

12)

$$\Delta x \cdot \Delta p \geq \hbar \rightarrow \Delta x \geq \frac{\hbar}{\Delta p}$$

$$\text{hier: } \frac{\Delta p}{p} = 10^{-3}$$

$$\Delta p = 10^{-3} p$$

Tischtennisball:

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{10^{-3} p} = \frac{\hbar}{10^{-3} m v} = 1,054 \cdot 10^{-29} \text{ m}$$

Elektron (relativistisch)

$$p = m v = m_0 \frac{\beta}{\sqrt{1-\beta^2}} c$$

$$\text{mit } \beta = \frac{v}{c} = 0,6$$

$$\Delta x \geq \frac{\hbar}{10^{-3} m_0 \frac{\beta}{\sqrt{1-\beta^2}} c} = 5,15 \cdot 10^{-10} \text{ m}$$

13) a)

$$\langle \vec{L}^2 \rangle = l(l+1) \hbar^2$$

$$\text{mit } l = 0, 1, 2, \dots, n-1$$

$$\langle L_z \rangle = m \hbar$$

$$\text{mit } m = 0, \pm 1, \pm 2, \dots, \pm l$$

$$n=5$$

$$l=0$$

$$m=0$$

$$l=1$$

$$m=0, \pm 1$$

$$l=2$$

$$m=0, \pm 1, \pm 2$$

$$l=4$$

$$m=0, \pm 1, \pm 2, \pm 3, \pm 4$$

$$n=6$$

$$l=0$$

$$m=0$$

$$\vdots$$

$$l=5$$

$$m=0, \pm 1, \pm 2, \dots, \pm 5$$

$$b) R_{1,0}(r) = 2 \left(\frac{Z}{a_0} \right)^{3/2} e^{-\frac{Zr}{a_0}} \stackrel{Z=1}{=} 2 \left(\frac{1}{a_0} \right)^{3/2} e^{-\frac{r}{a_0}}$$

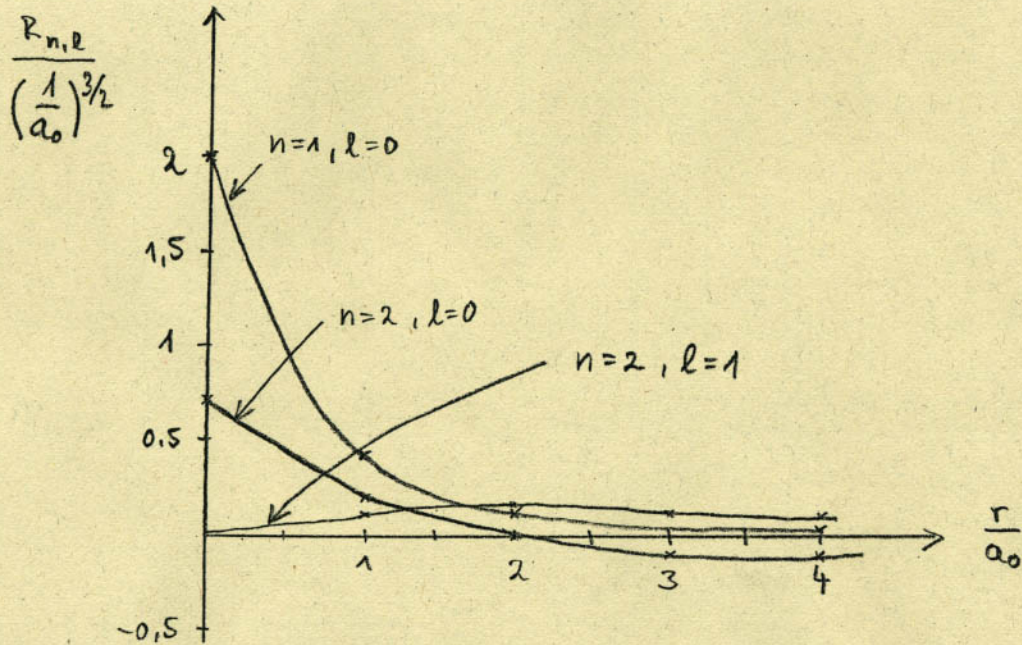
$\Rightarrow$ keine Nullstelle!

