

21) a) $n=2$ $l=1$ $s=\frac{1}{2}$

$\Rightarrow j = \frac{3}{2} = l + s$ oder

$m_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ (4-fach entartet)

$j = \frac{1}{2} = l - s$

$m_j = \frac{1}{2}, -\frac{1}{2}$ (2-fach entartet)

b) $W_{LS} = \lambda \vec{L} \cdot \vec{S}$

$\lambda = \frac{e^2 \mu_0}{8 \pi m_e} \langle r^{-3} \rangle$

(Faktor $\frac{1}{2}$ aus Thomas Korrektur)

- Berechnen $\langle r^{-3} \rangle$

$\psi_{n,l,m} = R_{n,l}(r) Y_{l,m}(\theta, \varphi)$

Integration über die normierten $Y_{l,m}(\theta, \varphi)$ ergibt 1, es bleibt

$\langle r^{-3} \rangle = \int r^{-3} R_{n,l}^2(r) r^2 dr$

$= \int \frac{1}{r} R_{n,l}^2(r) dr = \frac{4}{3} \left(\frac{1}{2a_0} \right)^3 \frac{1}{4a_0^2} \underbrace{\int_0^\infty \frac{1}{r} r^2 e^{-\frac{r}{a_0}} dr}_{= a_0^2}$ //

$n=2, l=1, \bar{z}=1$

Braunian $= a_0^2$

$= \frac{1}{3} \cdot \frac{1}{8} \cdot \frac{1}{a_0^3} = \frac{1}{24} \left(\frac{1}{a_0} \right)^3$

- Umschreiben von $\vec{L} \cdot \vec{S}$

$\vec{J} = \vec{L} + \vec{S}$

$\vec{J}^2 = \vec{L}^2 + \vec{S}^2 + 2 \vec{L} \cdot \vec{S} \Rightarrow \vec{L} \cdot \vec{S} = \frac{1}{2} (\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$

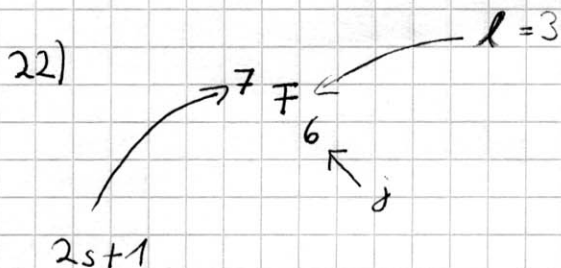
$= \frac{1}{2} \hbar^2 (j(j+1) - l(l+1) - s(s+1))$

$W_{LS} = \underbrace{\frac{e^2 \mu_0}{8 \pi m_e} \frac{1}{24} \left(\frac{1}{a_0} \right)^3 \frac{1}{2} \hbar^2 (j(j+1) - l(l+1) - s(s+1))}_{\frac{\lambda \hbar^2}{2}}$

$$j = \frac{3}{2} \quad W_{\frac{3}{2}} = \frac{\lambda \hbar^2}{2} \left(\frac{15}{4} - 2 - \frac{3}{4} \right) = \frac{\lambda \hbar^2}{2}$$

$$j = \frac{1}{2} \quad W_{\frac{1}{2}} = \frac{\lambda \hbar^2}{2} \left(\frac{3}{4} - 2 - \frac{3}{4} \right) = -2 \frac{\lambda \hbar^2}{2}$$

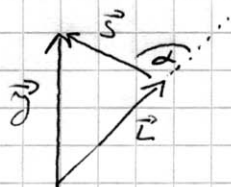
$$\Delta E = W_{\frac{3}{2}} - W_{\frac{1}{2}} = \frac{3}{2} \lambda \hbar^2 = 7,26 \cdot 10^{-24} \text{ J}$$



$$2s+1=7 \rightarrow s=3$$

$$l=3$$

$$j=6 (=l+s)$$



$$\vec{S} \cdot \vec{L} = |\vec{S}| |\vec{L}| \cdot \cos \alpha$$

$$\vec{S} \cdot \vec{L} = \frac{1}{2} (\vec{j}^2 - \vec{L}^2 - \vec{S}^2)$$

$$\cos \alpha = \frac{\frac{1}{2} [j(j+1) - l(l+1) - s(s+1)]}{\sqrt{s(s+1)} \sqrt{l(l+1)}}$$

