Blatt 6

20) a) $n=2 \quad l=1 \quad s=\frac{1}{2}$

$\Longrightarrow \quad j=\frac{3}{2}=l+s$ oder $j=\frac{1}{2}=l-s$

$m_j=\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ (4-fach entartet) $\quad m_j=\frac{1}{2}, -\frac{1}{2}$ (2-fach entartet)

b) $W_{LS}=\lambda \vec{L}\vec{S}$

$$\lambda=\frac{e^2\mu_0}{8\pi m_e^2}\langle r^{-3}\rangle$$ (Faktor $\frac{1}{2}$ der Thomaskorrektur)

- Berechne $\langle r^{-3}\rangle$

$$\psi_{n,l,m}=R_{n,l}(r)\,Y_{l,m}(\theta,\varphi)$$

Integration über die normierten $Y_{l,m}(\theta,\varphi)$ ergibt 1, es bleibt

$$\langle r^{-3}\rangle=\int r^{-3}R_{n,l}^2(r)\,r^2\,dr$$

$$=\int\frac{1}{r}R_{n,l}^2(r)\,dr \underset{n=2,\,l=1,\,Z=1}{=} \frac{4}{3}\left(\frac{1}{2a_0}\right)^3\frac{1}{4a_0^2}\underbrace{\int_0^\infty\frac{1}{r}r^2 e^{-\frac{r}{a_0}}dr}_{=a_0^2 \text{ Bronstein}}$$

$$=\frac{1}{3}\cdot\frac{1}{8}\frac{1}{a_0^3}=\frac{1}{24}\left(\frac{1}{a_0}\right)^3$$

- Umschreiben von $\vec{L}\vec{S}$

$$\vec{J}=\vec{L}+\vec{S}$$

$$\vec{J}^2=\vec{L}^2+\vec{S}^2+2\vec{L}\vec{S} \quad\Longrightarrow\quad \vec{L}\vec{S}=\frac{1}{2}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)$$

$$=\frac{1}{2}\hbar^2\left(j(j+1)-l(l+1)-s(s+1)\right)$$

$$W_{LS}=\underbrace{\frac{e^2\mu_0}{8\pi m_e^2}\frac{1}{24}\left(\frac{1}{a_0}\right)^3\frac{1}{2}\hbar^2}_{\frac{\lambda\hbar^2}{2}}\left(j(j+1)-l(l+1)-s(s+1)\right)$$

$$j = \frac{3}{2} \qquad W_{\frac{3}{2}} = \frac{\lambda \hbar^2}{2}\left(\frac{15}{4} - 2 - \frac{3}{4}\right) = \frac{\lambda \hbar^2}{2}$$

$$j = \frac{1}{2} \qquad W_{\frac{1}{2}} = \frac{\lambda \hbar^2}{2}\left(\frac{3}{4} - 2 - \frac{3}{4}\right) = -2\,\frac{\lambda \hbar^2}{2}$$

$$\Delta E = W_{\frac{3}{2}} - W_{\frac{1}{2}} = \frac{3}{2}\lambda \hbar^2 = 7{,}26 \cdot 10^{-24}\, J$$

22)

$\ell = 3$

$${}^{7}F_{6}$$

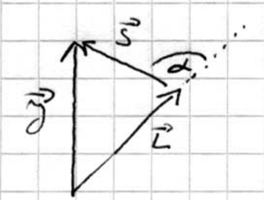
j

$2s+1$

$2s+1 = 7 \longrightarrow s = 3$

$\ell = 3$

$j = 6 \; (= \ell + s)$



$$\vec{S} \cdot \vec{L} = |\vec{S}||\vec{L}| \cdot \cos \alpha$$

$$\vec{S}\vec{L} = \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$$

$$\cos \alpha = \frac{\frac{1}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right]}{\sqrt{s(s+1)}\;\sqrt{\ell(\ell+1)}}$$

$$= \frac{1}{2}\,\frac{6 \cdot 7 - 2 \cdot 3 \cdot 4}{3 \cdot 4} = \frac{3}{4}$$

$$\Rightarrow \alpha = 41{,}4^\circ$$

21)

a)

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

$3\,p_{\frac{1}{2}}$ $\quad j = \frac{1}{2} \quad l = 1 \quad s = \frac{1}{2} \quad \Rightarrow \quad g = \frac{2}{3}$