$$R_{2,0}(r) \stackrel{Z=1}{=} 2 \left(\frac{1}{2a_0} \right)^{3/2} \left(1 - \frac{r}{2a_0} \right) \cdot e^{-\frac{r}{2a_0}}$$

$$\text{Nullstelle } r_0 = 2a_0$$

$$R_{2,1} = \frac{2}{\sqrt{3}} \left(\frac{1}{2a_0} \right)^{3/2} \frac{r}{2a_0} e^{-\frac{r}{2a_0}}$$

$\Rightarrow$ Nullstelle $r_0 = 0$



c) $\int r^2 R_{n,l}^2(r) dr \hat{=} \text{Aufenthaltswahrscheinlichkeit des Elektrons in einer Kugelschale von Radius } r \text{ und Dichte } dr$

$$n=1$$

$$\int_0^{\infty} r^2 R_{n,l}^2(r) dr = \int_0^{\infty} r^2 R_{1,0}^2(r) dr$$

muß natürlich 1 ergeben
(Normierung: irgendwo ist das Elektron)

Rechnung:

$$= \int_0^{\infty} r^2 4 \left(\frac{1}{a_0} \right)^3 e^{-\frac{2r}{a_0}} dr$$

$$= 4 \left(\frac{1}{a_0} \right)^3 \int_0^{\infty} r^2 e^{-\frac{2}{a_0} r} dr$$

Bronstein

$$\rightarrow 4 \left(\frac{1}{a_0} \right)^3 \left[e^{-\frac{2}{a_0} r} \cdot \left(\frac{r^2}{-\frac{2}{a_0}} - \frac{2r}{\frac{4}{a_0^2}} + \frac{2}{-\frac{8}{a_0^3}} \right) \right] \Big|_0^{\infty}$$

$$= \frac{4}{a_0^3} \cdot \left(0 - \frac{2}{-\frac{8}{a_0^3}} \right) = 1$$

d)

$$n=1$$

$$r^2 R_{1,0}^2(r) = r^2 \frac{4}{a_0^3} e^{-\frac{2r}{a_0}}$$

$$\frac{d}{dr} \left(r^2 R_{1,0}^2(r) \right)$$

$$= \frac{4}{a_0^3} \left(2r e^{-\frac{2r}{a_0}} - \frac{2r^2}{a_0} e^{-\frac{2r}{a_0}} \right) \stackrel{!}{=} 0$$

$$\Rightarrow r_1 = 0, r_2 = a_0$$

$$n=2, l=0$$

$$\frac{d}{dr} \left(r^2 R_{2,0}'(r) \right) = \frac{d}{dr} \left(r^2 \frac{1}{2a_0^3} \left(1 - \frac{r}{2a_0} \right)^2 e^{-\frac{r}{a_0}} \right)$$

$$= \frac{1}{2a_0^3} \left\{ 2 \left(r - \frac{r^2}{2a_0} \right) \left(1 - \frac{2r}{2a_0} \right) - \frac{1}{a_0} \left(r - \frac{r^2}{2a_0} \right)^2 \right\} e^{-\frac{r}{a_0}} = \frac{d}{dr} \left(\frac{1}{6a_0^3} \frac{r^4}{4a_0} e^{-\frac{r}{a_0}} \right)$$

$$= \frac{1}{2a_0^3} \left\{ \left(r - \frac{r^2}{2a_0} \right) \left(2 - \frac{2r}{a_0} - \frac{r}{a_0} + \frac{r^2}{2a_0^2} \right) \right\} e^{-\frac{r}{a_0}} = \frac{1}{24a_0^5} \left(4r^3 - \frac{r^4}{a_0} \right) e^{-\frac{r}{a_0}}$$