$$= \frac{1}{2} \frac{6 \cdot 7 - 2 \cdot 3 \cdot 4}{3 \cdot 4} = \frac{3}{4}$$

$$\Rightarrow \alpha = 41,4^\circ$$

23)

$$a) \quad g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

$$3p_{\frac{1}{2}} \quad j = \frac{1}{2} \quad l = 1 \quad s = \frac{1}{2} \quad \Rightarrow g = \frac{2}{3}$$

$$3s_{\frac{1}{2}} \quad j = \frac{1}{2} \quad l = 0 \quad s = \frac{1}{2} \quad \Rightarrow g = 2 \quad (\text{reiner Spinnmagnetismus})$$

$$3p_{\frac{3}{2}} \quad j = \frac{3}{2} \quad l = 1 \quad s = \frac{1}{2} \quad \Rightarrow g = \frac{4}{3}$$

b)

$$3p_{\frac{1}{2}} \rightarrow 3s_{\frac{1}{2}} : \lambda_{p_{\frac{1}{2}}} = 5895,92 \text{ \AA}$$

$$\frac{1}{\lambda_{p_{\frac{1}{2}}}} = 16960,88 \frac{1}{\text{cm}}$$

$$3p_{\frac{3}{2}} \rightarrow 3s_{\frac{1}{2}} : \lambda_{p_{\frac{3}{2}}} = 5889,95 \text{ \AA}$$

$$\frac{1}{\lambda_{p_{\frac{3}{2}}}} = 16978,07 \frac{1}{\text{cm}}$$

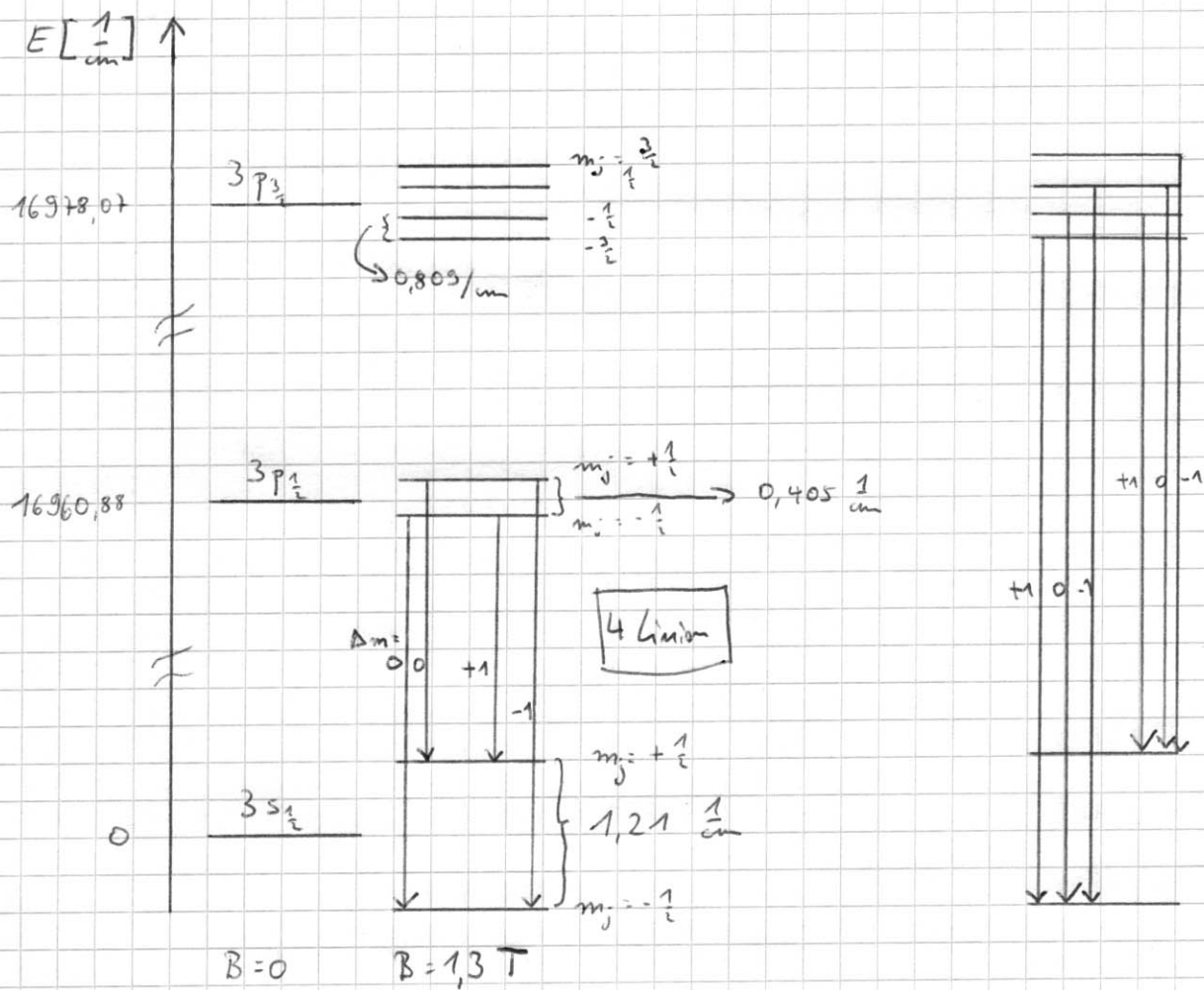
$$E_{\text{Zeeman}} = -\langle \mu_z \rangle B_z = + g_j m_j \mu_B B_z$$

