# Blatt 5

16)

a) Nur Winkelabhängigkeit : $Y_{l,m}(\theta,\varphi)$ , $l=2$, $\varphi=0$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1)$$

$\theta=0$ : $3\cos^2\theta - 1 = 2$
$\theta=90^\circ$ : $3\cos^2\theta - 1 = -1$
$3\cos^2\theta - 1 = 0$ : $\theta = 54{,}74^\circ$

$$Y_{2,\pm1} = \sqrt{\frac{15}{8\pi}}\,\cos\theta\sin\theta\, e^{\pm i\varphi}$$

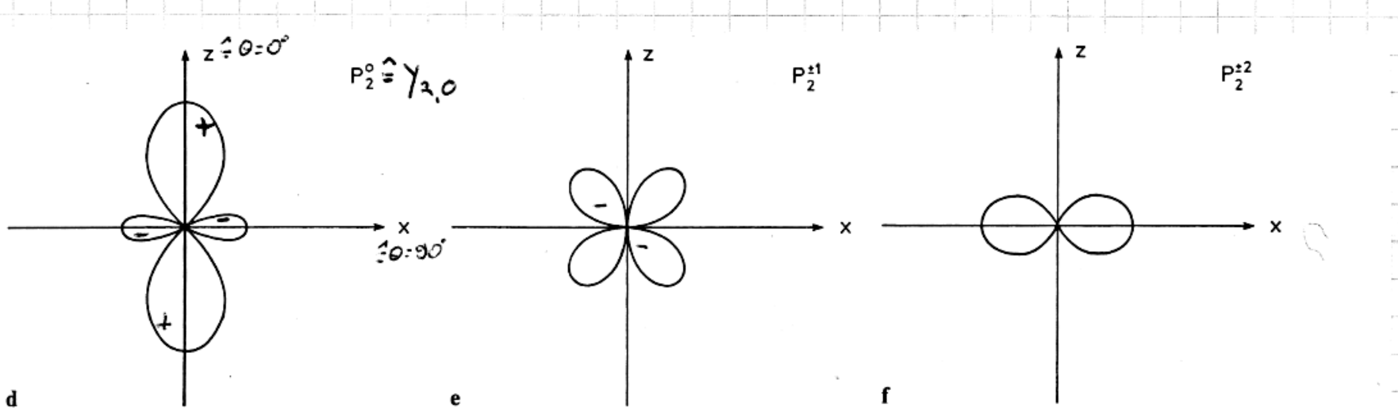
$=0$ für $\theta=0$

$=0$ für $\theta=90^\circ$

$=\frac{1}{2}$ für $\theta=45^\circ$

$=\frac{\sqrt{3}}{4} = 0{,}43$ für $\theta=30^\circ$

$$Y_{2,\pm2} = \sqrt{\frac{15}{32\pi}}\,\sin^2\theta\; e^{\pm i2\varphi}$$




aus Haken, Wolf: „Atom- und Quantenphysik“

b)

Kugelsymmetrisch, ungleich von Null für $r \Rightarrow 0$ : s-Orbital $(l=0)$

Aus Vorlesung: Anzahl nicht-trivialer Nullstellen:

$$n-(l+1) \overset{!}{=} 4 \implies n=5$$

$\implies$ 5s-Orbital

17) $L_z \hat{=} \frac{\hbar}{i}\partial_\varphi$

$$L_z\psi = \frac{\hbar}{i}\partial_\varphi\left[A r^2 e^{\frac{-r}{3a_0}} \sin\Theta\cos\Theta\, e^{i\varphi}\right]$$

$$= \frac{\hbar}{i}\, i\psi = \hbar\psi \implies \underline{\underline{m=1}},$$

$$\left[\text{denn } \langle L_z\rangle = \iiint \psi^* L_z \psi\, dV = m\hbar \underset{\text{s.o.}}{=} \iiint \psi^* \hbar\psi\, dV = \hbar \iiint \psi^*\psi\, dV = \hbar\right]$$

$$\langle \vec{L}^2 \rangle = l(l+1)\hbar^2$$ (nicht $\vec{L}\psi$, da nur Betrag und eine Komponente bestimmbar)

wie oben:

$$\vec{L}^2\psi = -\hbar^2\left(\underbrace{\frac{1}{\sin\Theta}\partial_\Theta(\sin\Theta\,\partial_\Theta)}_{1.\text{ Teil}} + \underbrace{\frac{1}{\sin^2\Theta}\partial^2_\varphi}_{2.\text{ Teil}}\right)\psi$$

$\vec{L}^2$ in Kugelkoordinaten, z.B. aus Nolting „QM 2"

1. Teil: $\frac{1}{\sin\Theta}\partial_\Theta(\sin\Theta\,\partial_\Theta)\sin\Theta\cos\Theta$

$$= \frac{1}{\sin\Theta}\partial_\Theta \sin\Theta\left(\cos^2\Theta - \sin^2\Theta\right)$$

$$= \frac{\cos\Theta}{\sin\Theta}\left(\cos^2\Theta - \sin^2\Theta\right) - 4\cos\Theta\sin\Theta$$