$3\,s_{\frac{1}{2}}$ $\quad j = \frac{1}{2} \quad l = 0 \quad s = \frac{1}{2} \quad \Rightarrow \quad g = 2$ $\quad$ (reiner Spinmagnetismus)

$3\,p_{\frac{3}{2}}$ $\quad j = \frac{3}{2} \quad l = 1 \quad s = \frac{1}{2} \quad \Rightarrow \quad g = \frac{4}{3}$

b)

$3\,p_{\frac{1}{2}} \to 3\,s_{\frac{1}{2}}$: $\lambda_{p\frac{1}{2}} = 5895{,}92\ \text{Å}$ $\qquad \frac{1}{\lambda_{p\frac{1}{2}}} = 16960{,}88\ \frac{1}{cm}$

$3\,p_{\frac{3}{2}} \to 3\,s_{\frac{1}{2}}$: $\lambda_{p\frac{3}{2}} = 5889{,}95\ \text{Å}$ $\qquad \frac{1}{\lambda_{p\frac{3}{2}}} = 16978{,}07\ \frac{1}{cm}$

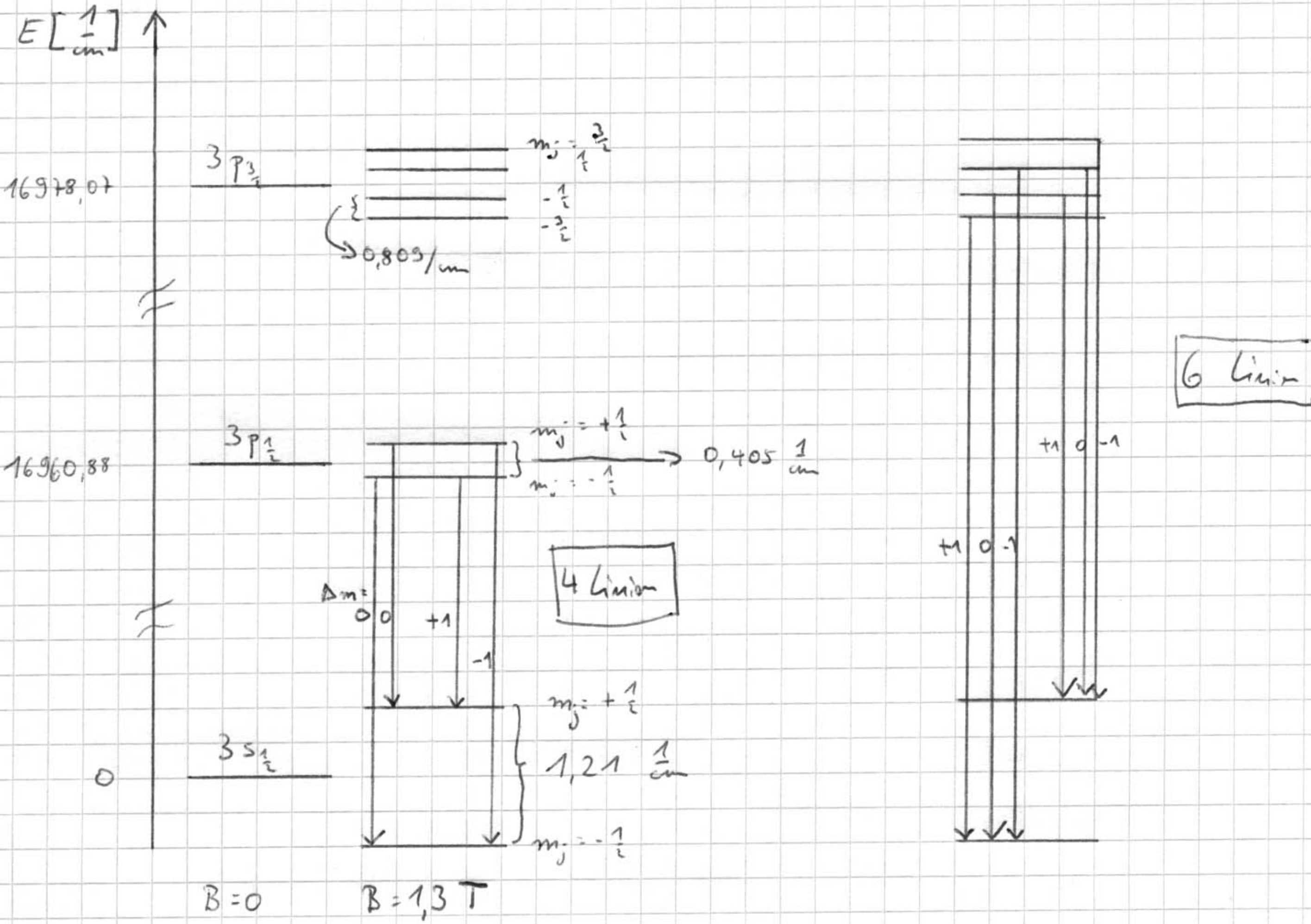
$$E_{Zeeman} = -\langle \mu_z \rangle B_z = +\, g_j\, m_j\, \mu_B\, B_z$$