$$\Delta E_{\text{Zeeman}} = g_j \mu_B B_z \quad \frac{\mu_B B_z}{h c} = 0,607 \frac{1}{\text{cm}} \quad (B_z = 1,3 \text{ T})$$

$$\Delta E_{p_{\frac{1}{2}}} = \frac{2}{3} \cdot 0,607 \frac{1}{\text{cm}} = 0,405 \frac{1}{\text{cm}}$$

$$\Delta E_{p_{\frac{3}{2}}} = \frac{4}{3} \cdot 0,607 \frac{1}{\text{cm}} = 0,809 \frac{1}{\text{cm}}$$

$$\Delta E_{s_{\frac{1}{2}}} = 2 \cdot 0,607 \frac{1}{\text{cm}} = 1,21 \frac{1}{\text{cm}}$$



c) Übergänge: $3p_{1/2} \rightarrow 3s_{1/2}$

$$\Delta E = \underbrace{E_{p_{1/2}} + \frac{2}{3} \mu_B B m_j}_{\text{obere Niveaus}} - \underbrace{(E_{s_{1/2}} + 2 \mu_B B m_j')}_{\text{untere Niveaus}} = \Delta E_{p_{1/2}} + \underbrace{0,405 \frac{1}{\text{cm}} m_j - 1,21 \frac{1}{\text{cm}} m_j'}_{\Delta E^*}$$

m_j	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
m_j'	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
$\Delta E^* [\frac{1}{\text{cm}}]$	0,403	-0,808	+0,808	-0,403
Δm	0	+1	-1	0

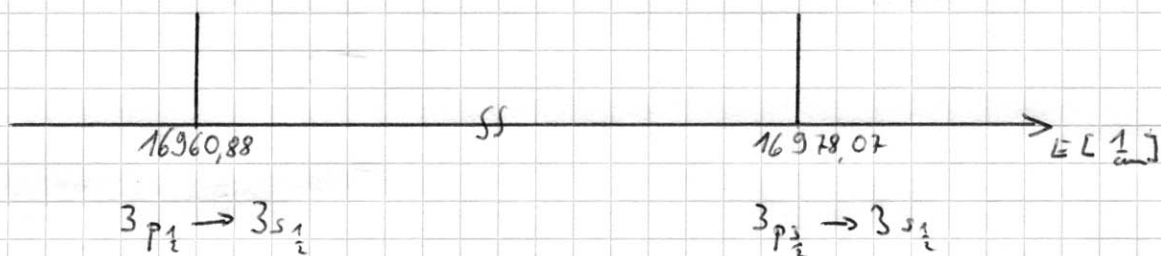
Übergang $3p_{3/2} \rightarrow 3s_{1/2}$

$$\Delta E = E_{p_{3/2}} + \frac{4}{3} \mu_B B m_j - (E_{s_{1/2}} + 2 \mu_B B m_j)$$

$$= \Delta E_{p_{3/2}} + \underbrace{0,809 \frac{1}{\text{cm}} m_j - 1,21 \frac{1}{\text{cm}} m_j}_{\Delta E^*}$$

m_j	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
m_j'	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\Delta E^* [\frac{1}{\text{cm}}]$	0,609	-0,201	-1,01	+1,01	+0,201	-0,609
Δm	-1	0	+1	-1	0	+1

$B=0$, Aufspaltung aufgrund Spin-Bahn-Kopplung



$B=1,3\text{T}$, anomale Zeemaneffekt ($\langle \omega_L \rangle \gg \langle \omega_{\text{Zeeman}} \rangle$)



d) Paschen-Back-Effekt

$E_{LS} \ll E_{SB}, E_{LB}$, dann ist Spin-Bahn-Kopplung aufgehoben, d.h. Spins magnetismus ($g=2$) und Bahnmagnetismus ($g=1$) werden getrennt betrachtet.