$$= \frac{1}{24a_0^5} \left\{ \left(r - \frac{r^2}{2a_0} \right) \left(2 - \frac{3r}{a_0} + \frac{r^2}{2a_0^2} \right) \right\} e^{-\frac{r}{a_0}} \stackrel{!}{=} 0 \quad \stackrel{!}{=} 0$$

$$\Rightarrow r - \frac{r^2}{2a_0} = 0 \Rightarrow r_1 = 0, r_2 = 2a_0$$

$$r^2 - 6a_0 r + 4a_0^2 = 0 \quad r_{1/2} = a_0 (3 \pm \sqrt{5})$$

$$n=2, l=1$$

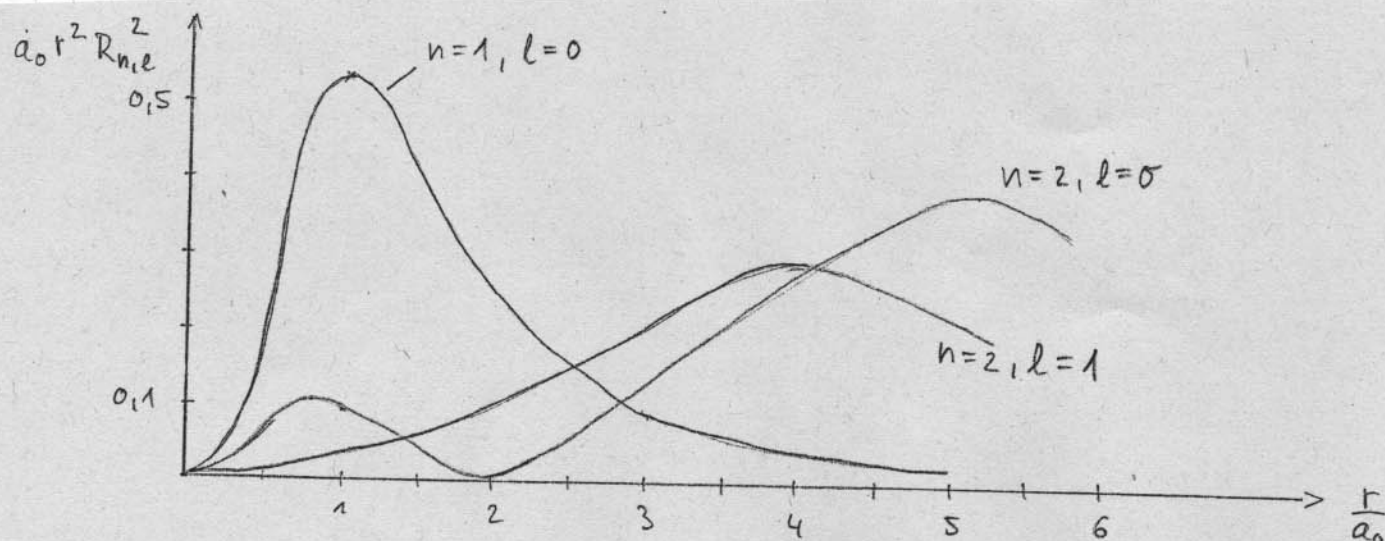
$$\frac{d}{dr} \left(r^2 R_{2,1}'(r) \right)$$

$$= \frac{d}{dr} \left(\frac{1}{6a_0^3} \frac{r^4}{4a_0} e^{-\frac{r}{a_0}} \right)$$

$$= \frac{1}{24a_0^5} \left(4r^3 - \frac{r^4}{a_0} \right) e^{-\frac{r}{a_0}}$$

$$\stackrel{!}{=} 0$$

$$\Rightarrow r_1 = 0 \quad r_2 = 4a_0$$



Keine scharfen Werte im Gegensatz zu Bohr ($r_n = a_0 n^2$)