$\Delta E_{Zeeman} = g_j\, \mu_B\, B_z$ $\qquad \frac{\mu_B B_z}{hc} = 0{,}607\ \frac{1}{cm}$ $\qquad (B_z = 1{,}3\,T)$

$$\Delta E_{p\frac{1}{2}} = \frac{2}{3} \cdot 0{,}607\ \frac{1}{cm} = 0{,}405\ \frac{1}{cm}$$

$$\Delta E_{p\frac{3}{2}} = \frac{4}{3}\ 0{,}607\ \frac{1}{cm} = 0{,}809\ \frac{1}{cm}$$

$$\Delta E_{s\frac{1}{2}} = 2 \cdot 0{,}607\ \frac{1}{cm} = 1{,}21\ \frac{1}{cm}$$



c) Übergänge: $3p_{\frac{1}{2}} \longrightarrow 3s_{\frac{1}{2}}$

$$\Delta E = \underbrace{E_{P\frac{1}{2}} + \frac{2}{3}\mu_B B m_j}_{\text{obere Niveau}} - \underbrace{\left(E_{S\frac{1}{2}} + 2\mu_B B m_j'\right)}_{\text{untere Niveau}} = \Delta E_{P\frac{1}{2}} \underbrace{+0{,}405 \tfrac{1}{cm} m_j - 1{,}21 \tfrac{1}{cm} m_j'}_{\Delta E^*}$$

m_j	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
m_j'	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
$\Delta E^* [\frac{1}{cm}]$	0,403	-0,808	+0,808	-0,403
Δm	0	+1	-1	0

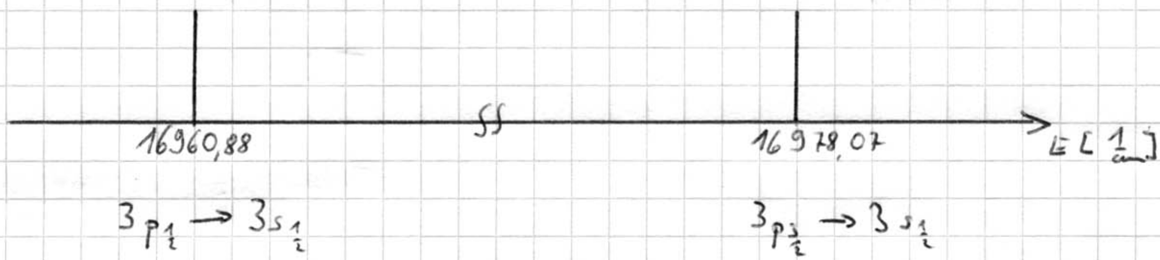
Übergänge $3p_{\frac{3}{2}} \rightarrow 3s_{\frac{1}{2}}$

$$\Delta E = E_{p\frac{3}{2}} + \frac{4}{3}\mu_B B m_j - \left(E_{s\frac{1}{2}} + 2\mu_B B m_j'\right)$$

$$= \Delta E_{p\frac{3}{2}} \underbrace{+ 0{,}809 \tfrac{1}{cm} m_j - 1{,}21 \tfrac{1}{cm} m_j'}_{\Delta E^*}$$

m_j	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
m_j'	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$\Delta E^*[\frac{1}{cm}]$	0,609	−0,201	−1,01	+1,01	+0,201	−0,609
Δm	−1	0	+1	−1	0	+1

B = 0, Aufspaltung aufgrund Spin-Bahn-Kopplung



B = 1,3 T, anomaler Zeemaneffekt $(\langle W_{LS}\rangle \gg \langle W_{Zeeman}\rangle)$

d) Paschen - Bach - Effekt

$E_{\vec{L}\vec{S}} \ll E_{\vec{S}\vec{B}_z}, E_{\vec{L}\vec{B}}$, dann ist Spin-Bahn-Kopplung <u>aufgehoben</u>, d.h. Spinmagnetismus ($g=2$) und Bahnmagnetismus ($g=1$) werden getrennt betrachtet.

