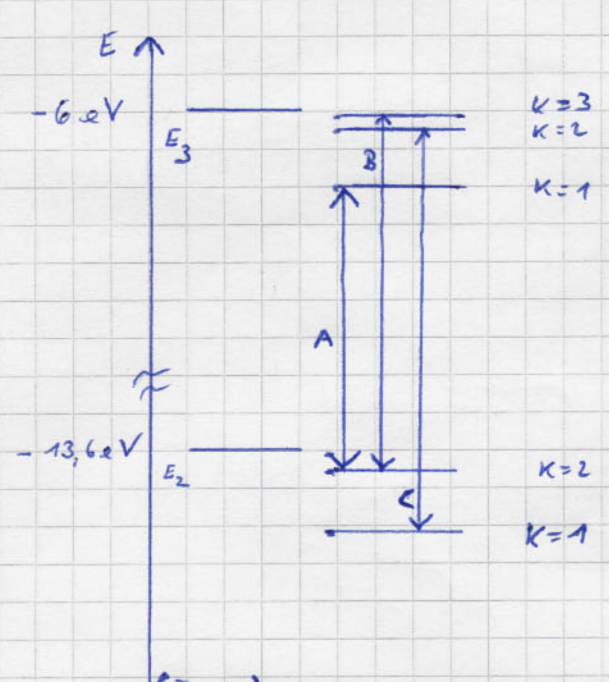
$b_{3,3} = \frac{9}{2} a_0 = a_3$



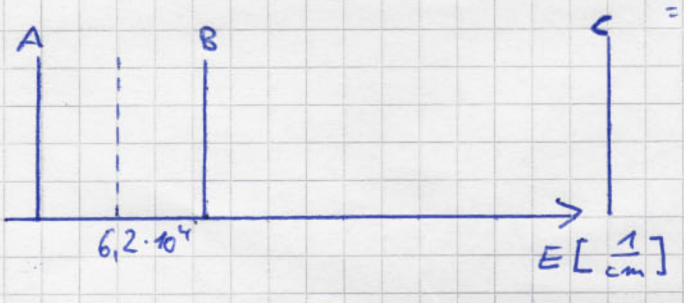
b)

n	k	$E_n = -hc \dfrac{R_\infty}{1+\frac{m_e}{m_K}} \dfrac{Z^2}{n^2}$	$\left(\dfrac{n}{k}-\dfrac{3}{4}\right)\dfrac{\alpha^2 Z^2}{n^2}$	$E_{n,k}\ \left[\frac{1}{cm}\right]$*
1	1	$-54,4\ eV$	$2,13\cdot 10^{-4}$	$43,8\cdot 10^{4}$
2	1	$-13,6\ eV$	$6,66\cdot 10^{-5}$	$11,0\cdot 10^{4}$
2	2	$-13,6\ eV$	$1,33\cdot 10^{-5}$	
3	1	$-6,0\ eV$	$5,33\cdot 10^{-5}$	$4,8\cdot 10^{4}$
3	2	$-6,0\ eV$	$1,77\cdot 10^{-5}$	
3	3	$-6,0\ eV$	$0,59\cdot 10^{-5}$	

* In der Spektroskopie gibt man häufig die Termwerte in Wellenzahlen $\bar{\nu} = \frac{E}{hc}$ an. $\frac{E}{\bar{\nu}}\ hc = 1,98648\cdot 10^{-23}\ \frac{J}{cm^{-1}}$



$$\frac{(E_3 - E_2)}{hc} = 6,2\cdot 10^4\ \frac{1}{cm}$$



c) Übergänge mit $\Delta k = \pm 1$

A: $E_{3,1} - E_{2,2}$

$$= -4,8\cdot 10^4 \left(1 + 5,33\cdot 10^{-5}\right)\frac{1}{cm}$$
$$+ 11,0\cdot 10^4\left(1 + 1,33\cdot 10^{-5}\right)\frac{1}{cm}$$
$$= 6,2\cdot 10^4\ \frac{1}{cm} - 1,1\ \frac{1}{cm}$$

B: $E_{3,3} - E_{2,2}$

$$= 6,2\cdot 10^4\ \frac{1}{cm} + 1,2\ \frac{1}{cm}$$

C: $E_{3,2} - E_{2,1}$

$$= 6,2\cdot 10^4\ \frac{1}{cm} + 6,5\ \frac{1}{cm}$$