14) $1s : n=1, l=0, m=0$

$$\Psi_{n,l,m} = R_{n,l}(r) Y_{l,m}(\vartheta, \varphi)$$

$$\langle \vec{r} \rangle = \iiint \vec{r} |\Psi_{n,l,m}|^2 dV = \underline{\underline{0}} \quad \text{für alle } \Psi_{n,l,m},$$

da $|\Psi|^2$ gerade und $\vec{r}(-x, -y, -z) = -\vec{r}(x, y, z)$ ungerade.

$$\langle r \rangle_{1,0} = \int_0^\infty \int_0^\pi \int_0^{2\pi} r \underbrace{\left(\frac{1}{\sqrt{4\pi}} \right)}_{Y_{0,0}} \underbrace{\cdot 2 \left(\frac{1}{a_0} \right)^{3/2} e^{-\frac{r}{a_0}}}_{R_{1,0}})^2 r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= \frac{1}{4\pi} \cdot 4 \cdot \frac{1}{a_0^3} \int_0^\infty r^3 e^{-\frac{2r}{a_0}} dr \underbrace{\int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\varphi}_{4\pi}$$

Klar, $Y_{l,m}$ sind normiert.

$$= \frac{4}{a_0^3} \int_0^\infty r^3 e^{-\frac{2r}{a_0}} dr = \underline{\underline{\frac{3}{2} a_0}}$$

mit $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$
und $\int x^2 e^{ax} dx =$

$$= e^{ax} \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right)$$

$$\Rightarrow \int \dots = \frac{3}{8} a_0^4$$

$\langle r^2 \rangle$: Betrachte nur Radialteil, da $Y_{l,m}$ normiert (s.o.)

$$\begin{aligned}\langle r^2 \rangle_{1,0} &= \int_0^\infty r^2 \left(2 \cdot \frac{1}{a_0^{3/2}} e^{-\frac{r}{a_0}} \right)^2 r^2 dr \\ &= \frac{4}{a_0^3} \underbrace{\int_0^\infty r^4 e^{-\frac{2r}{a_0}} dr}_{\frac{3}{4} a_0^5} = \underline{\underline{3 a_0^2}}\end{aligned}$$

15) Tröpfchenmodell: $r_K \approx 1,3 \cdot 10^{-15} \text{ m} \cdot \sqrt[3]{A}$

$$r_H = 1,3 \text{ fm} =$$

$$r_{Na} = 3,7 \text{ fm}$$

Aufenthaltswahrscheinlichkeit $\mathcal{P} = \iiint_{V_K} \psi^* \psi dV = \int_0^{r_K} r^2 R_{n,l}^2(r) dr$
 $\uparrow$
 $Y_{l,m}$ normiert (s.o.)

Annahme: $R_{n,0}(r) = \text{const.} = R_{n,0}(0)$ für $r \leq r_K$, s-Elektronen: $l=0$

$$\Rightarrow \mathcal{P}_K = R_{n,0}^2(0) \int_0^{r_K} r^2 dr = \frac{1}{3} r_K^3 R_{n,0}^2(0)$$

$$R_{1,0}(0) = 2 \left(\frac{Z}{a_0} \right)^{3/2}$$

$$R_{2,0}(0) = 2 \left(\frac{Z}{2a_0} \right)^{3/2}$$

$$R_{3,0}(0) = 2 \left(\frac{Z}{3a_0} \right)^{3/2}$$

$$R_{1,0}^2(0) = 4 \left(\frac{Z}{a_0} \right)^3$$

$$R_{2,0}^2(0) = \frac{1}{2} \left(\frac{Z}{a_0} \right)^3$$

$$R_{3,0}^2(0) = \frac{4}{27} \left(\frac{Z}{a_0} \right)^3$$

mit $Z=1$ für H
 $Z=11$ für Na

	1s	2s	3s
$\mathcal{P}_H$	$1,98 \cdot 10^{-14}$	$2,47 \cdot 10^{-15}$	$7,33 \cdot 10^{-16}$
$\mathcal{P}_{Na}^{10+}$	$6,07 \cdot 10^{-10}$	$7,59 \cdot 10^{-11}$	$2,25 \cdot 10^{-11}$