Auswahlregeln: $\Delta m_l = 0, \pm 1$
$\Delta m_s = 0$
für optische Übergänge

$$E_{zu} = -\langle \mu_z \rangle B_z$$

$$E_{P.B.} = 1\, \mu_B\, B \frac{\langle l_z \rangle}{\hbar} + 2\, \mu_B\, B \frac{\langle s_z \rangle}{\hbar}$$

$$= \mu_B B \, (m_l + 2 m_s)$$

Keine Kopplung zu j mehr, d.h. $3p \rightarrow 3s$

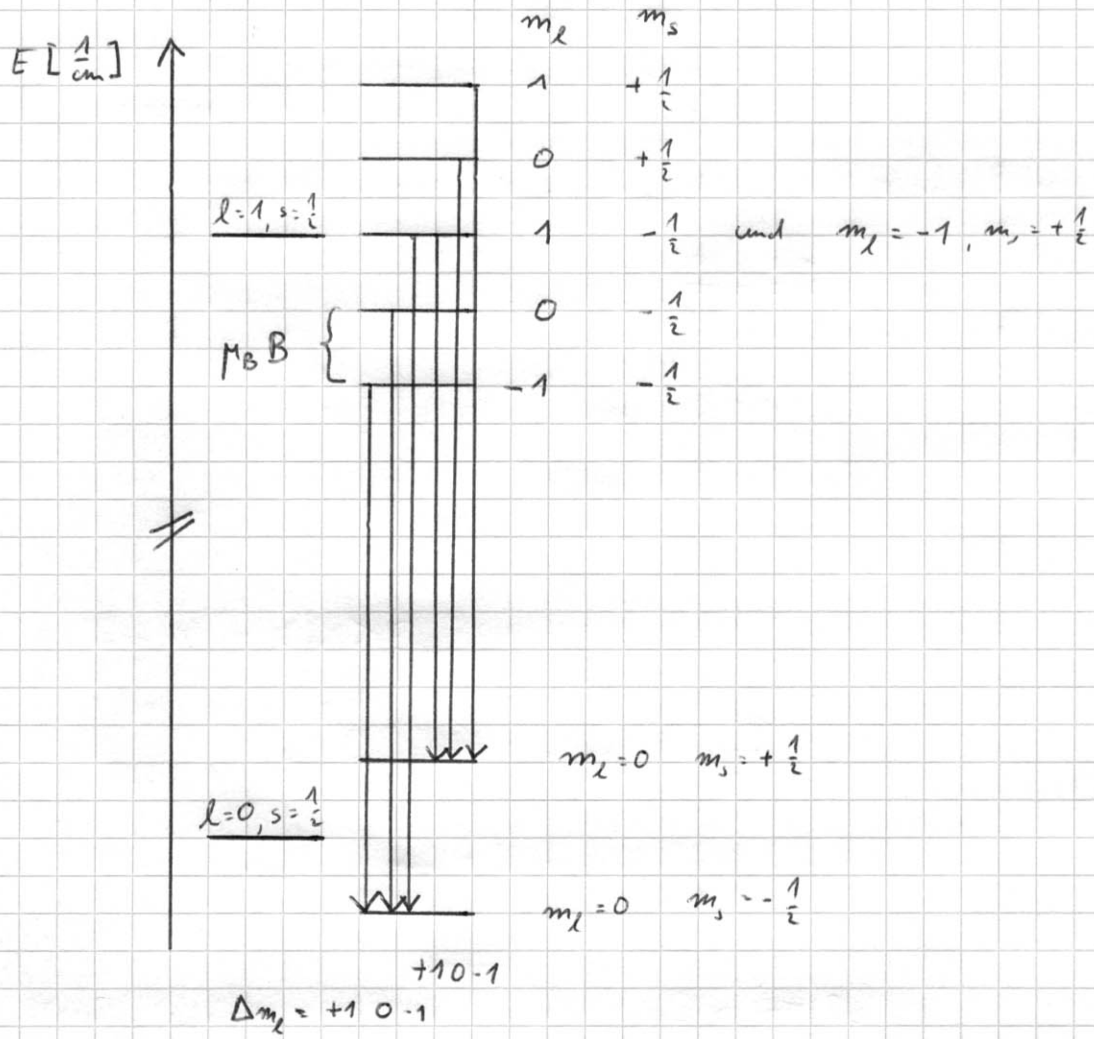
<u>$3p$</u>: $l = 1$, $s = \frac{1}{2}$

m_l	0	0	1	1	-1	-1
m_s	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$
$\frac{E_{P.B.}}{\mu_B}$	1	-1	2	0	0	-2