2. Teil $\frac{1}{\sin^2\Theta}\partial^2_\varphi \sin\Theta\cos\Theta\, e^{i\varphi} = -\frac{\cos\Theta}{\sin\Theta}e^{i\varphi}$

$$\Rightarrow \vec{L}^2\psi = A r^2 e^{-\frac{r}{3a_0}}\left[\frac{\cos\Theta}{\sin\Theta}\left(1-2\sin^2\Theta\right) - 4\cos\Theta\sin\Theta - \frac{\cos\Theta}{\sin\Theta}\right]e^{i\varphi}(-\hbar^2)$$

$$= 6\hbar^2\psi \overset{!}{=} l(l+1)\hbar^2\psi \rightarrow \underline{\underline{l=2}}$$

„$e^{-\frac{r}{3a_0}}$" $\implies n=3$

18) a) <u>orthogonal</u>: mit $\iint Y^*_{l,m}(u,\varphi)\, Y_{l',m'}(u,\varphi)\, du\, d\varphi = \delta_{l,l'}\,\delta_{m,m'}$

$d_{z^2}$ und $d_{x^2-y^2}$

$$\iint Y^*_{2,0}\, \frac{1}{\sqrt{2}}\,(Y_{2,-2}+Y_{2,2})\, du\, d\varphi = \frac{1}{\sqrt{2}} \iint [Y^*_{2,0}\, Y_{2,-2} + Y^*_{2,0}\, Y_{2,2}]\, du\, d\varphi$$

$$= \frac{1}{\sqrt{2}}\,[\delta_{2,2}\,\cancelto{0}{\delta_{0,-2}} + \delta_{2,2}\,\cancelto{0}{\delta_{0,2}}] = 0 \quad ✓$$

$d_{x^2-y^2}$ und $d_{xy}$

$$\iint \frac{1}{\sqrt{2}}\,(Y^*_{2,-2}+Y^*_{2,2})\left(\frac{i}{\sqrt{2}}\right)(Y_{2,-2}-Y_{2,2})\, du\, d\varphi$$

$$= \iint \frac{i}{2}\,(Y^*_{2,-2}\, Y_{2,-2} - Y^*_{2,2}\, Y_{2,2})\, du\, d\varphi = 0 \quad ✓$$

u.s.w. ...

<u>normiert</u>:

$$\iint Y^*_{2,0}\, Y_{2,0}\, du\, d\varphi = 1 \quad ✓ \text{ (klar)}$$

$$\iint \frac{1}{\sqrt{2}}\,(Y^*_{2,-2}+Y^*_{2,2})\,\frac{1}{\sqrt{2}}\,(Y_{2,-2}+Y_{2,2})\, du\, d\varphi$$

$$= \frac{1}{2}\iint [Y^*_{2,-2}\, Y_{2,2} + Y^*_{2,-2}\, Y_{2,2} + Y^*_{2,2}\, Y_{2,2}]\, du\, d\varphi$$

$$= \frac{1}{2}\cdot[1+0+0+1] = 1 \quad ✓$$

$$\iint -\frac{i}{\sqrt{2}}\,(Y^*_{2,-1}-Y^*_{2,1})\,\frac{i}{\sqrt{2}}\,(Y_{2,-1}-Y_{2,1})\, du\, d\varphi$$

$$= \frac{1}{2}\iint (Y^*_{2,-1}\, Y_{2,-1} + Y^*_{2,1}\, Y_{2,1})\, du\, d\varphi = 1 \quad ✓$$

usw.

b) $\vec{L}^2 \, Y_{l,m} = l\,(l+1)\,\hbar^2 \, Y_{l,m}$

$d_{z^2}$: $\langle \vec{L}^2 \rangle = \iint Y_{2,0}^* \, \vec{L}^2 \, Y_{2,0} \, du \, d\varphi = 2\cdot 3 \cdot \hbar^2 \cdot \underbrace{\iint Y_{2,0}^* \, Y_{2,0} \, du \, d\varphi}_{1}$

$= 2\cdot 3 \cdot \hbar^2$ ✓

$d_{x^2-y^2}$ $\langle \vec{L}^2 \rangle = \iint \frac{1}{\sqrt{2}} \left( Y_{2,-2}^* + Y_{2,2}^* \right) \vec{L}^2 \frac{1}{\sqrt{2}} \left( Y_{2,-2} + Y_{2,2} \right) du \, d\varphi$

$= \frac{1}{2} \cdot 2 \cdot 3 \cdot \hbar^2 \left[ 1 + 0 + 0 + 1 \right] = 2 \cdot 3 \cdot \hbar^2$ ✓

$d_{zy}$ $\langle \vec{L}^2 \rangle = \iint -\frac{i}{\sqrt{2}} \left( Y_{2,-1}^* - Y_{2,1}^* \right) \frac{i}{\sqrt{2}} \left( 2 \cdot 3 \cdot \hbar^2 \right) \left( Y_{2,-1} - Y_{2,1} \right) du \, d\varphi$

$= \frac{1}{2} \cdot 2 \cdot 3 \cdot \hbar^2 \cdot (1+1) = 2 \cdot 3 \cdot \hbar^2$ ✓

usw.