Auswahlregeln: $\Delta m_l = 0, \pm 1$
 $\Delta m_s = 0$ für optische Übergänge

$$E_{zu} = - \langle \mu_z \rangle B_z$$

$$E_{P.B.} = 1 \mu_B B \frac{\langle l_z \rangle}{\hbar} + 2 \mu_B B \frac{\langle s_z \rangle}{\hbar}$$

$$= \mu_B B (m_l + 2 m_s)$$

Keine Kopplung zu j mehr, d.h. $3p \rightarrow 3s$

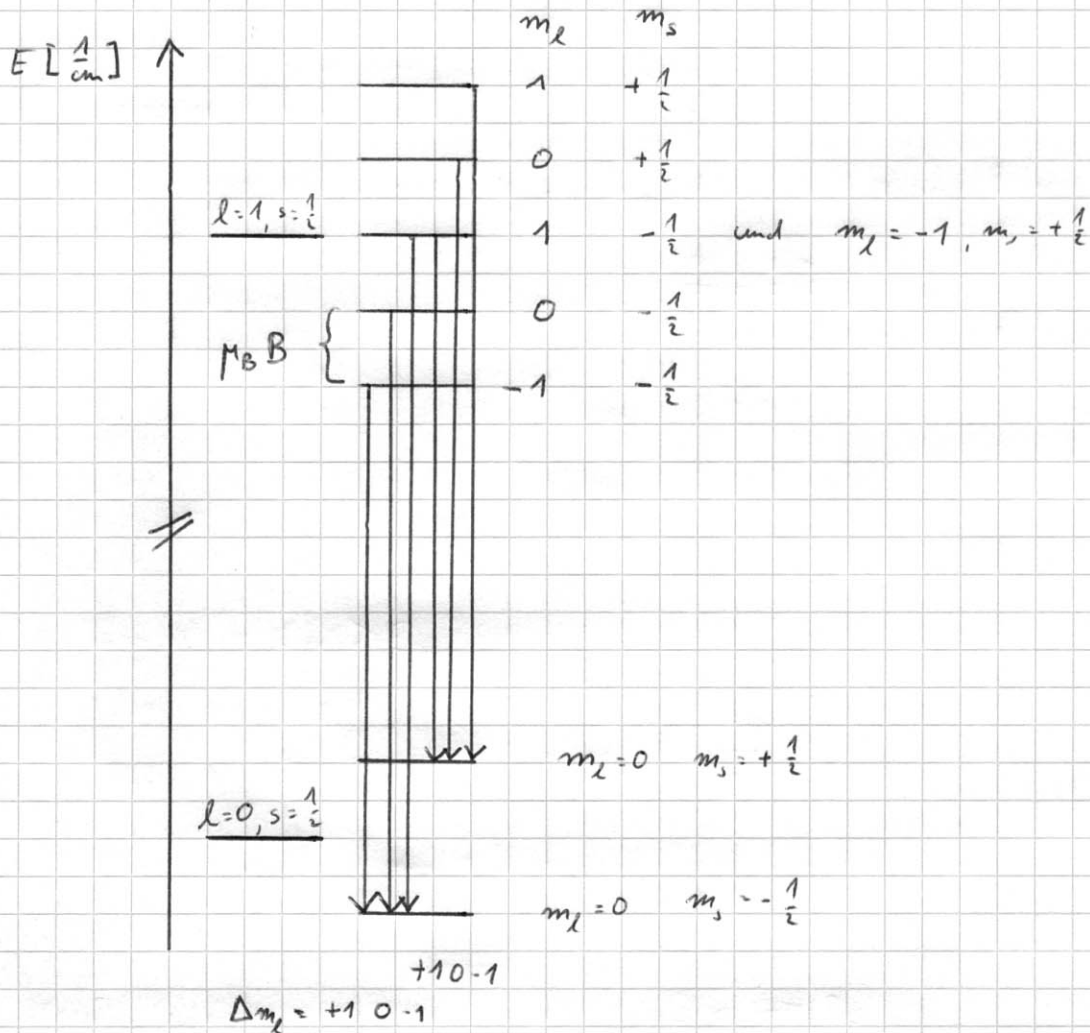
3p: $l=1, s=\frac{1}{2}$

m_l	0	0	1	1	-1	-1
m_s	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$
$\frac{E_{P.B.}}{\mu_B}$	1	-1	2	0	0	-2