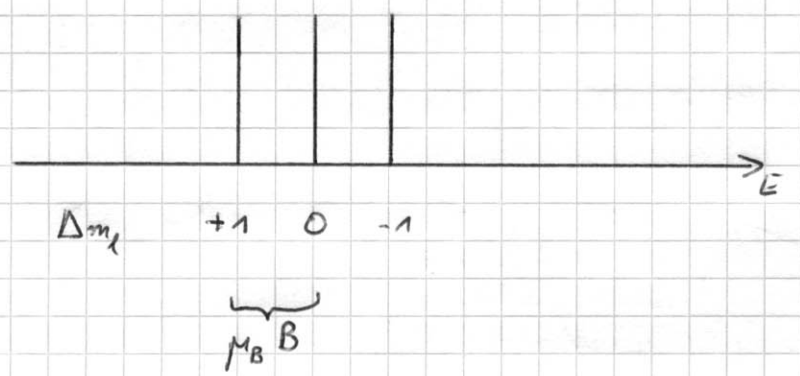
$\underbrace{0 \quad 0}_{\text{fallen zusammen (Entartung)}}$

$3s$: $l = 0$, $s = \frac{1}{2}$

m_l	0	0
m_s	$+\frac{1}{2}$	$-\frac{1}{2}$
$\frac{E_{P.B.}}{\mu_B B}$	1	-1



Paschen-Bach-Effekt: nur noch 3 Linien:



Aus $E_{\vec{L}\vec{S}} \ll E_{\vec{S}\vec{B}} \longrightarrow E_{\vec{L}\vec{S}} \ll \mu_B B$

$$E_{\vec{L}\vec{S}} = E_{P_{3/2}} - E_{P_{1/2}} = 16\,978{,}07\ \frac{1}{cm} - 16\,960{,}88\ \frac{1}{cm} = 17{,}19\ \frac{1}{cm} \mathrel{\hat{=}} 3{,}4 \cdot 10^{-22}\ J$$

$$\mu_B B \gg 3{,}4 \cdot 10^{-22}\ J$$

$$B \gg 36{,}7\ T$$

23) $\vec{F} = \vec{I} + \vec{J}$ $\qquad\qquad W_{HF} = A \cdot \vec{I} \cdot \vec{J}$

a) $F = J - I,\ J - I + 1,\ \dots,\ J + I$

mit $J = \frac{7}{2}$ und $I = \frac{3}{2}$: $F = 2, 3, 4, 5$

b) $\langle W_{Hf} \rangle = A \cdot \langle \vec{I}\,\vec{J} \rangle$ analog zu Spin-Bahn-Kopplung:

$$W_{HF} = \frac{1}{2} A \hbar^2 \left(F(F+1) - I(I+1) - J(J+1) \right)$$

Abstand benachbarter Niveaus:

$$\Delta W_{HF} = W_{HF}(F) - W_{HF}(F-1) = \frac{1}{2} A \hbar^2 \left[F(F+1) - (F-1) \cdot F \right]$$

$$= A \hbar^2 F$$

$$\frac{W_{HF}(F)}{A \hbar^2} = \frac{1}{2}\left(F(F+1) - \frac{15}{4} - \frac{63}{4} \right) = \frac{1}{2} F(F+1) - \frac{39}{4}$$

F	$\frac{W_{HF}}{A\hbar^2}$	$\nu = \frac{\Delta W_{HF}}{h} = \frac{A\hbar^2}{h} F$
2	$3 - \frac{39}{4} = -\frac{27}{4}$	
3	$6 - \frac{39}{4} = -\frac{15}{4}$	$\frac{3 \cdot 10^{-26}\,J}{6{,}626 \cdot 10^{-34}\,Js} \cdot 3 = 136\ MHz$
4	$10 - \frac{39}{4} = +\frac{1}{4}$	$181\ MHz$
5	$15 - \frac{39}{4} = +\frac{21}{4}$	$226\ MHz$