$d_{z^2}$: $\langle L_z \rangle = \iint Y_{2,0}^* \underbrace{L_z Y_{2,0}}_{0} \, du \, d\varphi = 0$ ✓

$d_{x^2-y^2}$ $\langle L_z \rangle = \iint \frac{1}{\sqrt{2}} \left( Y_{2,-2}^* + Y_{2,2}^* \right) L_z \frac{1}{\sqrt{2}} \left( Y_{2,-2} + Y_{2,2} \right) du \, d\varphi$

$= \frac{1}{2} \iint \left( Y_{2,-2}^* + Y_{2,2}^* \right) \left( -2\hbar \, Y_{2,-2} + 2\hbar \, Y_{2,2} \right) du \, d\varphi$

$= \frac{1}{2} \left( -2\hbar + 2\hbar \right) = 0$ ✓

$d_{zy}$ $\langle L_z \rangle = \iint \frac{i}{\sqrt{2}} \left( Y_{2,1}^* - Y_{2,-1}^* \right) \frac{i}{\sqrt{2}} \left( -\hbar \, Y_{2,-1} - \hbar \, Y_{2,1} \right) du \, d\varphi$

$= -\frac{1}{2} \hbar \, (1-1) = 0$ ✓

usw.

c) mit: $Y_{l,m}(\Theta,\varphi) = A_{l,m}\, P_l^{|m|}(\cos\Theta)\, e^{im\varphi}$

$$\Rightarrow Y_{l,m}(\Theta,\varphi+\alpha) = A_{l,m}\, P_l^{|m|}(\cos\Theta)\, e^{im(\varphi+\alpha)} = Y_{l,m}(\Theta,\varphi)\, e^{im\alpha}$$

$d_{x^2-y^2}$: $\frac{1}{\sqrt{2}}\left(Y_{2,-2}(\Theta,\varphi) + Y_{2,2}(\Theta,\varphi)\right) \longrightarrow \frac{1}{\sqrt{2}}\left(Y_{2,-2}(\Theta,\varphi-\frac{\pi}{4}) + Y_{2,2}(\Theta,\varphi-\frac{\pi}{4})\right)$

$$= \frac{1}{\sqrt{2}}\left(Y_{2,-2}(\Theta,\varphi)\underbrace{e^{i\frac{\pi}{2}}}_{i} + Y_{2,2}(\Theta,\varphi)\underbrace{e^{-i\frac{\pi}{2}}}_{-i}\right)$$

$$= \frac{i}{\sqrt{2}}\left(Y_{2,-2} - Y_{2,2}\right) \hat{=} d_{xy} \quad \checkmark$$

d) Bezeichnung nach Winkelabhängigkeit:

$$\frac{x}{r} = \sin\Theta\cos\varphi \qquad \frac{y}{r} = \sin\Theta\sin\varphi \qquad \frac{z}{r} = \cos\Theta$$

$$\Rightarrow p_z \sim \cos\Theta \qquad p_y \sim \sin\Theta\sin\varphi \qquad p_x \sim \sin\Theta\cos\varphi$$

mit

$Y_{1,0} \sim \cos\Theta$

$Y_{1,1} \sim \sin\Theta\, e^{i\varphi}$ und $\frac{1}{2i}\left(e^{i\varphi} - e^{-i\varphi}\right) = \sin\varphi$

$Y_{1,-1} \sim \sin\Theta\, e^{-i\varphi}$ $\qquad \frac{1}{2}\left(e^{i\varphi} + e^{-i\varphi}\right) = \cos\varphi$

folgt:

$$p_z = Y_{1,0}$$

$$p_y = \frac{i}{\sqrt{2}}\left(Y_{1,1} - Y_{1,-1}\right) \sim \frac{i}{\sqrt{2}}\sin\Theta\left(e^{i\varphi} - e^{-i\varphi}\right) \sim \sin\Theta\sin\varphi$$

$$p_x = \frac{1}{\sqrt{2}}\left(Y_{1,1} + Y_{1,-1}\right) \sim \frac{1}{\sqrt{2}}\sin\Theta\left(e^{i\varphi} + e^{-i\varphi}\right) \sim \sin\Theta\cos\varphi$$

19) Resonanz: $g \mu_B B_0 = h\nu$

$$g = \frac{h\nu}{\mu_B B_0} = \frac{h}{\mu_B} \frac{9{,}2\ \mathrm{GHz}}{327{,}74\ \mathrm{mT}} = 2{,}0056$$

Resonanzbedingung für $\nu = 9{,}80$ GHz für diesen $g$-Faktor:

$$B_0\,(9{,}80\ \mathrm{GHz}) = 349{,}11\ \mathrm{mT}$$