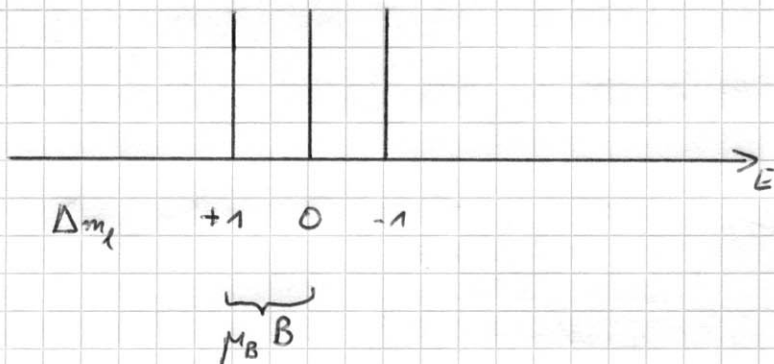
$\underbrace{\hspace{10em}}$
 fallen zusammen
 (Entartung)

3s: $l=0, s=\frac{1}{2}$

m_l	0	0
m_s	$+\frac{1}{2}$	$-\frac{1}{2}$
$\frac{E_{P.B.}}{\mu_B}$	1	-1



Paschen-Back-Effekt: nur noch 3 Linien:



$$\text{Aus } E_{LS} \ll E_{SB} \rightarrow E_{LS} \ll \mu_B B$$

$$E_{LS} = E_{P_{\frac{3}{2}}} - E_{P_{\frac{1}{2}}} = 16978,07 \frac{1}{\text{cm}} - 16960,88 \frac{1}{\text{cm}} = 17,19 \frac{1}{\text{cm}} \approx 3,4 \cdot 10^{-22} \text{ J}$$

$$\mu_B B \gg 3,4 \cdot 10^{-22} \text{ J}$$

$$B \gg 36,7 \text{ T}$$

$$24) \quad \vec{F} = \vec{J} + \vec{S}$$

$$W_{HF} = A \cdot \vec{S} \cdot \vec{J}$$

$$a) \quad F = J - S, J - S + 1, \dots, J + S$$

$$\text{mit } J = \frac{7}{2} \quad \text{und } S = \frac{3}{2} : \quad F = 2, 3, 4, 5$$

$$b) \quad \langle W_{HF} \rangle = A \cdot \langle \vec{S} \cdot \vec{J} \rangle \quad \text{analog zu Spin-Bahn-Kopplung:}$$

$$W_{HF} = \frac{1}{2} A h^2 (F(F+1) - S(S+1) - J(J+1))$$

Abstand benachbarter Niveaus:

$$\Delta W_{HF} = W_{HF}(F) - W_{HF}(F-1) = \frac{1}{2} A h^2 [F(F+1) - (F-1) \cdot F]$$

$$= A h^2 F$$

$$\frac{W_{HF}(F)}{A h^2} = \frac{1}{2} \left(F(F+1) - \frac{15}{4} - \frac{63}{4} \right) = \frac{1}{2} F(F+1) - \frac{39}{4}$$

F	$\frac{W_{HF}}{A h^2}$	$\nu = \frac{\Delta W_{HF}}{h} = \frac{A h^2}{h} F$
2	$3 - \frac{39}{4} = -\frac{27}{4}$	$\frac{3 \cdot 10^{-26} \text{ J}}{6,626 \cdot 10^{-34} \text{ J s}} \cdot 3 = 136 \text{ MHz}$ 181 MHz 226 MHz
3	$6 - \frac{39}{4} = -\frac{15}{4}$	
4	$10 - \frac{39}{4} = +\frac{1}{4}$	
5	$15 - \frac{39}{4} = +\frac{21}{4}$	

$$J = \frac{7}{2}, S = \frac{3}{2}$$

